Multiplying a Binomial by a Monomial

Why?
A square patio has a side length of $x$ meters. If you increase the length by 3 meters, what is the area of the new patio?

a. Write an expression to represent the new side length of the patio.

b. Write an expression to represent the area of the new patio.

Binomials You can use algebra tiles to model binomials, a polynomial with two terms.

The algebra tiles shown form a rectangle with a width of $x$ and a length of $x + 2$. They represent the product $x(x + 2)$.

The area of the rectangle represents the product. Since the rectangle consists of one $x^2$-tile and two $x$-tiles, $x(x + 2) = x^2 + 2x$.

EXAMPLE 1 Multiplying a Binomial by a Monomial

Use algebra tiles to find $x(x + 3)$.

Step 1 Make a rectangle with a width of $x$ and a length of $x + 3$. Use algebra tiles to mark off the dimensions on a product mat.

Step 2 Using the marks as a guide, fill in the rectangle with algebra tiles.

Step 3 The area of the rectangle is $x^2 + x + x + x$. In simplest form, the area is $x^2 + 3x$. So, $x(x + 3) = x^2 + 3x$.

Guided Practice
1. Use algebra tiles to find $2x(2x + 4)$.
In Example 1, each term in the binomial is multiplied by the monomial. This and other similar examples suggest that the Distributive Property can be used to multiply a binomial by a monomial.

**Key Concept**

**Multiplying a Binomial by a Monomial**

**Words**
To multiply a binomial by a monomial, use the Distributive Property.

**Symbols**
\[ a(b + c) = ab + ac \]

**Model**

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**EXAMPLE 2**

**Multiplying a Binomial by a Monomial**

Multiply. Use models if needed.

a. \( x(x + 5) \)

**Method 1** Use a model.

So, \( x(x + 5) = x^2 + 5x \).

**Method 2** Use the Distributive Property.

\[
x(x + 5) = x(x) + x(5) \quad \text{Distributive Property}
= x^2 + 5x \quad \text{Simplify}
\]

So, \( x(x + 5) = x^2 + 5x \).

b. \( x(2x + 3) \)

\[
x(2x + 3) = x(2x) + x(3) \quad \text{Distributive Property}
= 2x^2 + 3x
\]

c. \( 3x^3(2x^2 - 5x) \)

\[
3x^3(2x^2 - 5x) = 3x^3(2x^2) - 3x^3(5x) \quad \text{Distributive Property}
= 6x^5 - 15x^4 \quad \text{Simplify}
\]

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**Guided Practice**

2A. \( 7(2x + 5) \)

2B. \( 4x(3x^2 - 7) \)

2C. \( y^3(3y^2 + 2y) \)

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EXAMPLE 3  Simplifying Expressions

Simplify $5x(x + 2) - 2x(x)$.

$5x(x + 2) - 2x(x) = 5x^2 + 5x(2) - 2x(x)$

$= 5x^2 + 10x - 2x^2$

$= 3x^2 + 10x$

Guided Practice

3A. $-6a(a + 1) + 4a(2a)$

3B. $3n^2(n - 4) - 4(n^2 - 7)$

Real-World EXAMPLE 4

SPORTS  The length of a hockey rink is 115 feet longer than its width. If the perimeter of the rink is 570 feet, what are the dimensions of the rink?

Words  Perimeter equals twice the sum of the length and width.

Variable  Let $w$ represent width. So, $w + 115$ represents the length.

Expression

$570 = 2(w + w + 115)$

$570 = 2(2w + 115)$

$570 = 4w + 230$

$340 = 4w$

$85 = w$

So, the width of the rink is 85 feet and the length is 85 + 115 or 200 feet.

Guided Practice

4. SPORTS  The perimeter of a tennis court is 228 feet. The length of the court is 6 feet more than twice the width. What are the dimensions of the court?

Check Your Understanding

Find each product. Use models if needed.

1. $2(2x + 1)$
2. $x(2x + 2)$
3. $3a(a - 1)$
4. $-3n(5 - 2n)$
5. $4z(z^2 - 2z)$
6. $3(8y^3 + 3)$
Lesson 4
Multiplying a Binomial by a Monomial

Simplify.

