## Contents

### CHAPTER 1

**Foldables** ........................................ 1  
**Vocabulary Builder** .............................. 2  
1-1 Patterns and Inductive Reasoning ........ 4  
1-2 Points, Lines, and Planes ................. 7  
1-3 Postulates ..................................... 9  
1-4 Conditional Statements .................... 11  
1-5 Tools of the Trade ......................... 13  
1-6 A Plan for Problem Solving ............... 16  
**Study Guide** .................................... 19  

### CHAPTER 2

**Foldables** ........................................ 23  
**Vocabulary Builder** .............................. 24  
2-1 Real Numbers and Number Lines ....... 26  
2-2 Segments and Properties of Real Numbers .... 28  
2-3 Congruent Segments ...................... 31  
2-4 The Coordinate Plane ..................... 34  
2-5 Midpoints .................................... 37  
**Study Guide** .................................... 40  

### CHAPTER 3

**Foldables** ........................................ 43  
**Vocabulary Builder** .............................. 44  
3-1 Angles ......................................... 46  
3-2 Angle Measure ................................ 48  
3-3 The Angle Addition Postulate .......... 51  
3-4 Adjacent Angles & Linear Pairs ....... 54  
3-5 Comp. and Supp. Angles ................. 56  
3-6 Congruent Angles ......................... 58  
3-7 Perpendicular Lines ....................... 60  
**Study Guide** .................................... 62  

### CHAPTER 4

**Foldables** ........................................ 65  
**Vocabulary Builder** .............................. 66  
4-1 Parallel Lines and Planes ............... 68  
4-2 Parallel Lines and Transversals ....... 70  
4-3 Transversals and Corres. Angles ....... 73  
4-4 Proving Lines Parallel .................... 76  
4-5 Slope ......................................... 78  
4-6 Equations of Lines ......................... 80  
**Study Guide** .................................... 83  

### CHAPTER 5

**Foldables** ........................................ 87  
**Vocabulary Builder** .............................. 88  
5-1 Classifying Triangles ..................... 89  
5-2 Angles of a Triangle ...................... 93  
5-3 Geometry in Motion ....................... 95  
5-4 Congruent Triangles ...................... 98  
5-5 SSS and SAS ................................ 100  
5-6 ASA and AAS ................................ 102  
**Study Guide** .................................... 104  

### CHAPTER 6

**Foldables** ........................................ 107  
**Vocabulary Builder** .............................. 108  
6-1 Medians ....................................... 110  
6-2 Altitudes and Perp. Bisectors ........ 112  
6-3 Angle Bisectors of Triangles .......... 115  
6-4 Isosceles Triangles ....................... 117  
6-5 Right Triangles ................................ 119  
6-6 The Pythagorean Theorem ............... 121  
6-7 Distance on a Coordinate Plane ....... 123  
**Study Guide** .................................... 125  

### CHAPTER 7

**Foldables** ........................................ 129  
**Vocabulary Builder** .............................. 130  
7-1 Segments, Angles, Inequalities ....... 131  
7-2 Exterior Angle Theorem ................. 134  
7-3 Inequalities Within a Triangle ....... 137  
7-4 Triangle Inequality Theorem .......... 139  
**Study Guide** .................................... 142  

### CHAPTER 8

**Foldables** ........................................ 145  
**Vocabulary Builder** .............................. 146  
8-1 Quadrilaterals ............................... 148  
8-2 Parallelograms .............................. 150  
8-3 Tests for Parallelograms ................. 152  
8-4 Rectangles, Rhombi, and Squares .... 154  
8-5 Trapezoids ................................... 156  
**Study Guide** .................................... 159
Organizing Your Foldables

Make this Foldable to help you organize and store your chapter Foldables. Begin with one sheet of 11" × 17" paper.

**STEP 1** Fold
Fold the paper in half lengthwise. Then unfold.

**STEP 2** Fold and Glue
Fold the paper in half widthwise and glue all of the edges.

**STEP 3** Glue and Label
Glue the left, right, and bottom edges of the Foldable to the inside back cover of your Noteables notebook.

Reading and Taking Notes As you read and study each chapter, record notes in your chapter Foldable. Then store your chapter Foldables inside this Foldable organizer.
This note-taking guide is designed to help you succeed in *Geometry: Concepts and Applications*. Each chapter includes:

### Surface Area and Volume

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

**Begin with a plain piece of 11” × 11” paper.**

**STEP 1:** Fold the paper in thirds lengthwise.

**STEP 2:** Open and fold a 2” tab along the short side. Then fold the rest in fifths.

**STEP 3:** Draw lines along folds and label as shown.

**A Note-Taking Tip:** When taking notes, explain each new idea or concept in words and give one or more examples.

The **Chapter Opener** contains instructions and illustrations on how to make a Foldable that will help you to organize your notes.

**The Build Your Vocabulary table** allows you to write definitions and examples of important vocabulary terms together in one convenient place.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>axis</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>composite solid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cone</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cube</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cylinder (circular)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>edge</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>face</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>lateral area (LAFWALL)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>lateral edge</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>lateral face</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>net</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>oblique cone (non-PLR)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>oblique cylinder</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>oblique prism</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
12–5 Volumes of Pyramids and Cones

SOL: G.13 The student will use formulas for surface area and volume of three-dimensional figures to solve practical problems. Calculators will be used to find decimal approximations for results.

What You'll Learn
• Find the volumes of pyramids and cones.

Organize It

Use the boxes for notes in your Foldable.

Examples parallel the examples in your textbook.

Foldables feature reminds you to take notes in your Foldable.

Lessons cover the content of the lessons in your textbook. As your teacher discusses each example, follow along and complete the fill-in boxes. Take notes as appropriate.

Each lesson is correlated to the Virginia SOLs.
**NOTE-TAKING TIPS**

Your notes are a reminder of what you learned in class. Taking good notes can help you succeed in mathematics. The following tips will help you take better classroom notes.

- Before class, ask what your teacher will be discussing in class. Review mentally what you already know about the concept.
- Be an active listener. Focus on what your teacher is saying. Listen for important concepts. Pay attention to words, examples, and/or diagrams your teacher emphasizes.
- Write your notes as clear and concise as possible. The following symbols and abbreviations may be helpful in your note-taking.

<table>
<thead>
<tr>
<th>Word or Phrase</th>
<th>Symbol or Abbreviation</th>
<th>Word or Phrase</th>
<th>Symbol or Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>for example</td>
<td>e.g.</td>
<td>not equal</td>
<td>≠</td>
</tr>
<tr>
<td>such as</td>
<td>i.e.</td>
<td>approximately</td>
<td>≈</td>
</tr>
<tr>
<td>with</td>
<td>w/</td>
<td>therefore</td>
<td>’.</td>
</tr>
<tr>
<td>without</td>
<td>w/o</td>
<td>versus</td>
<td>vs</td>
</tr>
<tr>
<td>and</td>
<td>+</td>
<td>angle</td>
<td>∠</td>
</tr>
</tbody>
</table>

- Use a symbol such as a star (★) or an asterisk (*) to emphasis important concepts. Place a question mark (?) next to anything that you do not understand.
- Ask questions and participate in class discussion.
- Draw and label pictures or diagrams to help clarify a concept.
- When working out an example, write what you are doing to solve the problem next to each step. Be sure to use your own words.
- Review your notes as soon as possible after class. During this time, organize and summarize new concepts and clarify misunderstandings.

**Note-Taking Don’ts**

- Don’t write every word. Concentrate on the main ideas and concepts.
- Don’t use someone else’s notes as they may not make sense.
- Don’t doodle. It distracts you from listening actively.
- Don’t lose focus or you will become lost in your note-taking.
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with a sheet of 8\(\frac{1}{2}\)" \(\times\) 11" paper.

**STEP 1**
Fold
Fold lengthwise in fourths.

**STEP 2**
Draw
Draw lines along the folds and label each column *sequences*, *patterns*, *conjectures*, and *conclusions*.

**NOTE-TAKING TIP:** When you are taking notes, be sure to be an active listener by focusing on what your teacher is saying.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 1. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>collinear [co-LIN-ee-ur]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>compass</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>conclusion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>conditional statement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>conjecture [con-JEK-shoor]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>construction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>contrapositive [con-tra-PAS-i-tiv]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>converse</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>coplanar [co-PLAY-nur]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>counterexample</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>endpoint</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>formula</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>-------------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>hypothesis</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[hi-PA-the-sis]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>if-then statement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>inductive reasoning</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[in-DUK-tiv]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>inverse</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[in-VURS]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>line</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>line segment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>midpoint</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>noncollinear</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>noncoplanar</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>plane</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>point</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>postulate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[PAS-chew-let]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ray</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Find the next three terms of the sequence 11.2, 9.2, 7.2, . . .

Study the pattern in the sequence.

Each term is less than the term before it. Assume this pattern continues.

The next three terms are .

Your Turn: Find the next three terms of each sequence.

a. 3.7, 5.7, 7.7, . . .

b. 1, 3, 9, . . .
2 Find the next three terms of the sequence 101, 102, 105, 110, 117, \ldots.

\[
\begin{array}{cccccc}
101 & 102 & 105 & 110 & 117 \\
\text{box} & \text{box} & \text{box} & \text{box} & \text{box}
\end{array}
\]

Notice the pattern. To find the next three terms in the sequence, add , , and .

\[
\begin{array}{cccccccc}
101 & 102 & 105 & 110 & 117 & 126 & 137 & 150 \\
\text{box} & \text{box} & \text{box} & \text{box} & \text{box} & \text{box} & \text{box} & \text{box}
\end{array}
\]

The next three terms are .

Your Turn Find the next four terms in the sequence 51, 53, 57, 63, 71, 81, 93, \ldots.

\[
\begin{array}{cccccccc}
\text{box} & \text{box} & \text{box} & \text{box} & \text{box} & \text{box} & \text{box} & \text{box}
\end{array}
\]

3 Draw the next figure in the pattern. There are two patterns to study.

- The first pattern is size of the squares. The next square should be the area of the previous square.

- The second pattern is shaded or unshaded. The next square should be .
Your Turn

Draw the next figure in the pattern.

\[ \bigcirc \bigcirc \square \ldots \]

**BUILD YOUR VOCABULARY** (page 2)

A **conjecture** is a statement based on inductive reasoning.

An example that shows that a conjecture is not true is a **counterexample**.

**EXAMPLE**

Minowa studied the data below and made the following conjecture. Find a counterexample for her conjecture.

**Multiplying a number by \(-1\) produces a product that is less than \(-1\).**

<table>
<thead>
<tr>
<th>Number (\times)((-1))</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>5((-1))</td>
<td>(-5)</td>
</tr>
<tr>
<td>15((-1))</td>
<td>(-15)</td>
</tr>
<tr>
<td>100((-1))</td>
<td>(-100)</td>
</tr>
<tr>
<td>300((-1))</td>
<td>(-300)</td>
</tr>
</tbody>
</table>

The product of \(-2\) and \(-1\) is 2 but 2 is not less than \(-1\). So, the conjecture is **false**.

**Your Turn** Find a counterexample for this statement: 
**Division of a positive number by another positive number produces a quotient less than the dividend.**
Name two points on the line.

Two points are point \( A \) and point \( B \).

Give three names for the line.

Any two points on the line or the script letter can be used to name it. Three names are \( \text{L} \), \( \text{K} \), and \( \text{m} \).

Your Turn

Refer to the figure shown.

a. Name two points on the line.
   
   b. Give three names for the line.
3 Name three points that are collinear and three points that are noncollinear.
Points \( M, P, \) and \( Q \), are collinear. Points \( N, P, \) and \( Q \) are noncollinear.

4 Name three segments and one ray.
Three of the segments are \( \overline{DE}, \overline{EF}, \) and \( \overline{FG} \). One ray is ray \( \overrightarrow{BD} \).

Your Turn  Refer to the figure.

a. Name three collinear points and three noncollinear points.

b. Name three segments and one ray.

BUILD YOUR VOCABULARY  (pages 2–3)

A plane is a surface that extends without end in all directions.
Points that lie on the same plane are coplanar.
Points that do not lie on the same plane are noncoplanar.
Postulates are statements in geometry that are accepted as true.

Postulate 1–1 Two points determine a unique line.
Postulate 1–2 If two distinct lines intersect, then their intersection is a point.
Postulate 1–3 Three noncollinear points determine a unique plane.

In the figure, points \( K \), \( L \), and \( M \) are noncollinear.

1. Name all of the different lines that can be drawn through these points.
   There is only one line through each pair of points. Therefore, the lines that contain points \( K \), \( L \), and \( M \), taken two at a time, are \( \overline{KL} \), \( \overline{KM} \), and \( \overline{LM} \).

2. Name the intersection of \( \overline{KL} \) and \( \overline{KM} \).
   The intersection of \( \overline{KL} \) and \( \overline{KM} \) is \( \overline{K} \).

Your Turn Refer to the figure.

a. Name three different lines.

b. Name the intersection of \( \overline{AC} \) and \( \overline{BH} \).
**EXAMPLE 3**

Name all of the planes that are represented in the prism.

![Prism diagram]

There are eight points, $A$, $B$, $C$, $D$, $E$, $F$, $G$, and $H$.

There is only one plane that contains three noncollinear points. The different planes are planes $\text{ }$.

**Your Turn** Name four different planes in the figure.

**Postulate 1-4** If two distinct planes intersect, then their intersection is a line.

**EXAMPLE 4**

Name the intersection of plane $ABC$ and plane $DEF$.

The intersection is $\text{ }$.

**Your Turn** Name the intersection of plane $ABD$ and plane $DJK$. 

**HOMEWORK ASSIGNMENT**

Page(s): 
Exercises: 

Geometry: Concepts and Applications
Conditional Statements and Their Converse

**WHAT YOU’LL LEARN**

- Write statements in if-then form and write the converses of the statements.

**BUILD YOUR VOCABULARY** (pages 2–3)

- **If-then statements** join two statements based on a condition.
- If-then statements are also known as **conditional statements**.
- In a conditional statement the part following *if* is the **hypothesis**. The part following then is the **conclusion**.

**EXAMPLES**

1. **Identify the hypothesis and conclusion in this statement.**
   
   *If it is raining, then we will read a book.*
   
   Hypothesis: ________
   
   Conclusion: ________

2. **Write two other forms of this statement.**
   
   *If two lines are parallel, then they never intersect.*
   
   All ________ never intersect.
   
   Lines never ________ if they are ________.

