

Factoring Trinomials

8.A.11 Factor a trinomial in the form $ax^2 + bx + c$; $a = 1$ and c having no more than three sets of factors

To factor some trinomials, you will use the pattern for multiplying two binomials. Study the following example.

F	O	I	L	
$(x + 2)(x + 3) = (x \cdot x) + (x \cdot 3) + (x \cdot 2) + (2 \cdot 3)$				Use the FOIL method.
$= x^2 + 3x + 2x + 6$				Simplify.
$= x^2 + (3 + 2)x + 6$				Distributive Property
$= x^2 + 5x + 6$				Simplify.

Observe the following pattern in this multiplication.

$$\begin{aligned}
 (x + 2)(x + 3) &= x^2 + (3 + 2)x + (2 \cdot 3) \\
 (x + m)(x + n) &= x^2 + (n + m)x + mn \\
 &= x^2 + \underbrace{(m + n)}_{bx} x + \underbrace{mn}_c
 \end{aligned}$$

$b = m + n$ and $c = mn$

Notice that the coefficient of the middle term is the sum of m and n and the last term is the product of m and n . This pattern can be used to factor quadratic trinomials of the form $x^2 + bx + c$.

Key Concept	Factoring $x^2 + bx + c$
Words	To factor quadratic trinomials of the form $x^2 + bx + c$, find two integers, m and n , whose sum is equal to b and whose product is equal to c . Then write $x^2 + bx + c$ using the pattern $(x + m)(x + n)$.
Symbols	$x^2 + bx + c = (x + m)(x + n)$ when $m + n = b$ and $mn = c$.
Example	$x^2 + 5x + 6 = (x + 2)(x + 3)$, since $2 + 3 = 5$ and $2 \cdot 3 = 6$.

EXAMPLE 1

 b and c are positive

Factor $x^2 + 7x + 12$.

In this trinomial, $b = 7$ and $c = 12$. You need to find two numbers whose sum is 7 and whose product is 12. Make an organized list of the factors of 12, and look for the pair of factors whose sum is 7.

Factors of 12	Sum of Factors
1, 12	13
2, 6	8
3, 4	7

The correct factors are 3 and 4.

Since 3 and 4 are factors of 12 and their sum is 7 we know that $m = 3$ and $n = 4$, so $x^2 + 7x + 12 = (x + 3)(x + 4)$.

EXAMPLE 2 b is negative and c is positive**Factor $x^2 - 22x + 21$.**

In this trinomial, $b = -22$ and $c = 21$. This means that $m + n$ is negative and mn is positive. So m and n must both be negative. Therefore, make a list of the negative factors of 21, and look for the pair of factors whose sum is -22 .

Factors of 21	Sum of Factors
-1, -21	-22
-3, -7	-10

The correct factors are -1 and -21 .

So, $m = -1$ and $n = -21$.

$$\begin{aligned} x^2 - 22x + 21 &= (x + (-1))(x + (-21)) \\ &= (x - 1)(x - 21) \end{aligned}$$

EXAMPLE 3 b is positive and c is negative**Factor $x^2 + x - 20$.**

In this trinomial, $b = 1$ and $c = -20$. This means that $m + n$ is positive and mn is negative. So either m or n is negative, but not both. Therefore, make a list of the factors of -20 , where one factor of each pair is negative. Look for the pair whose sum is one.

Factors of -20	Sum of Factors
1, -20	-19
-1, 20	19
2, -10	-8
-2, 10	8
4, -5	-1
-4, 5	1

The correct factors are -4 and 5 .

So, $m = -4$ and $n = 5$.

$$x^2 + x - 20 = (x - 4)(x + 5)$$

EXAMPLE 4 b and c are negative**Factor $x^2 + 18x - 40$.**

Since $b = -18$ and $c = -40$, $m + n$ is negative and mn is negative. So either m or n is negative, but not both.

Factors of -40	Sum of Factors
1, -40	-39
-1, 40	39
2, -20	-18
-2, 20	18
4, -10	-6
-4, 10	6
5, -8	-3
-5, 8	3

The correct factors are 2 and -20.

So, $m = 2$, $n = -20$, and $x^2 - 18x - 40 = (x + 2)(x - 20)$.

EXERCISES**Factor each trinomial.**

1. $x^2 + 9x + 20$

2. $x^2 - 7x + 10$

3. $x^2 + 7x - 18$

4. $x^2 - 2x - 3$

5. $x^2 + 6x + 5$

6. $x^2 - 10x + 21$

7. $x^2 + x - 12$

8. $x^2 - 13x - 14$

9. $x^2 + 8x + 15$

10. $x^2 - 4x + 4$

11. $x^2 + 9x - 22$

12. $x^2 - 6x - 7$

13. $x^2 + 26x + 25$

14. $x^2 - 12x - 13$

15. $x^2 + 7x - 8$

16. $x^2 - 2x - 35$