7. \(7x(x + 2) + 3x(x - 5)\)

8. \(4y^2 - 8 + 10 - 2(y + 12)\)

9. \(-3(2a - 4) + 9 - 3(a + 1)\)

10. \(x(x + 3) + 5x + x(x + 5) + 9\)

11. **SPORTS** One of the world's largest swimming pools is the Orthlieb Pool in Casablanca, Morocco. It is 30 meters longer than 6 times its width. If the perimeter of the pool is 1110 meters, what are the dimensions of the pool?

**Practice and Problem Solving**

Find each product. Use models if needed.

12. \(2x(x + 4)\)

13. \(2x(2x + 2)\)

14. \(4x(-2x + 7x^2)\)

15. \(-5y(6 - 2y)\)

16. \(5d(d^2 + 3)\)

17. \(0.25x(4x - 8)\)

18. \(2a(a^2 - 5a) - 6(a^3 + 3a)\)

19. \(5t^2(t + 2) - 5t(4t^2 - 3t)\)

20. \(3n^2(n - 4) + 6n(3n^2 + n - 7) - 4(n - 7)\)

21. **FOOTBALL** The perimeter of an arena football field is 188 yards. The perimeter of an NFL football field is 346 yards. Use the information in the table to find the length and width of each field.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Arena</th>
<th>NFL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>(a - 38)</td>
<td>(n + 3)</td>
</tr>
<tr>
<td>Length</td>
<td>(a)</td>
<td>(2n + 20)</td>
</tr>
</tbody>
</table>

22. **MONUMENTS** The rectangular Reflecting Pool extends from the Lincoln Memorial to the World War II memorial in Washington, D.C. The length of the pool is 25 feet more than 12 times its width.

a. If the perimeter of the pool is 4392 feet, find the dimensions of the pool.

b. Suppose the width of the pool is 83.5 times the depth. Find the volume of the pool.

23. **SPORTS** A billiards table has a perimeter of 24 feet. The length of a billiards table is twice as long as its width. If the width of the billiards table is \(4x - 1\), what are the dimensions of the table?
ALGEBRA  Solve \( w(w + 12) = w(w + 14) + 12 \).

\[
egin{align*}
w(w + 12) &= w(w + 14) + 12 \\
w(w) + w(12) &= w(w) + w(14) + 12 \\
w^2 + 12w &= w^2 + 14w + 12 \\
12w &= 14w + 12 \\
-2w &= 12 \\
w &= -6
\end{align*}
\]

Write the equation.
Distributive Property
Simplify.
Subtract \( w^2 \) from each side.
Subtract 14w from each side.
Divide each side by \(-2\).

Solve each equation.

24. \( 4(y^2 - 3y) - 8 = 2y(2y + 4) + 32 \)

25. \( n(n - 7) = n^2 + 3(-2n + 1) - 1 \)

26. \( a(a + 2) + 3a = a(a - 3) + 8 \)

27. \( c(c + 8) - c(c + 3) - 23 = 3c + 11 \)

28. \( b(b + 10) + 6 = b(b + 6) - 2 \)

29. RECREATION  On the Caribbean island of Trinidad, children play a form of hopscotch called *Jumby*. The pattern for this game is shown at the right.

a. Suppose each rectangle is \( y + 5 \) units long and \( y \) units wide. Write an expression in simplest form for the area of the pattern.

b. If \( y \) represents 10 inches, find the area of the pattern.

H.O.T. Problems  Use Higher-Order Thinking Skills

30. OPEN ENDED  Write three different multiplication problems for which the product is \( 6a^2 + 8a \).

31. CHALLENGE  The product of \( x^2 - 2x + 1 \) and a monomial is \( 4x^3 - 8x^2 + 4x \). What is the monomial? Explain your reasoning.

32. ERROR ANALYSIS  Carmen and Paul are simplifying \( 5(p^2 + 2p - 2) - 4p(p - 1) \). Is either of them correct? Explain your reasoning.

\[
\begin{align*}
\text{Carmen:} & \quad 5(p^2 + 2p - 2) - 4p(p - 1) = \\
& \quad 5p^2 + 10p - 10 - 4p^2 + 4p = \\
& \quad p^2 + 14p - 10
\end{align*}
\]

\[
\begin{align*}
\text{Paul:} & \quad 5(p^2 + 2p - 2) - 4p(p - 1) = \\
& \quad 5p^2 + 10p - 10 - 4p^2 - 4p = \\
& \quad p^2 + 6p - 10
\end{align*}
\]

33. REASONING  Explain how to multiply \(-2n\) and \(3 - 5n\).

34. WRITING IN MATH  Describe how you can use models to multiply a monomial by a polynomial.