**Your Turn**

a. **Identify the hypothesis and conclusion in this statement.**
   
   *If you ski, then you like snow.*
   
   ________
   
   ________

b. **Write two other forms of this statement.**
   
   *If a figure is a rectangle, then it has four angles.*
   
   ________
   
   ________

**REVIEW IT**

How can you show that a conjecture is false? (Lesson 1-1)

_______

_______

_______

Geometry: Concepts and Applications
**Build Your Vocabulary** (page 2)

The converse of a conditional statement is formed by exchanging the ______.

**Example**

3. Write the converse of this statement.

*If today is Saturday, then there is no school.*

If there is ______, then ______.

**Your Turn** Write the converse of this statement.

*If it is \(-30^\circ \text{F}\), then it is cold.*

**Example**

4. Write the statement in if-then form. Then write the converse of the statement.

*Every member of the jazz band must attend the rehearsal on Saturday.*

If-then form: If a ______ is a member of the jazz band, then he or she must attend ______.

Converse: If a student ______ on Saturday, then he or she is a ______ member.

**Your Turn** Write the statement in if-then form. Then write the converse of the statement. *People who live in glass houses should not throw stones.*
**Tools of the Trade**

**WHAT YOU’LL LEARN**

- Use geometry tools.

**WHAT YOU’LL LEARN**

- Use geometry tools.

**BUILD YOUR VOCABULARY** (pages 2–3)

A **straightedge** is an object used to draw a blank line.

A **compass** is commonly used for drawing arcs and blank.

In geometry, figures drawn using only a blank and a blank are constructions.

The **midpoint** is the blank in the blank of a line segment.

**EXAMPLE**

1. Find two lines or segments in a classroom that appear to be parallel. Use a ruler to determine whether they are parallel.

The opposite sides of a textbook represent two segments that appear to be parallel.

- Choose two points on one side of the textbook.
- Place the 0 mark of the ruler on each point. Make sure the ruler is perpendicular to the side at each chosen point.
- Measure the distance to the second side. If the distances are blank, then the sides are blank.

**Your Turn** Find another pair of lines or segments in a classroom that appear to be parallel. Use a ruler or a yardstick to determine if they are parallel.
1. On the figure shown, mark a point $C$ on line $\ell$ that you judge will create $BC$ that is the same length as $AB$. Then measure to determine how accurate your guess was.

To draw an exact recreation of the length, place the point of a compass on point $B$. Place the point of the pencil on point __. Then draw a small arc on line $\ell$ without changing the setting of the compass. This duplicates the measure of __.

2. Use a compass and a straightedge to construct a six-pointed star.

Use the compass to draw a circle. Then using the same compass setting, put the compass point on the circle and draw a small arc on the circle.

Move the compass point to the arc and, without changing the compass setting, draw another arc along the circle. Continue until there are six arcs.

Draw two triangles by connecting alternating marks, resulting in a six-pointed star.

Remember It

An arc is part of a circle.
Your Turn

a. On the figure given, mark point $Z$ on line $m$ that you judge will create $WZ$ that is the same length as $XY$. Then measure to determine the accuracy of your guess.

b. Use a compass and a straightedge to construct a triangle with sides of equal length.
A Plan for Problem Solving

**What You’ll Learn**
- Use a four-step plan to solve problems that involve the perimeters and areas of rectangles and parallelograms.

**Build Your Vocabulary** (page 2)

A formula is an **that shows how certain quantities are related.**

**Key Concepts**

**Perimeter of a Rectangle**
The perimeter $P$ of a rectangle is the sum of the measures of its sides. It can also be expressed as two times the length $l$ plus two times the width $w$.

**Area of a Rectangle**
The area $A$ of a rectangle is the product of the length $l$ and the width $w$.

**Examples**

1. **a. Find the perimeter of a rectangle with length 12 centimeters and width 3 centimeters.**
   
   $P = 2l + 2w$
   
   $P = 2\square + 2\square$
   
   $P = \square + \square$ or $\square \text{ centimeters}$

   **b. Find the perimeter of a square with side 10 feet long.**
   
   $P = 2l + 2w$
   
   $P = 2(10) + 2(10)$
   
   $P = \square + \square$ or $\square \text{ feet}$

2. **a. Find the area of a rectangle with length 12 kilometers and width 3 kilometers.**
   
   $A = lw$
   
   $A = \square \square$ square kilometers

   **b. Find the area of a square with sides 10 yards long.**
   
   $A = lw$
   
   $A = \square \square$ square yards
What is the difference between perimeter and area?

a. Find the perimeter of a rectangle with length 11 meters and width 4 meters.

b. Find the perimeter of a square with sides 7 centimeters long.

c. Find the area of a rectangle with length 14 inches and width 4 inches.

d. Find the area of a square with sides 11 feet long.

Find the area of a parallelogram with a height of 4 meters and a base of 5.5 meters.

\[ A = bh \]

\[ A = \square \text{ square meters} \]

Your Turn Find the area of a parallelogram with a height of 6.4 inches and a base length of 10 inches.
A door is 3-feet wide and 6.5-feet tall. Chad wants to paint the front and back of the door. A one-pint can of paint will cover about 15 ft². Will two one-pint cans of paint be enough?

**EXPLORE** You know the dimensions of the door and that one-pint can of paint covers about \(15\) ft\(^2\).

**PLAN** Use the formula for the area of a to find the total area of the two sides of the door to be covered with paint.

**SOLVE** Area of both sides of the door

\[ A = 2l \times h \]

\[ A = 2(3)(6.5) = 39 \text{ ft}^2 \]

One pint covers 15 ft\(^2\). Two one-pint cans cover 2(15) or \(30\) ft\(^2\). So, two one-pint cans will be enough.

**EXAMINE** Since the area of one side of the door is \((3)(6.5)\) or 19.5 ft\(^2\) the answer is reasonable.

Chad will need \(2\) one-pint cans of paint.

---

**Your Turn** A building contractor needs to build a rectangular deck with an area of 484 ft\(^2\). The side lengths must be whole numbers. The perimeter must be less than 260 ft. What are the possible dimensions for the deck?
1. Find the next three terms in the sequence.

1. 1, 1, 2, 3, 5, . . .

2. –1, 2, –4, 8, –16, . . .

3. Draw the next figure in the pattern.

4. Use the figure to match the example to the correct term.

4. collinear points

5. segment

6. plane

7. ray

a. GFC

b. PB

c. AD

d. PE

e. GBE
Complete the sentence.

8. A(n) **postulate** is a statement in geometry that is accepted as true without proof.

Identify three planes in the figure shown.

9. □
10. □
11. □

12. Refer to the above figure. Where do planes \( ACF \) and \( DEF \) intersect? □
   - a. point \( F \)
   - b. \( \overline{DF} \)
   - c. plane \( DEF \)
   - d. point \( D \)

Underline the correct term that completes each sentence.

13. The “if” part of the **if-then statement** is the hypothesis/conclusion.

14. The “then” part of the **if-then statement** is the hypothesis/conclusion.

15. Rewrite the statement in **if-then form**.
   
   *Students who complete all assignments score higher on tests.*

16. Write the converse of the statement.
   
   *If it is Saturday, then there is no school.*
Match the geometry tool to its function.

17. compass
   a. to plot points
18. straightedge
   b. to draw arcs and circles
19. protractor
   c. to measure angles
20. patty paper
   d. to draw lines in constructions
   e. to find the midpoint in constructions

21. Indicate whether the statement is true or false.

   A conjecture is a special drawing that is created using only a straightedge and compass.

Complete each sentence.

22. The ________ is the distance around the edges of a figure.

23. The formula for the area of a rectangle is ________.

24. ________ is the formula to find the area of a parallelogram.

25. Find the area of a rectangle with length 8 feet and width 9 feet.

26. A framer must frame a piece of art. The frame is $1\frac{1}{2}$ inches wide, and its outer edge measures 24 inches by 36 inches. What is the area of the piece of art displayed in the center of the frame?
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
  • You are probably ready for the Chapter Test.
  • You may want to take the Chapter 1 Practice Test on page 45 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
  • You should complete the Chapter 1 Study Guide and Review on pages 42–44 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 1 Practice Test on page 45 of your textbook.

☐ I asked for help from someone else to complete the review of all or most lessons.
  • You should review the examples and concepts in your Study Notebook and Chapter 1 Foldable.
  • Then complete the Chapter 1 Study Guide and Review on pages 42–44 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 1 Practice Test on page 45 of your textbook.

Student Signature       Parent/Guardian Signature

Teacher Signature
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

**Begin with a sheet of notebook paper.**

**STEP 1**
Fold
Fold lengthwise to the holes.

**STEP 2**
Cut
Cut along the top line and then cut 10 tabs

**STEP 3**
Label
Label each tab with a highlighted term from the chapter. Store the Foldable in a 3-ring binder.

**NOTE-TAKING TIP:** When taking notes, it is helpful to record the main ideas as you listen to your teacher, or read through a lesson.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 2. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>absolute value</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>betweenness</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>bisect</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>congruent segments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>coordinate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>coordinate plane</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>coordinates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>greatest possible error</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>measure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>measurements</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>-----------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>midpoint</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ordered pair</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>origin</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>percent of error</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>precision</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>quadrants</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>theorem</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>unit of measure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vector</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x-axis</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x-coordinate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y-axis</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y-coordinate</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. a rational number between 6 and 8 with a 2-digit repeating pattern

   Sample answer: 7.3232323232... 

2. an irrational number greater than 5

   Sample answer: 5.4344334443... 

**Your Turn**

For each situation, write a real number with ten digits to the right of the decimal point.

a. a rational number between −4 and −1 with a 3-digit repeating pattern

b. an irrational number less than −7 

**Postulate 2-2 Distance Postulate**

For any two points on a line and a given unit of measure, there is a unique positive real number called the measure of the distance between the points.

**Postulate 2-3 Ruler Postulate**

The points on a line can be paired with the real numbers so that the measure of the distance between corresponding points is the positive difference of the numbers.
Use the number line below to find $CE$.

The coordinate of $C$ is ____, and the coordinate of $E$ is ____.

$$CE = \left| -1 - \frac{1}{3} \right| = \left| -1 \frac{1}{3} \right|$$

$$= -1 \frac{1}{3} \text{ or } \frac{2}{3}$$

Erin traveled on I-85 from Durham, North Carolina, to Charlotte. The Durham entrance to I-85 that she used is at the 173-mile marker, and the Charlotte exit she used is at the 39-mile marker. How far did Erin travel on I-85?

$$|173 - 39| = |134| = 134$$

She traveled 134 miles on I-85.

Your Turn

a. Refer to Example 3. Find $AE$.

b. Rahmi's drive starts at the 263-mile marker of I-35 and finishes at the 287-mile marker. How far did Rahmi drive on I-35?
Segments and Properties of Real Numbers

**WHAT YOU’LL LEARN**

- Apply properties of real numbers to the measure of segments.

**Foldables**

**ORGANIZE IT**

On the third tab of your Foldable write Measure and on the fourth tab write Unit of Measure. Under each tab, explain the differences between the terms and give examples of each.

---

**Build Your Vocabulary** (page 24)

Point $R$ is **between** points $P$ and $Q$ if and only if $R$, $P$ and $Q$ are and $PR + RQ = PQ$. 

---

**Example 1**

Points $K$, $L$, and $J$ are collinear. If $KL = 31$, $JL = 16$, and $JK = 47$, determine which point is between the other two.

Check to see which two measures add to equal the third.

$KL + JL = JK$

Therefore, is between and .

**Your Turn** Points $A$, $B$, and $C$ are collinear. If $AB = 54$, $BC = 33$, and $AC = 21$, determine which point is between the other two.

---

**Example 2**

If $FG = 12$ and $FJ = 47$, find $GJ$.

$FG + GJ = FJ$

Definition of betweenness

$12 + GJ = 47$

Substitution Property

$GJ = \underline{\text{?}}$

---

© Glencoe/McGraw-Hill
If \(BE = 17\) and \(AE = 25\), find \(AB\).

Use a ruler to draw a segment 8 centimeters long. Then find the length of the segment in inches.

Use a metric ruler to draw the segment. Mark a point and call it \(X\). Then put the 0 point at point \(X\) and draw a line segment extending to the 8 centimeter mark. Mark the endpoint \(Y\).

The length of \(XY\) is \(\) centimeters.
Use a customary ruler to measure $\overline{XY}$ in inches. Put the 0 point at $X$ and measure the distance to $Y$.

The length of $\overline{XY}$ is about ___ inches.

**Your Turn** Use a ruler to draw a segment 3 centimeters long. Then find the length of the segment in inches.
Use the figure below to determine whether each statement is true or false. Explain your reasoning.

a. $\overline{DE} \cong \overline{GH}$

Because $DE = 4$ and $GH = \overline{\text{ segment }}$, $\overline{\text{ segment }} = \overline{\text{ segment }}$.

So, $\overline{\text{ segment }} \cong \overline{\text{ segment }}$ is a true statement.

b. $\overline{EF} \cong \overline{FG}$

Because $EF = \overline{\text{ segment }}$ and $FG = \overline{\text{ segment }}$, $EF \neq FG$. So, $\overline{EF}$ is not congruent to $\overline{FG}$, and the statement is false.

Your Turn

Use the figure below to determine whether each statement is true or false. Explain your reasoning.

a. $\overline{AE} \cong \overline{BG}$

b. $\overline{DG} \cong \overline{FJ}$

Theorems are statements that can be justified by using reasoning.
Determine whether the statement is true or false. Explain your reasoning.

\[ \overline{CD} \text{ is congruent to } \overline{CD}. \]

Congruence of segments is \(\) , so \(\) .

Therefore, the statement is \(\) .

**Your Turn** Determine whether the statement is true or false. Explain your reasoning.

\[ \overline{MN} \text{ is congruent to } \overline{NM}. \]

**Build Your Vocabulary** (pages 24–25)

A unique point on every segment that separates the segment into \(\) segments of \(\) length is known as the \textit{midpoint}.

To \textit{bisect} something means to separate it into two \(\) parts.
In the figure, $K$ is the midpoint of $JL$. Find the value of $d$.

![Diagram]

You need to find the value of $d$. Since $K$ is the midpoint of $JL$, $JK = KL$. Write and solve an equation involving $d$, and solve for $d$.

\[
JK = KL \\
\frac{d + 5}{2d} = \frac{5}{3}
\]

Substitution

Subtraction Property of Equality

\[
d = \frac{10}{3}
\]

Your Turn In the figure, $D$ is the midpoint of $XY$. Find the value of $a$.

![Diagram]
The Coordinate Plane

**WHAT YOU’LL LEARN**
- Name and graph ordered pairs on a coordinate plane.

**BUILD YOUR VOCABULARY** (pages 24–25)

The ____ of the grid used to locate points is known as the **coordinate plane**.

