

Logic

5.PS.9 Understand the basic language of logic in mathematical situations (and, or, not)

A **statement** is any sentence that is true or false, but not both. The truth or falsity of a statement is called its **truth value**. Statements are often represented using a letter such as p or q .

p : The state fruit of New York is the apple. *This statement is true.*

The **negation** of a statement has the opposite meaning as well as an opposite truth value. For example, the negation of the statement above is *not p*. This is written symbolically as $\sim p$.

not p: The state fruit of New York is not the apple. *This statement is false.*

Two or more statements can be joined to form a **compound statement**. Consider the following two statements.

p : The state fruit of New York is the apple.

q : New York City is sometimes referred to as “The Big Apple”.

These two statements can be joined by the word *and*.

p and q : The state fruit of New York is the apple, *and* New York City is sometimes referred to as “The Big Apple”.

By joining p and q we have formed a conjunction. A **conjunction** is a compound statement formed by joining two or more statements with the word *and*. A conjunction is true only if both p and q are true. The way this is written symbolically is: $p \wedge q$. This is read “ p and q .”

Statements can also be joined by the word *or*. This type of statement is a **disjunction**. Consider the following statements:

p : Alex would like cake for dessert.

q : Alex would like ice cream for dessert.

p or q : Alex would like cake for dessert, *or* Alex would like ice cream for dessert.

A disjunction is true if at least one of the statements is true. In the case of p or q above, the disjunction is true if Alex would like either cake or ice cream or both for dessert. The disjunction is false only if Alex would like neither cake nor ice cream for dessert. The way this is written symbolically is: $p \vee q$. This is read “ p or q .”

TRUTH TABLES A convenient method for organizing the truth values of statements is to use a **truth table**.

Negation	
p	$\sim p$ (not p)
T	F
F	T

If p is a true statement, then $\sim p$ is a false statement.
 If p is a false statement, then $\sim p$ is a true statement.

Truth tables can also be used to determine truth values of compound statements.

A conjunction is true only when both statements are true.

Conjunction		
p	q	$p \wedge q$ (p and q)
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction		
p	q	$p \vee q$ (p or q)
T	T	T
T	F	T
F	T	T
F	F	F

A disjunction is false only when both statements are false.

EXERCISES

For Exercises 1–12, use the statements below to write a compound statement for each conjunction or disjunction. Then find its truth value.

p : An isosceles triangle has three congruent sides.

q : $5 < 7$

r : An acute angle measures greater than 0° and less than 90° .

s : $1 + 1 = 3$

1. p and q

2. p or q

3. p and r

4. r and s

5. q or r

6. q and s

7. $p \wedge s$

8. $q \wedge r$

9. $r \vee p$

10. $s \vee q$

11. $(p \wedge q) \vee s$

12. $s \vee (q \text{ and } r)$