The ____ number line is the **y-axis**.

The **x-axis** is the ____ number line.

The two axes separate the coordinate plane into ____ regions known as **quadrants**.

The two axes ____ at a ____ called the origin.

An ordered pair of real numbers, called the **coordinates** of a point, locates a ____ on the coordinate plane.

The ____ number of the ordered pair is called the **x-coordinate**.

The **y-coordinate** is the ____ number of the ordered pair.

**ORGANIZE IT**

On the seventh tab of your Foldable, write **Coordinate Plane**. Under the tab, draw a coordinate plane, labeling the four quadrants and the two axes.

On the eighth tab of your Foldable, write **Ordered Pair and Coordinates**. Under the tab, give an example of an ordered pair. Label the x-coordinate and the y-coordinate for the pair.

**EXAMPLES**

1. **Graph point K at (−4, 1).**

   Start at the origin. Move ____ units to the left. Then, move ____ unit up.

   Label this point K.
**Postulate 2-4**
Completeness Property for Points in the Plane
Each point in a coordinate plane corresponds to exactly one ordered pair of real numbers. Each ordered pair of real numbers corresponds to exactly one point in a coordinate plane.

**Your Turn**
Graph point \( L \) at \((1, -4)\).

**2** Name the coordinates of points \( L \) and \( M \).

Point \( L \) is \( \square \) units to the right of the origin and \( \square \) unit below the origin. Its coordinates are \( \square \).

Point \( M \) is \( \square \) to the left of the origin and \( \square \) units above the origin. Its coordinates are \( \square \).

**WRITE IT**
Explain how to graph any ordered pair \((x, y)\). Describe which direction you move when \( x \) or \( y \) are either positive or negative.

---

**Your Turn**
Name the coordinates of points \( P \) and \( Q \).
Theorem 2-4
If $a$ and $b$ are real numbers, a vertical line contains all points $(x, y)$ such that $x = a$, and a horizontal line contains all points $(x, y)$ such that $y = b$.

**Example**

1. Graph $y = -2$.

The graph of $y = -2$ is a ________ line that intersects the $y$-axis at ________.

**Your Turn**

Graph $x = -1$.

**Homework Assignment**

Page(s): __________
Exercises: __________
2–5 Midpoints

SOL G.2.a The student will use . . . coordinate methods to solve problems involving symmetry and transformation. This will include investigating and using formulas for finding distance, midpoint, and slope.

Theorem 2-5  Midpoint Formula for a Number Line
On a number line, the coordinate of the midpoint of a segment whose endpoints have coordinates \( a \) and \( b \) is
\[
\frac{a + b}{2}
\]

Theorem 2-6  Midpoint Formula for a Coordinate Plane
On a coordinate plane, the coordinates of the midpoint of a segment whose endpoints have coordinates \((x_1, y_1)\) and \((x_2, y_2)\) are
\[
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).
\]

Example

1. Find the coordinate of the midpoint of \( \overline{AB} \).

Use the Midpoint Formula to find the coordinate of the midpoint of \( AB \).
\[
\frac{a + b}{2} = \frac{-4 + 1}{2} = \frac{-3}{2} = -1.5 \text{ or } -\frac{3}{2}
\]

The coordinate of the midpoint is \( \boxed{-1.5} \).

Your Turn

Find the coordinate of the midpoint of \( \overline{OK} \).
Find the coordinates of \( D \), the midpoint of \( \overline{CE} \), given endpoints \( C(2, 1) \) and \( E(16, 8) \).

Use the Midpoint Formula to find the coordinates of \( D \).

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{2 + 16}{2}, \frac{1 + 8}{2} \right)
\]

\[
= \left( \frac{18}{2}, \frac{9}{2} \right)
\]

The coordinates of \( D \) are \( \left( \frac{18}{2}, \frac{9}{2} \right) \).

**Your Turn** Find the coordinates of \( Y \), the midpoint of \( \overline{XZ} \), given endpoints \( X(-3, 5) \) and \( Z(6, -1) \).

Suppose \( L(2, -5) \) is the midpoint of \( \overline{KM} \) and the coordinates of \( K \) are \((-4, -3)\). Find the coordinates of \( M \).

Let \((x_1, y_1)\) or \((-4, -3)\) be the coordinates of \( K \) and let \((x_2, y_2)\) be the coordinates of \( M \). So, \(x_1 = \) and \(y_1 = \). Use the Midpoint Formula.

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{2}{2}, \frac{-5}{2} \right)
\]
Remember It

The x-coordinate of the midpoint is the average of the x-coordinates of the endpoints. The y-coordinate of the midpoint is the average of the y-coordinates of the endpoints.

\[
\frac{-4 + x_2}{2} = \phantom{2} \quad \text{Replace } x_1 \text{ with } -4.
\]

\[
\frac{-4 + x_2}{2} = 2 \quad \text{Multiply each side by } 2.
\]

\[
\frac{-4 + x_2}{2} = \phantom{2} \quad \text{Add to isolate the variable.}
\]

\[
x_2 = \phantom{2}
\]

\[
\frac{-3 + y_2}{2} = \phantom{2} \quad \text{Replace } x_1 \text{ with } -3.
\]

\[
\frac{-3 + y_2}{2} = -5 \quad \text{Multiply each side by } 2.
\]

\[
\frac{-3 + y_2}{2} = \phantom{2} \quad \text{Add to isolate the variable.}
\]

\[
y_2 = \phantom{2}
\]

The coordinates of \( M \) are \((\phantom{2}, \phantom{2})\).

Your Turn

Suppose \( S\left(\frac{1}{2}, -\frac{3}{2}\right) \) is the midpoint of \( RT \) and the coordinates of \( R \) are \((-2, -5)\). Find the coordinates of \( T \).
Choose the term that best completes the statement.

1. The set of non-negative integers is also called the set of [natural/whole] numbers.

2. The quotient of two integers, where the denominator is not zero, is a(n) [rational/irrational] number.

3. Decimals that do not repeat or terminate are called [rational/irrational] numbers.

Find.

4. \(|-4 - 1|\)  
5. \(|-(12)|\)  
6. \(|11 + 2|\)

2-2

Segments and Properties of Real Numbers

7. Points \(X, Y,\) and \(Z\) are collinear. If \(XY = 10\) and \(XZ = 3,\) find \(YZ.\)

8. Points \(A, B,\) and \(C\) are collinear. If \(AB = 6, BC = 8,\) and \(AC = 14,\) which point is between the other two points?

9. Points \(M, N,\) and \(P\) are collinear. If \(P\) lies between \(M\) and \(N,\) \(MP = 2,\) and \(PN = 1,\) find \(MN.\)
2-3

**Congruent Segments**

Complete the statement.

10. Two segments are **congruent** if they are equal in length.

11. When a segment is separated into two congruent segments, the segment is **bisected**.

12. Statements known as **postulates** can be justified using logical reasoning.

13. Points $A$, $B$, and $C$ are collinear. If $AC \cong CB$, then the point $C$ is the **endpoint** of $AB$.

2-4

**The Coordinate Plane**

Refer to the graph and name the ordered pair for each point.

14. point $P$

15. point $L$

16. point $A$

Graph and label the following points on the above coordinate plane.

17. point $N$ $(-4, 2)$

18. point $E$ $(3, 1)$

19. point $S$ $(1, -5)$

2-5

**Midpoints**

20. On a number line, if $X = -2$ and $Y = 4$, what is the coordinate of midpoint $Z$?

21. Find the coordinates of the midpoint of a segment whose endpoints are $(-5, -1)$ and $(-3, 3)$.

22. Find the coordinates of the other endpoint of a segment whose midpoint has coordinates $(4, 5)$ and second endpoint at $(2, -1)$.
ARE YOU READY FOR
THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
   • You are probably ready for the Chapter Test.
   • You may want to take the Chapter 2 Practice Test on page 85 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
   • You should complete the Chapter 2 Study Guide and Review on pages 82–84 of your textbook.
   • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   • You may also want to take the Chapter 2 Practice Test on page 85 of your textbook.

☐ I asked for help from someone else to complete the review of all or most lessons.
   • You should review the examples and concepts in your Study Notebook and Chapter 2 Foldable.
   • Then complete the Chapter 2 Study Guide and Review on pages 82–84 of your textbook.
   • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   • You may also want to take the Chapter 2 Practice Test on page 85 of your textbook.

Visit geomconcepts.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 2.

Student Signature

Parent/Guardian Signature

Teacher Signature
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with a sheet of plain $8\frac{1}{2}'' \times 11''$ paper.

**STEP 1**
Fold
Fold in half lengthwise.

**STEP 2**
Fold
Fold again in thirds.

**STEP 3**
Open
Open and cut along the second fold to make three tabs.

**STEP 4**
Label
Label as shown. Make another 3-tab fold and label as shown.

**NOTE-TAKING TIP:** When you take notes, listen or read for main ideas. Then record those ideas in simplified form for future reference.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 3. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>acute angle</td>
<td></td>
<td>[a-KYOOT]</td>
<td></td>
</tr>
<tr>
<td>adjacent angles</td>
<td></td>
<td>[uh-JAY-sent]</td>
<td></td>
</tr>
<tr>
<td>angle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>angle bisector</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>complementary angles</td>
<td></td>
<td>[kahm-pluh-MEN-tuh-ree]</td>
<td></td>
</tr>
<tr>
<td>congruent angles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>degrees</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>exterior</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>interior</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>linear pair</td>
<td></td>
<td>[LiN-ee-ur]</td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>---------------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>obtuse angle [ob-TOOS]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>opposite rays</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>perpendicular [PER-pun-DI-kyoo-lur]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>protractor</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>quadrilateral [KWAD-ruh-LAT-er-ul]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>right angle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sides</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>straight angle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>supplementary angles [SUP-luh-MEN-tuh-ree]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vertex [VER-teks]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vertical angles</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Angles

**What You'll Learn**
- Name and identify parts of an angle.

**Build Your Vocabulary** (pages 44–45)

**Opposite rays** are two rays that are part of the same line and have only their endpoints in common.

The figure formed by two opposite rays is referred to as a **straight angle**.

Any case where two rays have a common endpoint is known as an **angle**.

The common endpoint is called the **vertex**.

The two rays that make up the angle are called the **sides** of the angle.

**Example**

**1. Name the angle in four ways. Then identify its vertex and its sides.**

The angle can be named in four ways:

- **\( \angle HBD \)**
- **\( \angle ZHY \)**
- **\( \angle ZYH \)**
- **\( \angle YHZ \)**

Its vertex is **\( B \)**. Its sides are **\( \overrightarrow{BH} \)** and **\( \overrightarrow{BD} \)**.

**Your Turn** Name the angle in four ways. Then identify its vertex and its sides.

**2. Name all angles having **D** as their vertex.**

There are **3** distinct angles with vertex **D**.

**Review It**

Name the sides of **\( \angle ABC \)**. (Lesson 1-2)

- **\( \overrightarrow{AB} \)**
- **\( \overrightarrow{BC} \)**
Your Turn Name all angles having A as their vertex.

Build Your Vocabulary (page 44)

An angle separates a \( \angle \) into \( \triangle \) parts: the interior of the angle, the exterior of the angle, and the angle itself.

Examples

Tell whether each point is in the interior, exterior, or on the angle.

3 Point A: Point A is on the \( \triangle \) of the angle.

4 Point B: Point B is on the \( \triangle \) of the angle.

5 Point C: Point C is \( \triangle \).

Your Turn Tell whether each point is in the interior, exterior, or on the angle.

a. Point T

b. Point N

c. Point D
### Example 1

**Use a protractor to measure \( \angle KLM \).**

**STEP 1** Place the center point of the protractor on vertex \( L \).
Align the straightedge with side \( LM \).

**STEP 2** Use the scale that begins with 0 at \( LM \). Read where \( \overrightarrow{LK} \) crosses this scale.

Angle \( \angle KLM \) measures \( \square \).  

**Your Turn** Use a protractor to measure \( \angle XYZ \).

### Example 2

**Find the measures of \( \angle DHE \), \( \angle EHG \), and \( \angle FHG \).**

\[
\begin{align*}
\angle DHE &= \square \quad \overrightarrow{HD} \text{ is at } 0^\circ \text{ on the left.} \\
\angle EHG &= \square \quad \overrightarrow{HG} \text{ is at } 0^\circ \text{ on the right.} \\
\angle FHG &= \square \quad \overrightarrow{HG} \text{ is at } 0^\circ \text{ on the right.}
\end{align*}
\]
Your Turn  Find $m\angle PQR$, $m\angle RQS$, and $m\angle SQT$.

Postulate 3-2  Protractor Postulate
On a plane, given $\overline{AB}$ and a number $r$ between 0 and 180, there is exactly one ray with endpoint $A$, extending on each side of $\overline{AB}$ such that the degree measure of the angle formed is $r$.

**EXAMPLE**

3 Use a protractor to draw an angle having a measure of $35^\circ$.

**STEP 1** Draw $\overline{BC}$.

**STEP 2** Place the center point of the protractor on $B$. Align the mark labeled with the ray.

**STEP 3** Locate and draw point $A$ at the mark labeled . Draw $\overline{BA}$.

Your Turn Use a protractor to draw an angle having a measure of $78^\circ$.

**REMEMBER IT**
Read $m\angle PQR = 75$ as the degree measure of angle $PQR$ is 75.

**REMEMBER IT**
The symbol $\perp$ is used to indicate a right angle.

**BUILD YOUR VOCABULARY** (pages 44–45)

- A **right angle** has a degree measure of 90.
- The degree measure of an **acute angle** is greater than 0 and less than 90.
- An **obtuse angle** has a degree measure greater than 90 and less than 180.
- A three-sided closed figure with three interior angles is a **triangle**.
- A four-sided closed figure with four interior angles is a **quadrilateral**.
Classify each angle as acute, obtuse, or right.

4. Classify each angle as acute, obtuse, or right.

5. Classify each angle as acute, obtuse, or right.

Your Turn

Classify each angle as acute, obtuse, or right.

a. 

b. 

Example

6. The measure of $\angle A$ is 100. Solve for $x$.

You know that $m \angle A = 100$

and $m \angle A = 90 + 10$.

Write and solve an equation.

$\underline{\boxed{100 - \boxed{90}}} = \boxed{3x} + 10 - \boxed{90}

\text{Substitution}$

$\underline{\boxed{\underline{\underbrace{\text{Subtract } \boxed{90} \text{ from each side.}}}}}$

$\underline{\boxed{\underline{\underbrace{\text{Divide each side by } \underline{\boxed{3x}}}}}}$

$\underline{\boxed{\boxed{\frac{90}{3x}}}} = x$

Your Turn

The measure of $\angle N$ is 135. Solve for $x$.

$\boxed{\boxed{(7x - 5)^{\circ}}}$
### EXAMPLES

1. If \( m \angle KNL = 110 \) and \( m \angle LNM = 25 \), find \( m \angle KNM \).

   \[
   m \angle KNM = m \angle KNL + m \angle LNM
   \]
   \[
   = 110 + 25 \quad \text{Substitution}
   \]
   \[
   = 135 \quad \text{So, } m \angle KNM = 135.
   \]

2. Find \( m \angle 2 \) if \( m \angle 1 = 75 \) and \( m \angle ABC = 140 \).

   \[
   m \angle 2 = m \angle ABC - m \angle 1
   \]
   \[
   = 140 - 75 \quad \text{Substitution}
   \]
   \[
   = 65 \quad \text{So, } m \angle 2 = 65.
   \]

3. Find \( m \angle JKL \) and \( m \angle LKM \) if \( m \angle JKM = 140 \).

   \[
   m \angle JKL + m \angle LKM = m \angle JKM
   \]
   \[
   4x + (2x - 10) = 140 \quad \text{Substitution}
   \]
   \[
   6x - 10 = 140 + 6x \quad \text{Combine like terms.}
   \]
   \[
   6x = 150 \quad \text{Add } 6x \text{ to each side.}
   \]
   \[
   x = 25 \quad \text{Divide each side by } 6.
   \]
Replace $x$ with $\square$ in each expression.

$m\angle JKL = 4x$  \hspace{1cm}  $m\angle LKM = 2x - 10$

\[= 4 \square \hspace{1cm} = 2 \square - 10\]

Therefore, $m\angle JKL = \square$ and $m\angle LKM = \square$.

**Your Turn**

**a.** If $m\angle ABC = 95$ and $m\angle CBD = 65$, find $m\angle ABD$.

**b.** If $m\angle XYZ = 110$ and $m\angle XYW = 22$, find $m\angle WYZ$.

**c.** Find $m\angle RSZ$ and $m\angle ZST$ if $m\angle RST = 135$. 
If $\overline{FD}$ bisects $\angle CFE$ and $m\angle CFE = 70$, find $m\angle 1$ and $m\angle 2$.

Since $\overline{FD}$ bisects $\angle CFE$, $m\angle 1 = m\angle 2$.

$$m\angle 1 + m\angle 2 = m\angle CFE$$

Postulate 3–3

$$m\angle 1 + m\angle 2 = 70$$

Replace $m\angle CFE$ with $70$.

$$2\left(m\angle 1\right) = 70$$

Combine like terms.

$$2(m\angle 1) = 70$$

Divide each side by $2$.

$$m\angle 1 = \frac{70}{2}$$

$m\angle 1 = \frac{70}{2}$.

Since $m\angle 1 = m\angle 2$, $m\angle 2 = \frac{70}{2}$.

Your Turn If $\overline{EG}$ bisects $\angle FEH$ and $m\angle FEH = 98$, find $m\angle 1$ and $m\angle 2$. 

BUILD YOUR VOCABULARY (page 44)

A _______ that divides an angle into _______ angles of equal _______ is called the angle bisector.

A line segment that divides an angle into _______ angles of equal _______ is called the angle bisector.
Adjacent angles share a common side and a vertex, but have no points in common.

When the noncommon sides of adjacent angles form a linear pair, the angles are said to form a linear pair.

**Examples**

Determine whether \( \angle 1 \) and \( \angle 2 \) are adjacent angles.

1. \( \angle 1 \) and \( \angle 2 \). They have the same but no.

2. \( \angle 1 \) and \( \angle 2 \). They have the same and a common with no interior points in common.

3. \( \angle 1 \) and \( \angle 2 \). They have a but no common.

**Your Turn** Determine whether \( \angle 1 \) and \( \angle 2 \) are adjacent angles.

a. \( \angle 1 \) and \( \angle 2 \). 

b. \( \angle 1 \) and \( \angle 2 \). 

c. \( \angle 1 \) and \( \angle 2 \).
4. Name the angle that forms a linear pair with $\angle TCM$.

$\angle TCE$ and $\angle TCM$ have a common side $\text{___}$, the same vertex $\text{___}$, and opposite rays $\text{___}$ and $\text{___}$.

So, $\angle TCE$ forms a linear pair with $\angle TCM$.

5. Do $\angle 1$ and $\angle TCE$ form a linear pair? Justify your answer.

$\text{___}$, they are not $\text{___}$ angles.

Your Turn Refer to Examples 4 and 5.

a. Name the angle that forms a linear pair with $\angle HCE$.

b. Determine if $\angle TCA$ and $\angle TCH$ form a linear pair. Justify your answer.

6. List at least two models of linear pairs in your classroom or home.

Your Turn List at least two models of adjacent angles on your school playground.
Complementary and Supplementary Angles

**WHAT YOU'LL LEARN**

- Identify and use complementary and supplementary angles.

**BUILD YOUR VOCABULARY** (pages 44–45)

**Complementary angles** are two angles whose degree measures total 90.

**Supplementary angles** are two angles whose degree measures total 180.

**EXAMPLES**

1. **Use the figure to name a pair of nonadjacent supplementary angles.**

   
   
   
   
   

   \[ m\angle AGB + \boxed{55^\circ} = \boxed{180^\circ} \], and they have the same vertex \boxed{A}, but \boxed{AGB} sides. Therefore, \( \angle AGB \) and \boxed{AGB} are nonadjacent supplementary angles.

2. **Use the above figure to find the measure of an angle that is supplementary to \( \angle BGC \).**

   Let \( x = \) measure of angle supplementary to \( \angle BGC \).

   \[
   m\angle BGC + x = 180 \\
   \boxed{35^\circ} + x = 180 \\
   35 + x - \boxed{35^\circ} = 180 - \boxed{35^\circ} \\
   x = \boxed{145^\circ}
   \]

   Defn. of Supplementary Angles

   Subtract \boxed{35} from each side.
**Your Turn**

a. In the figure, name a pair of nonadjacent supplementary angles.

b. In the figure, find an angle with a measure supplementary to $\angle BAF$.

---

**EXAMPLE**

3. Angles $C$ and $D$ are supplementary. If $m\angle C = 12x$ and $m\angle D = 4(x + 5)$, find $x$. Then find $m\angle C$ and $m\angle D$.

\[
m\angle C + m\angle D = 180 \quad \text{Defn. of Supplementary Angles}
\]

\[
12x + 4(x + 5) = 180 \quad \text{Substitution}
\]

\[
12x + 4x + 20 = 180 \quad \text{Distributive Property}
\]

\[
16x + 20 = 180 \quad \text{Combine like terms.}
\]

\[
\frac{16x + 20}{16} = \frac{180}{16} \quad \text{Divide each side by 16.}
\]

\[
x = \boxed{10}
\]

Replace $x$ with $10$ in each expression.

\[
m\angle C = 12x
\]

\[
= 12 \boxed{10} \quad \text{or} \quad \boxed{120}
\]

\[
m\angle D = 4(x + 5)
\]

\[
= 4 \left( \boxed{10} + 5 \right) \quad \text{or} \quad \boxed{120}
\]

---

**REMEMBER IT**

Supplementary angles must be adjacent angles to form a linear pair.

---

**HOMEWORK ASSIGNMENT**

Page(s):

Exercises:

---

© Glencoe/McGraw-Hill
Congruent Angles

**SOL G.3** The student will solve practical problems involving complementary, supplementary, and congruent angles that include vertical angles, angles formed when parallel lines are cut by a transversal, and angles in polygons.

**WHAT YOU’LL LEARN**
- Identify and use congruent and vertical angles.

**BUILD YOUR VOCABULARY** (pages 44–45)

**Congruent angles** have the same measure.

When two lines, \([\angle A \cong \angle B]\), angles are formed. There are two pairs of nonadjacent angles. These pairs are **vertical angles**.

**Theorem 3-1** Vertical Angle Theorem
Vertical angles are congruent.

**EXAMPLES**

Find the value of \(x\) in each figure.

1. \[x - 100^\circ\]
   
   The angles are **vertical angles**.
   
   So, \(x = \) \[
   
   \]
   
   Since the angles are vertical angles, they are congruent.

2. \[(x - 18)^\circ\]

   \[75^\circ\]

   \[x - 18 = \]

   \[x - 18 + \quad = 75 + \quad\]

   Add \([\_\_\_\_]\) to each side.

   \[x = \]

**Your Turn** Find the value of \(x\) in each figure.

a.

b.

**REMEMBER IT**

The notation \(\angle A \equiv \angle B\) is read as angle \(A\) is congruent to angle \(B\).
EXAMPLES

3 Suppose \( \angle A \cong \angle B \) and \( m\angle B = 47 \). Find the measure of an angle that is supplementary to \( \angle A \).

Since \( \angle A \cong \angle B \), their supplements are congruent.

The supplement of \( \angle B \) is \( 180 - 47 \) or \( \boxed{133} \). So, the measure of an angle that is supplementary to \( \angle A \) is \( \boxed{133} \).

4 In the figure, \( \angle 1 \) is supplementary to \( \angle 2 \), \( \angle 3 \) is supplementary to \( \angle 2 \), and \( m\angle 2 = 105 \). Find \( m\angle 1 \) and \( m\angle 3 \).

\( \angle 1 \) and \( \angle 2 \) are supplementary.

So, \( m\angle 1 = \boxed{75} \) - 105 or \( \boxed{180} \) - 105. \( \angle 3 \) and \( \angle 2 \) are supplementary. So, \( m\angle 3 = \boxed{75} \) - 105 or \( \boxed{180} \) - 105.

YOUR TURN

a. Suppose \( \angle X \cong \angle Y \) and \( m\angle Y = 82 \). Find the measure of an angle that is supplementary to \( \angle X \).

b. In the figure, \( \angle 1 \) is supplementary to \( \angle 2 \) and \( \angle 4 \). If \( m\angle 4 = 54 \), find \( m\angle 1 \), \( m\angle 2 \), and \( m\angle 3 \).
Perpendicular Lines

Refer to the figure to determine whether each of the following is true or false.

1. \( QS \perp OP \)
   - \( QS \) and \( OP \) do not form angles. Therefore, they are not perpendicular.

2. \( \angle 7 \) is an obtuse angle.
   - \( \angle 7 \) forms an angle with an acute angle.

Remember It
Read \( p \perp q \) as line \( p \) is perpendicular to line \( q \).

Your Turn
In the figure \( WY \perp ZT \). Determine whether each of the following is true or false.

a. \( m \angle WZU + m \angle UZT = 90 \)
   - True

b. \( \angle SZY \) is obtuse.
   - True
3 Find \( m\angle 1 \) and \( m\angle 2 \) if \( \overline{AC} \perp \overline{BD} \), \( m\angle 1 = 8x - 2 \) and \( m\angle 2 = 16x - 4 \).

Since \( \overline{AC} \perp \overline{BD} \), \( \angle AED \) is a right angle.

\[
m\angle AED = 90
\]
Definition of perpendicular lines

\[
\angle 1 + \angle \boxed{} = \angle AED
\]
Angle Addition Postulate

\[
m\angle 1 + m\angle \boxed{} = m\angle AED
\]

Substitution

\[
(8x - 2) + (16x - 4) = 90
\]
Substitution

\[
24x - 6 = 90
\]
Combine like terms.

\[
24x - 6 + 6 = 90 + 6
\]
Add 6 to each side.

\[
24x = 96
\]

\[
\frac{24x}{24} = \frac{96}{24}
\]
Divide each side by 24.

\[
x = \boxed{}
\]

Replace \( x \) with \( \boxed{} \) to find \( m\angle 1 \) and \( m\angle 2 \).

\[
m\angle 1 = 8x - 2 \]

\[
\quad = 8 \left( \boxed{} \right) - 2
\]

\[
= 32 - 2 \text{ or } 30
\]

\[
m\angle 2 = 16x - 4 \]

\[
\quad = 16 \left( \boxed{} \right) - 4
\]

\[
= 64 - 4 \text{ or } 60
\]

Therefore, \( m\angle 1 = 30 \) and \( m\angle 2 = 60 \).

**Your Turn** Find \( m\angle 3 \) and \( m\angle 4 \) if \( \overline{AC} \perp \overline{BF} \), \( m\angle 3 = 7x + 6 \) and \( m\angle 4 = 12x + 27 \).
Indicate whether the statement is **true** or **false**.

1. \( XY \) and \( YZ \) are the sides of \( \angle XYZ \).  
   - [ ]

2. The vertex of an angle is a point where two rays intersect.  
   - [ ]

3. A straight angle is also a line.  
   - [ ]

### 3-2 Angle Measure

Use a protractor to measure the specified angles. Then, classify them as **acute**, **right**, or **obtuse** angles.

4. \( \angle BAC \)  
   - [ ]

5. \( \angle CAE \)  
   - [ ]

6. \( \angle DAE \)  
   - [ ]

### 3-3 The Angle Addition Postulate

7. If \( m\angle QPR = 30 \) and \( m\angle RPS = 51 \), find \( m\angle QPS \).  
   - [ ]

8. If \( m\angle QPX = 137 \) and \( m\angle QPR = 30 \), find \( m\angle RPX \).  
   - [ ]
9. In the figure $\overline{QN}$ and $\overline{QP}$ are opposite rays. Name the angles that form a linear pair.

10. If $m\angle 1 = 36$, what is the measure of its complement?

11. What is the measure of an angle supplementary to $m\angle 1 = 36$?

Lines $m$ and $n$ intersect at point $P$. What is the measure of each of the four angles formed?

12. 
13. 
14. 
15. 

If $\overline{WZ}$ is constructed perpendicular to $\overline{XY}$, list six terms that describe $\angle XWZ$ and $\angle YWZ$. 

16. 
17. 
18. 
19. 
20. 
21. 
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
   • You are probably ready for the Chapter Test.
   • You may want to take the Chapter 3 Practice Test on page 137 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
   • You should complete the Chapter 3 Study Guide and Review on pages 134–136 of your textbook.
   • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   • You may also want to take the Chapter 3 Practice Test on page 137.

☐ I asked for help from someone else to complete the review of all or most lessons.
   • You should review the examples and concepts in your Study Notebook and Chapter 3 Foldable.
   • Then complete the Chapter 3 Study Guide and Review on pages 134–136 of your textbook.
   • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   • You may also want to take the Chapter 3 Practice Test on page 137.

Visit geoconcepts.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 3.

Student Signature

Parent/Guardian Signature

Teacher Signature
Parallels

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with three sheets of plain 8 1/2" × 11" paper.

**STEP 1** Fold
Fold in half along the width.

**STEP 2** Open
Open and fold the bottom to form a pocket. Glue edges.

**STEP 3** Repeat
Repeat steps 1 and 2 three times and glue all three pieces together.

**STEP 4** Label
Label each pocket with the lesson names. Place an index card in each pocket.

**NOTE-TAKING TIP:** When taking notes, it is often a good idea to write in your own words a summary of the lesson. Be sure to paraphrase key points.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 4. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>alternate exterior angles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>alternate interior angles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>consecutive interior angles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>corresponding angles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>exterior angles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>finite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>great circle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>interior angles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>line</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>-------------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>line of latitude</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>line of longitude</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>linear equation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>parallel lines [PARE-uh-lel]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>parallel planes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>skew lines [SKYOO]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>slope</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>slope-intercept form</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>transversal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y-intercept</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Build Your Vocabulary** (pages 66–67)

**Parallel lines** are two lines in the same that do not intersect.

**Parallel planes** are the same apart at all points and intersect.

Lines that do not and are not in the plane are said to be **skew lines**.

**Examples**

Name the parts of the prism shown below. Assume segments that look parallel are parallel.

1. **all planes parallel to plane SKL**
   
   Plane is parallel to plane SKL.

2. **all segments that intersect MT**
   
   Intersect MT.

3. **all segments parallel to MT**

   Is parallel to MT.

4. **all segments skew to MT**

   Are skew to MT.
Your Turn
Name the parts of the prism shown below. Assume segments that look parallel are parallel.

![Prism Diagram]

a. all segments parallel to $\overline{RS}$

b. all segments that intersect $\overline{RS}$

c. a pair of parallel planes

d. all segments skew to $\overline{XT}$

**REMEMBER IT**
A plane that passes through points $A$, $B$, $C$ and $D$ can be named using any three of the points.

**HOMEWORK ASSIGNMENT**
Page(s):
Exercises:
Parallel Lines and Transversals

**WHAT YOU'LL LEARN**
- Identify the relationships among pairs of interior and exterior angles formed by two parallel lines and a transversal.

**BUILD YOUR VOCABULARY** (pages 66–67)

A line, line segment, or ray that intersects two or more lines at different is known as a transversal.

**Interior angles** lie in between the two lines.

**Alternate interior angles** are on sides of the transversal.

**Consecutive interior angles** are on the side of the transversal.

**Exterior angles** lie the two lines.

**Alternate exterior angles** are on sides of the transversal.

**EXAMPLES**

Identify each pair of angles as alternate interior, alternate exterior, consecutive interior, or vertical.

1. $\angle 3$ and $\angle 5$
   $\angle 3$ and $\angle 5$ are interior angles on the same side as the transversal, so they are angles.

2. $\angle 1$ and $\angle 8$
   $\angle 1$ and $\angle 8$ are exterior angles on opposite sides of the transversal, so they are angles.
Identify each pair of angles as alternate interior, alternate exterior, consecutive interior, or vertical.

a. \( \angle 3 \) and \( \angle 5 \)

b. \( \angle 3 \) and \( \angle 6 \)

Theorem 4-1 Alternate Interior Angles
If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.

Theorem 4-2 Consecutive Interior Angles
If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.

Theorem 4-3 Alternate Exterior Angles
If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.

EXAMPLE

In the figure, \( p \parallel q \), and \( r \) is a transversal. If \( m\angle 6 = 115 \), find \( m\angle 7 \).

\( \angle 6 \) and \( \angle 7 \) are alternate angles, so by Theorem 4-3, they are congruent. Therefore, \( m\angle 7 = \) ________.

Your Turn
If \( m\angle 1 = 50 \), find \( m\angle 8 \).
In the figure, $AB \parallel CD$, and $t$ is a transversal. If $m\angle 6 = 128$, find $m\angle 7$, $m\angle 8$, and $m\angle 9$.

$\angle 6$ and $\angle 8$ are consecutive interior angles, so by Theorem 4-2 they are supplementary.

$$m\angle 6 + m\angle 8 = 180$$

Replace $m\angle 6$.

$$128 + m\angle 8 = 180 -$$ Subtract 128 from each side.

$$m\angle 8 =$$

$\angle 7$ and $\angle 8$ are alternate interior angles, so by Theorem 4-1 they are congruent. Therefore, $m\angle 7 =$.

$\angle 6$ and $\angle 9$ are angles, so by Theorem 4-1 they are congruent. Therefore, $m\angle 9 =$.

Your Turn

In the figure, $n \parallel m$, and $a$ is a transversal. If $m\angle 6 = 73$, find $m\angle 1$, $m\angle 4$, and $m\angle 7$. 

Your Turn
Transversals and Corresponding Angles

SOL G.3 The student will solve practical problems involving complementary, supplementary, and congruent angles that include vertical angles, angles formed when parallel lines are cut by a transversal, and angles in polygons.

WHAT YOU’LL LEARN

• Identify the relationships among pairs of corresponding angles formed by two parallel lines and a transversal.

build your vocabulary (page 66)

When a __________________ crosses two lines, an interior angle and an exterior angle that are on the __________________ side of the transversal are called corresponding angles.

EXAMPLE

1 Lines a and b are cut by transversal c. Name two pairs of corresponding angles.

Corresponding angles lie on the same __________________ of the transversal and have __________________ vertices. Two pairs of corresponding angles are ________________________________.

Postulate 4-1 Corresponding Angles

If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.

EXAMPLES

In the figure, a \parallel b, and k is a transversal.

2 Which angle is congruent to \( \angle 1 \)? Explain your answer.

\( \angle 1 \equiv \underline{\hspace{2cm}} \), since \( \underline{\hspace{2cm}} \) angles are congruent (Postulate \( \underline{\hspace{2cm}} \)).
Find the measure of $\angle 1$ if $m \angle 4 = 60$.

$m \angle 1 = m \angle 3$

$\angle 3$ and $\angle 4$ are a linear pair, so they are supplementary.

$m \angle 3 + m \angle 4 = 180$

$m \angle 3 + 60 = 180$ 

Replace $m \angle 4$ with $60$.

$m \angle 3 + 60 - 60 = 180 - 60$ 

Subtract 60 from each side.

$m \angle 3 = 120$

$m \angle 1 = 120$ 

Substitution

Your Turn

a. Refer to the figure in Example 1. Name two different pairs of corresponding angles.

b. Refer to the figure in Example 2. Which angle is congruent to $\angle 2$? Explain your answer.

c. Refer to the figure in Example 2. Find the measure of $\angle 2$ if $m \angle 3 = 145$.

Theorem 4-4 Perpendicular Transversal
If a transversal is perpendicular to one of two parallel lines, it is perpendicular to the other.
In the figure, $p \parallel q$, and transversal $r$ is perpendicular to $q$. If $m\angle 2 = 3(x + 2)$, find $x$.

$p \perp r$

$\angle 2$ is a right angle.

\[
m\angle 2 = \underline{3(x + 2)}
\]

Definition of perpendicular lines

Definition of right angles

Given

Replace $m\angle 2$ with $\underline{3(x + 2)}$.

Distributive Property

Subtract 6 from each side.

\[
90 - \underline{3(x + 2)} = 3x + 6 - \underline{6}
\]

Divide each side by $\underline{3}$.

\[
\frac{84}{3} = \frac{3x}{3}
\]

$\underline{x} = x$

Your Turn In the figure, $a \parallel b$ and $r$ is a transversal. If $m\angle 1 = 3x - 5$ and $m\angle 2 = 2x + 35$, find $x$. 
Proving Lines Parallel

SOL G.4 The student will use the relationships between angles formed by two lines cut by a transversal to determine if two lines are parallel and verify, using algebraic and coordinate methods as well as deductive proofs.

What You’ll Learn

• Identify conditions that produce parallel lines and construct parallel lines.

Organize It

Use the index card labeled Proving Lines Parallel to record the postulates, theorems, and important concepts in this lesson. Record examples to help you remember the main idea.

Example

1. If \( m \angle 1 = 5x + 10 \) and \( m \angle 2 = 6x - 4 \), find \( x \) so that \( a \parallel b \).

From the figure, you know that \( \angle 1 \) and \( \angle 2 \) are corresponding angles. According to Postulate 4-2, if \( m \angle 1 = m \angle 2 \), then \( a \parallel b \).

\[
\begin{align*}
m \angle 1 &= m \angle 2 \\
5x - 5x + 10 &= 6x - 5x - 4 \\
10 &= x - 4 \\
10 + 4 &= x - 4 + 4 \\
x &= 4
\end{align*}
\]

Your Turn

Find \( c \) so that \( r \parallel s \).

Theorem 4-5 In a plane, if two lines are cut by a transversal so that a pair of alternate interior angles is congruent, then the two lines are parallel.

Theorem 4-6 In a plane, if two lines are cut by a transversal so that a pair of alternate exterior angles is congruent, then the two lines are parallel.

Theorem 4-7 In a plane, if two lines are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the two lines are parallel.

Theorem 4-8 In a plane, if two lines are perpendicular to the same line, then the two lines are parallel.
EXAMPLE

2 Identify the parallel segments in the letter E.

\( \angle FEC \) and \( \angle DCA \) are corresponding angles.

\[ m\angle FEC = m\angle DCA \]  
Both angles measure 68°.

\[ EF \parallel CD \]  
Postulate 4-2

\( \angle BAC \) and \( \angle DCE \) are corresponding angles.

\[ m\angle BAC = m\angle DCE \]  
Both angles measure 112°.

\[ AB \parallel CD \]  
Postulate 4-2

\[ AB \parallel CD \parallel EF \]  
Transitive Property

Your Turn Identify the parallel lines in the figure.

EXAMPLE

3 Find the value of \( x \) so that \( KL \parallel MN \).

\( PQ \) is a transversal for \( KL \) and \( MN \). If \( (9x)° = (10x - 8)° \), then \( KL \parallel MN \) by Theorem 4-6.

\[
9x = 10x - 8
\]

\[
9x - 9x = 10x - 9x - 8
\]

\[
0 = x - 8
\]

\[
0 + 8 = x - 8 + 8
\]

\[
x = 8
\]

Thus, if \( x = 8 \), then \( KL \parallel MN \).

Your Turn Find \( c \) so that \( r \parallel s \).

\[
(15c + 12)°
\]

\[
132°
\]
**WHAT YOU’LL LEARN**

- Find the slopes of lines and use slope to identify parallel and perpendicular lines.

**BUILD YOUR VOCABULARY** (page 67)

**Slope** is the ratio of the vertical change to the horizontal change, or the \( \frac{\text{rise}}{\text{run}} \), as you move from one point on the line to another.

**EXAMPLES**

**Definition of Slope**

The slope \( m \) of a line containing two points with coordinates \((x_1, y_1)\) and \((x_2, y_2)\) is the difference in the \(y\)-coordinates divided by the difference in the \(x\)-coordinates.

**Find the slope of each line.**

1. \( m = \frac{0 - 2}{2 - 0} = \frac{-2}{2} = -1 \)

2. \( m = \frac{-2 - (-2)}{2 - (-3)} = \frac{-2 + 2}{5} = \frac{0}{5} = 0 \)

**Your Turn**

Find the slope of each line.

a. \( m = \frac{4 - (-3)}{-2 - 4} = \frac{4 + 3}{-2 - 4} = \frac{7}{-6} = -\frac{7}{6} \)

b. \( m = \frac{4 - 0}{-2 - (-3)} = \frac{4}{1} = 4 \)
Postulate 4-3
Two distinct nonvertical lines are parallel if and only if they have the same slope.

Postulate 4-4
Two nonvertical lines are perpendicular if and only if the product of their slopes is $-1$.

**Example 3**

Given $A(-2, -\frac{1}{2})$, $B\left(2, \frac{1}{2}\right)$, $C(5, 0)$, and $D(4, 4)$, prove that $\overrightarrow{AB} \perp \overrightarrow{CD}$.

First, find the slopes of $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

$$\text{Slope of } \overrightarrow{AB} = \frac{\frac{1}{2} - -\frac{1}{2}}{2 - (-2)} = \frac{\frac{1}{2} + \frac{1}{2}}{2 + 2} = \frac{1}{2}$$

$$\text{Slope of } \overrightarrow{CD} = \frac{4 - 0}{4 - 5} = \frac{4}{-1} = -4$$

The product of the slopes for $\overrightarrow{AB}$ and $\overrightarrow{CD}$ is $(-4)$ or $-4$. Therefore, $\overrightarrow{AB} \perp \overrightarrow{CD}$.

**Your Turn**

Given $A(-3, -4)$, $B(-1, 7)$, $C(2, -5)$, and $D(4, 6)$, prove that $\overrightarrow{AB} \parallel \overrightarrow{CD}$.
**Equations of Lines**

**Build Your Vocabulary** (pages 66–67)

The graph of a linear equation is a straight line.

The y-value of the point where the line crosses the is called the y-intercept.

The slope-intercept form of a linear equation is written as , where \( m \) is the slope and \( b \) is the y-intercept.

**Examples**

Name the slope and y-intercept of the graph of each equation.

1. \( y = \frac{2}{3}x + 6 \)
   - The slope is .
   - The y-intercept is .

2. \( y = 0 \)
   - The slope is .
   - The y-intercept is .

3. \( x = 7 \)
   - The graph is a line. The slope is undefined. There is no y-intercept.

4. \( 3y + 12 = 6x \)
   - Rewrite the equation in slope-intercept form by solving for \( y \).
     
     \[
     3y + 12 = 6x \\
     3y + 12 - 12 = 6x - 12 \\
     3y = 6x - 12 \\
     \frac{3y}{3} = \frac{6x - 12}{3} \\
     y = 2x - 4
     \]
   - Subtract 12 from each side.
   - Divide each side by 3.
   - Simplify. This is written in slope-intercept form.
   - The slope \( m = \) .
   - The y-intercept is .
Your Turn  Name the slope and y-intercept of the graph of each equation.

a. \( y = -6x + 13 \)

b. \( y = 8 \)

c. \( x = 7 \)

d. \( 4x + 3y = 5 \)

Graph 2\(x - y = 4\) using the slope and y-intercept.

First, rewrite the equation in slope-intercept form.

\[
2x - y = 4 \\
2x - y - 2x = 4 - 2x \\
-y = 4 - 2x \\
\frac{-y}{-1} = \frac{4 - 2x}{-1} \\
y = \frac{4 - 2x}{-1} \quad \text{Slope-intercept form}
\]

The y-intercept is \(-4\). So, the point \((0, -4)\) is on the line. Since the slope is 2, or \(\frac{2}{1}\), plot a point by using a rise of \(\square\) units (up) and a run of \(\square\) unit (right). Draw a line through the two points.
Write an equation of the line parallel to the graph of \( y = -2x + 3 \) that passes through the point at \((0, 1)\).

Because the lines are parallel, they must have the same slope. So, \( m = \) ___.

To find \( b \), use the ordered pair \((0, 1)\) and substitute for \( m \), \( x \), and \( y \) in the slope-intercept form.

\[
y = mx + b
\]

\[
1 = \underline{m}(0) + b \quad m = \underline{\text{___}}, \quad (x, y) = \underline{\text{___}}
\]

\[
1 = 0 + b
\]

\[
\underline{b} = b
\]

The value of \( b \) is ___. So, the equation of the line is ___.

Your Turn

a. Write an equation of the line parallel to the graph of \(-5x + y = 6\) that passes through the point \((-1, 3)\).

b. Write an equation of the line perpendicular to the graph of \( y = -2x + 1 \) that passes through the point \((4, -5)\).
Choose the term that best completes each sentence.

1. (Skew/Parallel) lines always lie on the same plane.
2. (Perpendicular/Skew) lines never have any points in common.
3. (Parallel/Perpendicular) lines never intersect.

Refer to the figure and match the term with the best representative angle pair. Angle pairs cannot be matched more than once.

4. consecutive interior angles
5. exterior angles
6. alternate interior angles
7. alternate exterior angles

- a. $\angle 2$ and $\angle 7$
- b. $\angle 3$ and $\angle 6$
- c. $\angle 4$ and $\angle 6$
- d. $\angle 1$ and $\angle 7$
- e. $\angle 3$ and $\angle 4$
In the figure, \( \ell \parallel m \), and transversal \( r \) is perpendicular to \( m \). Name all angles congruent to the given angle.

8. \( \angle 4 \)

9. \( \angle 3 \)

10. \( \angle 9 \)

Refer to the above figure to find the measure of the specified angle if \( m \angle 3 = 40 \).

11. \( \angle 4 \)

12. \( \angle 5 \)

13. \( \angle 8 \)

14. \( \angle 2 \)

Find the values of \( a \), \( b \), and \( c \) so that \( \ell \parallel m \parallel n \).

15. \( a = \)

16. \( b = \)

17. \( c = \)

18. Name the parallel lines.
A wheelchair access ramp must be added to a home. One plan showed a ramp that started 30 feet away from the entrance. The entrance was 3 feet higher than ground level. The second plan started the ramp 15 feet from the same 3-foot high entrance.

19. What is the slope of each ramp?

20. Which slope is steeper?

21. Given \( A(0, 4), B(3, 6), C(1, 2), \) and \( D(3, -1) \), determine whether \( \overline{AB} \) and \( \overline{CD} \) are parallel, perpendicular, or neither.

**4-6**

**Equations of Lines**

Identify the slope and \( y \)-intercept of each equation.

22. \( y = -6x + \frac{1}{2} \)

23. \( 5x - 4y = 7 \)

24. \( y = -2 \)

25. \( x = 5 \)

26. Write an equation of a line parallel to \( y = 3x + 2 \) that passes through the point \((-1, -4)\).
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
  • You are probably ready for the Chapter Test.
  • You may want to take the Chapter 4 Practice Test on page 183 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
  • You should complete the Chapter 4 Study Guide and Review on pages 180–182 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 4 Practice Test on page 183.

☐ I asked for help from someone else to complete the review of all or most lessons.
  • You should review the examples and concepts in your Study Notebook and Chapter 4 Foldable.
  • Then complete the Chapter 4 Study Guide and Review on pages 180–182 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 4 Practice Test on page 183.

Student Signature
Parent/Guardian Signature
Teacher Signature
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

**Begin with a sheet of plain $8\frac{1}{2} \times 11''$ paper.**

**STEP 1**
**Fold**
Fold in half lengthwise.

**STEP 2**
**Fold**
Fold the top to the bottom.

**STEP 3**
**Open**
Open and cut along the second fold to make two tabs.

**STEP 4**
**Label**
Label each tab as shown.

**NOTE-TAKING TIP:** When you take notes, define new terms and write about the new concepts you are learning in your own words. Then, write your own examples that use the new terms and concepts.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 5. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>acute triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>base</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>base angles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>congruent triangles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>corresponding parts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>equiangular triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>equilateral triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>image</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>included angle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>included side</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>isometry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>isosceles triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>----------------------</td>
<td>--------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>legs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mapping</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>obtuse triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>preimage</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>reflection</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>right triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rotation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>scalene triangle</td>
<td></td>
<td></td>
<td>[SKAY-leen]</td>
</tr>
<tr>
<td>transformation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>translation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vertex</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vertex angle</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Classify each triangle by its angles and by its sides.

1. The triangle is a __________ triangle.

2. The triangle is an __________ triangle.

**WHAT YOU’LL LEARN**

- Identify the parts of triangles and classify triangles by their parts.

**ORGANIZE IT**

Draw examples of acute, obtuse, right, scalene, isosceles, and equilateral triangles in your notes.

**BUILD YOUR VOCABULARY**

The side that is opposite the vertex angle in an __________ triangle is called the **base**.

In an isosceles triangle, the two angles formed by the __________ and one of the congruent __________ are called **base angles**.

The congruent sides in an isosceles triangle are the **legs**.

The vertex of each angle of a __________ is a **vertex** of the triangle.

The angle formed by the __________ sides in an __________ triangle is called the **vertex angle**.
Your Turn
Classify each triangle by its angles and by its sides.

a.  

b.  

EXAMPLE

Find the measures of \( XY \) and \( YZ \) of isosceles triangle \( XYZ \) if \( \angle X \) is the vertex angle.

Since \( \angle X \) is the vertex angle, \( \text{ } \equiv \text{ } \).

So, \( XY = \text{ } \). Write and solve an equation.

\[
XY = \text{ }
\]

Substitution

\[
2n + 2 - \text{ } = 10 - \text{ } \quad \text{Subtract } \text{ } \text{ from each side.}
\]

Divide each side by \( \text{ } \).

The value of \( n \) is \( \text{ } \).
To find the measures of $XY$ and $YZ$, replace $n$ with $\square$ in the expression for each measure.

$XY = 2n + 2$

$= 2(\square) + 2$

$= \square + 2$

$YZ = 2n - 2$

$= 2(\square) - 2$

$= \square - 2$

Therefore, $XY = \square$ and $YZ = \square$.

**Your Turn** Triangle $DEF$ is an isosceles triangle with base $EF$. Find $DE$ and $EF$. 

![Diagram of triangle DEF with sides labeled]
**Theorem 5-1 Angle Sum Theorem**
The sum of the measures of the angles of a triangle is 180.

---

1. Find $m\angle P$ in $\triangle MNP$ if $m\angle M = 80$ and $m\angle N = 45$.

   \[
   m\angle P + m\angle M + m\angle N = 180 \quad \text{Angle Sum Theorem}
   \]
   \[
   m\angle P + 80 + 45 = 180 \quad \text{Substitution}
   \]
   \[
   m\angle P + 125 = 180
   \]
   \[
   m\angle P = 180 - 125 = 55 \quad \text{Subtract.}
   \]

2. Find the value of each variable in $\triangle ABC$.

   $\angle ABC$ is a vertical angle to the given angle measure of 75. Since vertical angles are congruent, $m\angle ABC = 75 = x$.

   \[
   m\angle ABC + m\angle BCA + m\angle CAB = 180 \quad \text{Angle Sum Theorem}
   \]
   \[
   75 + 58 + 58 = 180 \quad \text{Substitution}
   \]
   \[
   133 + y = 180
   \]
   \[
   133 + y - 133 = 180 - 133 \quad \text{Subtract.}
   \]
   \[
   y = 47
   \]

Therefore, $x = 75$ and $y = 47$.

---

**Your Turn** Find the value of each variable.

---

**WHAT YOU’LL LEARN**
- Use the Angle Sum Theorem

**EXAMPLES**

---

**REVIEW IT**

What does it mean when two angles are complementary? (Lesson 3-5)
Theorem 5-2
The acute angles of a right triangle are complementary.

Theorem 5-3
The measure of each angle of an equiangular triangle is 60.

3 Find \( m\angle J \) and \( m\angle K \) in right triangle \( JKL \).

\[ m\angle J + m\angle K = 90 \]
\[ (x + 15) + (x + 9) = 90 \]

\[ 2x + 24 = 90 \]

\[ 2x = 66 \]

\[ x = 33 \]

Therefore, \( m\angle J = \) and \( m\angle K = \).

**Your Turn** Find the value of \( a, b, \) and \( c \).

When all three angles in a triangle are congruent, the triangle is said to be equiangular.
Identify each motion as a translation, reflection, or rotation.

1. A figure from one position to another without turning it is called a translation.

2. A figure over a line creates the mirror image of the figure, or a reflection.

3. A figure around a fixed point creates a rotation.

Your Turn Identify each motion as a translation, reflection, or rotation.

- a. 
- b. 

WHAT YOU’LL LEARN
- Identify translations, reflections, and rotations and their corresponding parts.

BUILD YOUR VOCABULARY (page 89)

- a figure from one position to another without turning it is called a translation.
- a figure over a line creates the mirror image of the figure, or a reflection.
- a figure around a fixed point creates a rotation.

EXAMPLES

1. a figure from one position to another without turning it is called a translation.

2. a figure over a line creates the mirror image of the figure, or a reflection.

3. a figure around a fixed point creates a rotation.

Your Turn Identify each motion as a translation, reflection, or rotation.

- a. 
- b. 

© Glencoe/McGraw-Hill
In the figure, \( \triangle RST \rightarrow \triangle XYZ \) by a translation.

3. Name the image of \( \angle T \).
\( \triangle RST \rightarrow \triangle XYZ \) \( \angle T \) corresponds to \( \underline{\text{ }} \).

4. Name the side that corresponds to \( \overline{XY} \).
\( \triangle RST \rightarrow \triangle XYZ \)
Point \( R \) corresponds to point \( \underline{\text{ }} \).
Point \( S \) corresponds to point \( \underline{\text{ }} \).
So, \( \underline{\text{ }} \) corresponds to \( \underline{\text{ }} \).

**Your Turn** In the figure, \( \triangle QRS \rightarrow \triangle DEF \) by a rotation.

a. Name the angle that corresponds to \( \angle R \).

b. Name the side that corresponds to \( \overline{QR} \).
Identify the type(s) of transformations that were used to complete the work below.

Some figures can be moved to another without turning or flipping. Other figures have been turned around a point with respect to the original.

Therefore, the transformations are and .

**Your Turn** Identify the type(s) of transformations that were used to complete the work below.
If \( \triangle ABC \cong \triangle FDE \), name the congruent angles and sides. Then draw the triangles, using arcs and slash marks to show congruent angles and sides.

Name the three pairs of congruent angles by looking at the order of the vertices in the statement \( \triangle ABC \cong \triangle FDE \).

\[
\angle A \cong \underline{\angle A}, \quad \angle B \cong \underline{\angle B},
\]
and \( \angle C \cong \underline{\angle C} \).

Since \( A \) corresponds to \( D \), and \( B \) corresponds to \( E \),
\[
\underline{AB} \cong \underline{DE}.
\]

Since \( B \) corresponds to \( D \), and \( C \) corresponds to \( E \),
\[
\underline{BC} \cong \underline{DE}.
\]

Since \( \underline{AC} \) corresponds to \( \underline{F} \), and \( \underline{C} \) corresponds to \( \underline{E} \),
\[
\underline{AC} \cong \underline{FE}.
\]
The corresponding parts of two congruent triangles are marked on the figure. Write a congruence statement for the two triangles.

List the congruent angles and sides.

\[ \angle L \equiv \square \quad \angle M \equiv \angle R \quad \angle N \equiv \square \]

\[ LN \equiv ST \quad \square \equiv TR \quad \square \equiv SR \]

The congruence statement can be written by matching the \square of the \square angles. Therefore,

\[ \triangle L MN \equiv \square \]

Your Turn

a. If \( \triangle ACB \equiv \triangle ECD \), name the congruent angles and sides. Then draw the triangles, using arcs and slash marks to show congruent angles and sides.

b. Write another congruence statement for the two triangles other than the one given above.
SSS and SAS

SOL G.5.b The student will prove two triangles are congruent or similar, given information in the form of a figure or statement, using algebraic and coordinate as well as deductive proofs.

**Postulate 5-1 SSS Postulate**
If three sides of one triangle are congruent to three corresponding sides of another triangle, then the triangles are congruent.

**EXAMPLE**

1. In two triangles, $DF \cong UV$, $FE \cong VW$, and $DE \cong UW$. Write a congruence statement for the two triangles.

   Draw a pair of triangles. Identify the congruent parts with $\text{ }$. Label the vertices of one triangle.

   Use the given information to label the in the second triangle.

   By SSS, $\triangle \cong \triangle$.

**Your Turn** In two triangles, $CB \cong EF$, $CA \cong ED$, and $BA \cong FD$. Write a congruence statement for the two triangles.

**Build Your Vocabulary** (page 88)
In a triangle, the formed by two given is the included angle.
Determine whether the triangles shown at the right are congruent. If so, write a congruence statement and explain why the triangles are congruent. If not, explain why not.

There are three pairs of sides, \( \overline{RS} \), \( \overline{ST} \), \( \overline{TX} \), and \( \overline{RT} \), \( \overline{SY} \), \( \overline{SZ} \) and \( \overline{RX} \).

Therefore, \( \overline{RS} \cong \overline{ST} \), \( \overline{TX} \cong \overline{RX} \) and \( \overline{RT} \cong \overline{SY} \), \( \overline{SZ} \) by Postulate 5-2 SAS Postulate.

**Postulate 5-2 SAS Postulate**
If two sides and the included angle of one triangle are congruent to the corresponding sides and included angle of another triangle, then the triangles are congruent.

**Example 2**
Determine whether the triangles shown at the right are congruent. If so, write a congruence statement and explain why the triangles are congruent. If not, explain why not.

There are three pairs of sides, \( \overline{RS} \), \( \overline{ST} \), \( \overline{TX} \), and \( \overline{RT} \), \( \overline{SY} \), \( \overline{SZ} \) and \( \overline{RX} \).

Therefore, \( \overline{RS} \cong \overline{ST} \), \( \overline{TX} \cong \overline{RX} \) and \( \overline{RT} \cong \overline{SY} \), \( \overline{SZ} \) by Postulate 5-2 SAS Postulate.

**Your Turn**
Determine whether the triangles to the right are congruent. If so, write a congruence statement and explain why the triangles are congruent. If not, explain why not.
ASA and AAS

SOL G.5.b The student will prove two triangles are congruent or similar, given information in the form of a figure or statement, using algebraic and coordinate as well as deductive proofs.

Build Your Vocabulary (page 88)

The of the triangle that falls between two given is called the included side and is the one common side to both angles.

Postulate 5-3 ASA Postulate
If two angles and the included side of one triangle are congruent to the corresponding angles and included side of another triangle, then the triangles are congruent.

Example
1. In \( \triangle DEF \) and \( \triangle ABC \), \( \angle D \cong \angle C \), \( \angle E \cong \angle B \), and \( DE \cong CB \). Write a congruence statement for the two triangles.

   Draw a pair of \( \triangle DEF \) and \( \triangle ABC \) triangles. Mark the congruent parts with and . Label the vertices of one triangle \( D, E, \) and \( F \).

   Locate \( C \) and \( B \) on the unlabeled triangle in the same positions as \( A \) and \( B \). The unassigned vertex is \( \). Therefore, \( \triangle A \cong \triangle F \).

Your Turn
In \( \triangle RST \) and \( \triangle XYZ \), \( \overline{ST} \equiv \overline{XZ} \), \( \angle S \equiv \angle X \), and \( \angle T \equiv \angle Z \). Write a congruence statement for the two triangles.
Theorem 5-4  AAS Theorem
If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and nonincluded side of another triangle, then the triangles are congruent.

**Example**

2 $\triangle XYZ$ and $\triangle QRS$ each have one pair of sides and one pair of angles marked to show congruence. What other pair of angles needs to be marked so the two triangles are congruent by AAS?

If $\angle Q$ and $\angle X$ are marked $\underline{\text{ }}$, and $\underline{\text{ }} \equiv \underline{\text{ }}$, then $\underline{\text{ }}$ and $\underline{\text{ }}$ would have to be congruent for the triangles to be congruent by $\underline{\text{ }}$.

**Your Turn** $\triangle ACB$ and $\triangle FED$ each have one pair of sides and one pair of angles marked to show congruence. What other pair of angles needs to be marked so the two triangles are congruent by AAS?
5–1
Classifying Triangles

Complete each statement.

1. The sum of the measures of a triangle’s interior angles is ________.

2. The ________ angle is the angle formed by two congruent sides of an isosceles triangle.

3. The ________ angles of a right triangle are complementary.

4. A triangle with no congruent sides is ________.

5–2
Angles of a Triangle

Find the value of each variable.

5. \[
\begin{align*}
\triangle & = 3x
\end{align*}
\]

6. \[
\begin{align*}
\triangle & = y \\
& = 61^\circ
\end{align*}
\]
**5–3**

**Geometry in Motion**

Suppose \( \triangle SRN \rightarrow \triangle CDA \).

7. Which angle corresponds to \( \angle S \)?

8. Name the preimage of \( \overline{AD} \).

9. Identify the transformation that occurred in the mapping.

**5–4**

**Congruent Triangles**

If \( \triangle ABC \cong \triangle QRS \), name the corresponding congruent parts.

10. \( \angle B \)  
11. \( \overline{AC} \)  
12. \( \overline{RQ} \)  
13. \( \angle C \)

**5–5**

**SSS and SAS**

14. The pairs of triangles at the right are congruent. Write a congruence statement and the reason the triangles are congruent.

**5–6**

**ASA and AAS**

Underline the best term to make the statement true.

15. [Mapping/Congruence] of triangles is explained by SSS, SAS, ASA, and AAS.

16. AAS indicates two angles and their [included/nonincluded] side.
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
   • You are probably ready for the Chapter Test.
   • You may want to take the Chapter 5 Practice Test on page 223 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
   • You should complete the Chapter 5 Study Guide and Review on pages 220–222 of your textbook.
   • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   • You may also want to take the Chapter 5 Practice Test on page 223 of your textbook.

☐ I asked for help from someone else to complete the review of all or most lessons.
   • You should review the examples and concepts in your Study Notebook and Chapter 5 Foldable.
   • Then complete the Chapter 5 Study Guide and Review on pages 220–222 of your textbook.
   • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   • You may also want to take the Chapter 5 Practice Test on page 223 of your textbook.

Visit geomconcepts.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 5.
More About Triangles

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

**NOTE-TAKING TIP:** As you read a lesson, take notes on the materials. Include definitions, concepts, and examples. After you finish each lesson, make an outline of what you learned.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 6. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>altitude</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>angle bisector</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>centroid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>circumcenter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[SIR-kum-SEN-tur]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>concurrent</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Euler line</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hypotenuse</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[hi-PA-tin-oos]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>---------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>incenter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>leg</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>median</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>nine-point circle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>orthocenter</td>
<td></td>
<td></td>
<td>[OR-tho-SEN-tur]</td>
</tr>
<tr>
<td>perpendicular bisector</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pythagorean Theorem</td>
<td></td>
<td></td>
<td>[puh-THA-guh-REE-uhn]</td>
</tr>
<tr>
<td>Pythagorean triple</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In $\triangle ABC$, $\overline{CE}$ and $\overline{AD}$ are medians.

If $CD = 2x + 5$, $BD = 4x - 1$, and $AE = 5x - 2$, find $BE$.

Since $\overline{CE}$ and $\overline{AD}$ are medians, $D$ and $E$ are midpoints. Solve for $x$.

\[ CD = BD \]
\[ 2x + 5 = 4x - 1 \]
\[ 5 = 2x - 1 \]
\[ 5 + 2x - 1 = 6 \]
\[ 6 = 2x \]
\[ x = 3 \]

Use the values for $x$ and $AE$ to find $BE$.

\[ AE = BE \]
\[ 5x - 2 = BE \]
\[ 5(3) - 2 = 15 - 2 = BE \]
\[ 13 = BE \]

Your Turn In $\triangle OPS$, $\overline{ST}$ and $\overline{QP}$ are medians. If $PT = 3x - 1$, $OT = 2x + 1$, and $OQ = 4x - 2$, find $SQ$. 

\[ PT = 3x - 1 \]
\[ OT = 2x + 1 \]
\[ OQ = 4x - 2 \]
The three medians of a triangle intersect at a common point known as the **centroid**.

When three or more lines or segments meet at the same point, they are said to be **concurrent**.

**Theorem 6-1**
The length of the segment from the vertex to the centroid is twice the length of the segment from the centroid to the midpoint.

**EXAMPLES**

In \(\triangle XYZ\), \(XP\), \(ZN\), and \(YM\) are medians.

2. **Find** \(YQ\) **if** \(QM = 4\).

   Since \(QM = \square\), \(YQ = 2 \cdot \square\) or \(\square\).

3. **If** \(QZ = 18\), **what is** \(ZN\)?

   Since \(QZ = 18\) and \(QZ = \frac{2}{3} \cdot ZN\), solve the equation
   
   \[
   18 = \frac{2}{3} \cdot ZN
   \]
   
   \[
   \frac{3}{2}(18) = \frac{3}{2} \left(\frac{2}{3}ZN\right)
   \]
   
   Multiply each side by \(\square\).
   
   \[
   \square = ZN
   \]

**Your Turn**  
In \(\triangle EFG\), \(FA\), \(GB\), and \(EC\) are medians.

a. **Find** \(EO\) **if** \(CO = 3\).

b. **If** \(FA = 18\), **what are the measures of** \(AO\) **and** \(OP\)?
An altitude of a triangle is a perpendicular segment with one endpoint at a vertex and the other endpoint on the opposite side of that vertex.

**Examples**

1. Is $\overline{AD}$ an altitude of the triangle?
   - $\overline{AD}$ is a perpendicular segment. So, $\overline{AD}$ is an altitude of the triangle.

2. Is $\overline{GJ}$ an altitude of the triangle?
   - $\overline{GJ} \perp \overline{FH}$, $J$ is a vertex, and $G$ is on the side opposite $G$.
   - So, $\overline{GJ}$ is an altitude of the triangle.

**Your Turn**

a. Is $\overline{BD}$ an altitude of the triangle?

b. Is $\overline{XY}$ an altitude of the triangle?
Is $MN$ a perpendicular bisector of a side of the triangle?

Since $N$ is the midpoint of $KL$, $MN$ is a bisector of side $KL$. $MN$ perpendicular to $KL$, so $MN$ is a perpendicular bisector in $\triangle KLM$.

Is $AD$ a perpendicular bisector of a side of the triangle?

$AD \perp BC$ but $D$ the midpoint of $BC$. So, $AD$ a perpendicular bisector of side $BC$ in $\triangle ABC$.

**Your Turn**

a. Is $BD$ a perpendicular bisector of the triangle?

b. Is $LM$ a perpendicular bisector of the triangle?
Tell whether $MN$ is an altitude, a perpendicular bisector, both, or neither.

$: MN \perp KL$ but $N$ is the midpoint of $KL$. So, $MN$ is a perpendicular bisector of side $KL$ in $\triangle KLM$.

**Your Turn** Tell whether $XO$ is an altitude, a perpendicular bisector, both, or neither.
In \( \triangle ABC \), \( AC \) bisects \( \angle BAD \). If \( m\angle 1 = 41 \), find \( m\angle 2 \).

Since \( AC \) bisects \( \angle BAD \), \( m\angle 1 = \) \( \boxed{41} \).

Since \( m\angle 1 = \) \( \boxed{41} \), \( m\angle 2 = \) \( \boxed{20.5} \).

In \( \triangle KMN \), \( NL \) bisects \( \angle KNM \). If \( \angle KNM \) is a right angle, find \( m\angle 2 \).

\[
m\angle 2 = \frac{1}{2} (m\angle KNM)
\]

\[
m\angle 2 = \frac{1}{2} (\boxed{90})
\]

\[
m\angle 2 = \boxed{45}
\]

In \( \triangle WYZ \), \( ZX \) bisects \( \angle WZY \). If \( m\angle 1 = 55 \), find \( m\angle WZY \).

\[
m\angle WZY = 2(m\angle 1)
\]

\[
m\angle WZY = 2(\boxed{55})
\]

\[
m\angle WZY = \boxed{110}
\]

Your Turn

a. In \( \triangle XYZ \), \( \overline{YW} \) bisects \( \angle XYZ \).

If \( m\angle 2 = 33 \), find \( m\angle 1 \).
b. In \(\triangle NOM\), \(\overline{OP}\) bisects \(\angle NOM\). If \(\angle NOM = 85\), find \(m \angle 4\).

\[ \begin{align*} \angle HIG &= \angle FIG \\ 4x + 1 &= 5x - 5 \quad \text{(Distributive Property)} \\ 4x + 1 - 4x &= 5x - 5 - 4x \\ 1 &= x - 5 \\ 1 + 5 &= x - 5 + 5 \\ x &= \quad \text{(Add)} \\ m \angle HIG &= 4x + 1 = 4(\quad) + 1 = \quad + 1 = \quad \end{align*} \]

Your Turn: In \(\triangle JKL\), \(KM\) is an angle bisector. Find \(m \angle JKM\).
Find the values of the variables.

In the top triangle, find the value of base angle $x$. Since the triangle is isosceles, and one base angle $= 35$, $x = \boxed{35}$.

In the bottom triangle, find the value of base angle $y$. Since the other base angle $= 45$, $y = \boxed{45}$.

Your Turn Find the values of the variables.

Theorem 6-4 Converse of Isosceles Triangle Theorem
If two angles of a triangle are congruent, then the sides opposite those angles are congruent.
In \(\triangle DEF\), \(\angle 1 \cong \angle 2\) and \(m\angle 1 = 28\).

Find \(m\angle F\), \(DF\), and \(EF\).

First, find \(m\angle F\). You know that \(m\angle 1 = 28\). Since \(\angle 1 \cong \angle 2\), \(m\angle 2 = 28\).

\[
m\angle 1 + m\angle 2 + m\angle F = 180
\]

Replace \(m\angle 1\) and \(m\angle 2\).

\[
m\angle 1 + m\angle F = 180
\]

Subtract.

\[
m\angle F = \underline{56}
\]

Next, find \(DF\). Since \(\angle 1 \cong \angle 2\), Theorem 6-4 states that \(DF \cong EF\).

\[
DF = EF
\]

Replace \(DF\) and \(EF\).

\[
2x + 2 = 3x - 3
\]

Subtract.

\[
2 = x - 3
\]

Add.

\[
x = \underline{5}
\]

By replacing \(x\) with 5, you find that \(DF = 2x + 2 = 2(5) + 2 = 10 + 2\) or \(\underline{12}\). \(EF = 3x - 3 = 3(5) - 3 = 15 - 3\) or \(\underline{12}\).

**Your Turn** Find the values of the variables.

**Theorem 6-5** A triangle is equilateral if and only if it is equiangular.
Determine whether each pair of right triangles is congruent by LL, HA, LA, or HL. If it is not possible to prove that they are congruent, write \textit{not possible}.

There is one pair of congruent angles, $\angle DFE \cong \angle GFE$. The hypotenuses are congruent, $DF \cong GF$.

Therefore, $\triangle DEF \cong \triangle GEF$ by ____.
There is one pair of acute angles, \( \angle Z \cong \angle L \). There is one pair of opposite, \( \overline{XZ} \cong \overline{KL} \).

Therefore, \( \triangle YXZ \cong \triangle JKL \) by \( \triangle \).

**Write It**

Which test for congruence is used to establish the LA Theorem? Explain.

---

**Your Turn**

Determine whether each pair of right triangles is congruent by \( LL, HA, LA, \) or \( HL \). If it is not possible to prove that they are congruent, write \textit{not possible}.

a.

---

b.

---
Find the length of the hypotenuse of the right triangle. Use the Pythagorean Theorem to find the length of the hypotenuse.

\[ c^2 = a^2 + b^2 \]

Where \( c \) is the hypotenuse, and \( a \) and \( b \) are the legs of the right triangle.

Replace \( a \) and \( b \).

\[ c^2 = 16^2 + 12^2 \]
\[ c^2 = 256 + 144 \]
\[ c^2 = 400 \]

Take the square root of each side.

\[ c = \sqrt{400} \]
\[ c = 20 \]

The length is 20 ft.

Your Turn

a. Find the length of the hypotenuse of the right triangle.

\[ c^2 = 9^2 + 40^2 \]
\[ c^2 = 81 + 1600 \]
\[ c^2 = 1681 \]

Take the square root of each side.

\[ c = \sqrt{1681} \]
\[ c = 41 \]

The length is 41 in.
The lengths of three sides of a triangle are 4, 5, and 6 meters. Determine whether this triangle is a right triangle.

Since the longest side is meters, use as , the measure of the hypotenuse.

\[ c^2 = a^2 + b^2 \]  
\[ 6^2 = 4^2 + 5^2 \]  
\[ 36 = 16 + 25 \]  
\[ 36 = 41 \]

Since , the triangle a right triangle.

**Your Turn** The lengths of three sides of a triangle are 5, 12, and 13 yards. Determine whether this triangle is a right triangle.

**Theorem 6-10  Converse of the Pythagorean Theorem**

If is the measure of the longest side of a triangle, and are the lengths of the other two sides, and , then the triangle is a right triangle.

**EXAMPLE**

The lengths of three sides of a triangle are 5, 12, and 13 yards. Determine whether this triangle is a right triangle.

Since the longest side is meters, use as , the measure of the hypotenuse.

\[ c^2 = a^2 + b^2 \]  
\[ 13^2 = 5^2 + 12^2 \]  
\[ 169 = 25 + 144 \]  
\[ 169 = 169 \]

Since , the triangle a right triangle.

**Homework Assignment**

Page(s):  
Exercises: 
Use the Distance Formula to find the distance between $A(6, 2)$ and $B(4, -4)$. Round to the nearest tenth, if necessary.

Use the Distance Formula. Replace $(x_1, y_1)$ with $(6, 2)$ and $(x_2, y_2)$ with $(4, -4)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance Formula

$$AB = \sqrt{(4 - 6)^2 + (-4 - 2)^2}$$

Substitution

$$AB = \sqrt{(-2)^2 + (-6)^2}$$

$$AB = \sqrt{4 + 36}$$

$$AB = \sqrt{40}$$

$$AB \approx 6.3$$

Your Turn

a. Use the Distance Formula to find the distance between $M(2, 2)$ and $N(-6, -4)$. Round to the nearest tenth, if necessary.
b. Determine whether \( \triangle TRI \) with vertices \( T(-4, 1), R(2, 5), \) and \( I(2, -2) \) is isosceles.

EXAMPLE

2 Akio took a ride in a hot-air balloon. The flight began 4 miles north of his house. The balloon landed 3 miles south and 2 miles east of his house. If the balloon traveled in a straight line between the starting and ending points of the flight, what was the length of Akio’s balloon ride?

Suppose Akio’s house is located at the origin (0, 0). If the balloon ride began 4 miles north of his house, it began at \((x_1, y_1)\), or \((0, 4)\). Since the balloon landed 3 miles south and 2 miles east of his house, it landed at \((x_2, y_2)\) at \((2, -3)\). Use the Distance Formula to find the length of the balloon ride.

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
= \sqrt{(2 - 0)^2 + (-3 - 4)^2} \\
= \sqrt{2^2 + (-7)^2} \\
= \sqrt{4 + 49} = \text{ miles.}
\]

Akio’s balloon ride was approximately \( \) miles.

Your Turn Marcelle went to a friend’s house to complete a homework project after school instead of going directly home. The school lies 2 blocks north of her home. Her friend’s house is located 3 blocks west and 1 block north of her home. If Marcelle traveled in a straight path from school to her friend’s home, what was the length of her walk?
**6-1 Medians**

Complete the sentence.

1. The midpoint of a side of a triangle and the vertex of the opposite angle are endpoints of a __________.

2. A triangle’s medians are __________ at the centroid.

3. In $\triangle ABC$, $\overline{BD}$ is a median and $BD = 6$. What is $BE$?

**6-2 Altitudes and Perpendicular Bisectors**

For the triangles shown, state whether $AB$ is an altitude, a perpendicular bisector, both, or neither.

4. __________

5. __________

6. __________
6-3 Angle Bisectors of Triangles

7. In \( \triangle JKL \), \( KH \) bisects \( \angle JKL \). If \( m \angle 1 = 12 \), find \( m \angle JKL \).

8. What is the value of \( x \) so that \( BD \) is an angle bisector?

6-4 Isosceles Triangles

Indicate whether the statement is true or false.

9. The vertex angle of an isosceles triangle is opposite one of the congruent sides.

10. An isosceles triangle must be equiangular.

For each triangle, find the values of the variables.

11.

12.
Determine whether each pair of right triangles is congruent by \( LL, HA, LA, \text{ or } HL \). If it is not possible to prove that they are congruent, write \( not possible \).

13. \[ \text{Diagram of two right triangles} \]

14. \[ \text{Diagram of a right triangle} \]

The Pythagorean Theorem

Find the missing measure in each right triangle. Round to the nearest tenth, if necessary.

15. \[ \text{Diagram of a right triangle with sides 5, c, and 8} \]

16. \[ \text{Diagram of a right triangle with sides 12, 10, and a} \]

Distance on the Coordinate Plane

Use the Distance Formula to find the distance between each pair of points. Round to the nearest tenth, if necessary.

17. \( G(-3, 1), H(4, 5) \)

18. \( R(-1, 2), S(5, -6) \)

19. \( A(12, 0), B(0, 5) \)

20. Andre walked 2 blocks west of his home to school. After school, he walked to the store which is 1 block east and 1 block north of his home. About how far apart are the school and the store?
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
  • You are probably ready for the Chapter Test.
  • You may want to take the Chapter 6 Practice Test on page 271 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
  • You should complete the Chapter 6 Study Guide and Review on pages 268–270 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 6 Practice Test on page 271.

☐ I asked for help from someone else to complete the review of all or most lessons.
  • You should review the examples and concepts in your Study Notebook and Chapter 6 Foldable.
  • Then complete the Chapter 6 Study Guide and Review on pages 268–270 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 6 Practice Test on page 271.

Visit geomconcepts.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 6.

Student Signature

Parent/Guardian Signature

Teacher Signature
Triangle Inequalities

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

**NOTE-TAKING TIP:** When you take notes, define new vocabulary words, describe new ideas, and write examples that help you remember the meanings of the words and ideas.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 7. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>exterior angle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>inequality</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[IN-ee-KWAL-a-tee]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>remote interior angles</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
WHAT YOU'LL LEARN
• Apply inequalities to segment and angle measurements.

BUILD YOUR VOCABULARY (page 130)
Statements that contain the symbols $\leq$ or $\geq$ compare quantities or measures that do not have the same value and are called inequalities.

POSTULATE 7-1  Comparison Property
For any two real numbers $a$ and $b$, exactly one of the following statements is true: $a < b$, $a = b$, or $a > b$.

EXAMPLE
Refer to the number line and replace $\circ$ in $DR \cdot LN$ with $<$, $>$, or $\leq$ to make a true sentence.

$DR \cdot LN$

$-0 \circ -2$

$-3 \circ -4$

$3 \cdot 4$

$3 \geq 4$

YOUR TURN Refer to the number line and replace $\circ$ in $PR \cdot QS$ with $<$, $>$, or $\geq$ to make a true sentence.

THEOREM 7-1
If point $C$ is between points $A$ and $B$, and $A$, $C$, and $B$ are collinear, then $AB > AC$ and $AB > CB$.

THEOREM 7-2
If $EP$ is between $ED$ and $EF$, then $\angle DEF > \angle DEP$ and $\angle DEF > \angle PEF$. 

7–1 Segments, Angles, and Inequalities
Refer to the figure. Determine whether each statement is true or false.

2. \(AB > JK\)

\[AB = \quad \text{and } JK = \quad \]

\[48 > \quad \text{Substitution}\]

This is because \(48\) is greater than \(\quad\).

3. \(m\angle AHC \neq m\angle HKL\)

\[m\angle AHC = \quad \text{and } m\angle HKL = \quad \]

\[45 \neq \quad \text{Substitution}\]

This is because \(45\) is not greater than or equal to \(\quad\).

Your Turn Refer to the figure. Determine whether each statement is true or false.

a. \(XY < XZ\)


b. \(m\angle XYZ < m\angle ZXY\)
In the figure, $m\angle C > m\angle A$. If each of these measures were divided by 5, would the inequality still be true?

Replace $m\angle C$ with [ ] and $m\angle A$ with [ ].

Divide each side by [ ]

The inequality still holds [ ] because [ ] is greater than [ ].

Your Turn: In $\triangle XYZ$, $m\angle X > m\angle Z$. If each of these measures doubled, would this inequality still hold true?
Exterior Angle Theorem

SOL G.9 The student will use measures of interior and exterior angles of polygons to solve problems. Tessellations and tiling problems will be used to make connections to art, construction, and nature.

**WHAT YOU’LL LEARN**

- Identify exterior angles and remote interior angles of a triangle and use the Exterior Angle Theorem.

**BUILD YOUR VOCABULARY** (page 130)

An **exterior angle** of a triangle is an angle that forms a pair with one of the angles of the triangle.

**Remote interior angles** of a triangle are the angles that do not form a linear pair with the angle.

**EXAMPLE**

1. Name the remote interior angles with respect to $\angle 4$.

   Angle $\underline{1}$ forms a $\underline{\text{linear pair}}$ with $\angle 2$. Therefore, $\underline{\angle 2}$ and $\underline{\angle 3}$ are remote $\underline{\text{angles}}$ with respect to $\angle 4$.

**Your Turn** Name the remote interior angles with respect to $\angle 2$.

**Theorem 7-3 Exterior Angle Theorem**

The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.
2 In the figure, if $m\angle 1 = 145$ and $m\angle 5 = 82$, what is $m\angle 3$?

$$m\angle 1 = m\angle 5 + m\angle 3$$

Exterior Angle Theorem

$$145 = \phantom{m\angle 3} + m\angle 3$$

Replace $m\angle 1$ with 145 and $m\angle 5$ with 82.

$$145 - \phantom{m\angle 3} = 82 + m\angle 3 - \phantom{m\angle 3}$$

Subtract from each side.

$$\phantom{m\angle 3} = m\angle 3$$

3 In the figure, if $m\angle 6 = 8x$, $m\angle 3 = 12$, and $m\angle 2 = 4(x + 5)$, find the value of $x$.

$$m\angle 6 = m\angle 3 + m\angle 2$$

Exterior Angle Theorem

$$\phantom{8x} = \phantom{12} + 4(x + 5)$$

Replace $m\angle 6$ with $8x$, $m\angle 3$ with 12 and $m\angle 2$ with $4(x + 5)$.

$$8x = 12 + \phantom{4(x + 5)} + \phantom{4(x + 5)}$$

Combine terms.

$$8x = \phantom{4(x + 5)} + 4x$$

Subtract from each side.

$$8x - \phantom{4(x + 5)} = 32 + 4x - \phantom{4(x + 5)}$$

$$\phantom{4(x + 5)} = 32$$

Divide each side by $4x$.
REMEMBER IT

The measures of the angles in any triangle have a sum of 180 degrees.

Your Turn

a. Find the measure of $\angle 1$ in the figure.

b. In the figure, if $m\angle 6 = 10x + 3$, $m\angle 3 = 6x - 6$, and $m\angle 12 = 49$, find the value of $x$.

EXAMPLE

4 Name the two angles in $\triangle CDE$ that have measures less than 82.

The measure of the exterior angle with respect to $\angle 1$ is $\boxed{\text{ }}$. Angles $\boxed{\text{ }}$ and $\boxed{\text{ }}$ are its remote interior angles. By Theorem $\boxed{\text{ }}$, $82 > m\angle \boxed{\text{ }}$ and $82 > m\angle \boxed{\text{ }}$. Therefore, $\boxed{\text{ }}$ and $\boxed{\text{ }}$ have measures less than 82.

Your Turn

Name the two angles in $\triangle JKL$ that have measures less than 117.

Theorem 7-5

If a triangle has one right angle, then the other two angles must be acute.
In $\triangle KLM$, list the angles in order from least to greatest measure.

Write the segment measures in order from greatest to least. Then, use Theorem to write the measures of the angles opposite those sides in the same order.

$$KM < KL < LM$$

$$m\angle < m\angle < m\angle$$

Therefore, the angles in order from least to greatest are $\angle$, $\angle$, and $\angle$.

In $\triangle QPS$, list the angles in order from least to greatest measure.
Identify the side of $\triangle KLM$ with the greatest measure.

Write the angle measures in order from least to __________.

Then, use Theorem to write the measures of the sides opposite those angles in the same order.

\[ m\angle M < m\angle L < m\angle K \]

Therefore, __________ has the greatest measure.

Your Turn In $\triangle XYZ$, list the sides in order from least to greatest measure.

---

**Theorem 7-8**

In a right triangle, the hypotenuse is the side with the greatest measure.
Determine if the three numbers can be the measures of a triangle.

6, 7, 9

All possible cases true. Sides with these measures form a triangle.

1, 7, 8

All possible cases true. Sides with these measures form a triangle.

Your Turn Determine if the three numbers can be the measures of the sides of a triangle.

a. 15, 40, 19

b. 4, 18, 21
What are the greatest and least possible whole-number measures for a side of a triangle whose other two sides measure 4 feet and 6 feet?

Let \( x \) be the measure of the third side of the triangle. \( x \) is greater than the difference of the measures of the two other sides.

\[
x > 6 - \\
x > \\
\]

\( x \) is less than the sum of the measures of the two other sides.

\[
x < 6 + \\
x < \\
\]

Therefore, \( 6 < x < 10 \).

If the measures of two sides of a triangle are 12 meters and 14 meters, find the range of possible measures of the third side.

Let \( x \) be the measure of the third side of the triangle. \( x \) is greater than the difference of the measures of the two other sides.

\[
x > 14 - \\
x > \\
\]

\( x \) is less than the sum of the measures of the two other sides.

\[
x < 14 + \\
x < \\
\]

Therefore, \( 2 < x < 26 \).
Your Turn

a. What are the greatest and least possible whole-number measures for a side of a triangle whose other two sides measure 23 cm and 29 cm?

b. If the measures of two sides of a triangle are 11 inches and 3 inches, find the range of possible measures of the third side.
BRINGING IT ALL TOGETHER

STUDY GUIDE

<table>
<thead>
<tr>
<th>FOLDABLES</th>
<th>VOCABULARY</th>
<th>BUILD YOUR VOCABULARY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use your Chapter 7 Foldable to help you study for your chapter test.</td>
<td>To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 7, go to: <a href="http://www.glencoe.com/sec/math/t_resources/free/index.php">www.glencoe.com/sec/math/t_resources/free/index.php</a></td>
<td>You can use your completed Vocabulary Builder (page 130) to help you solve the puzzle.</td>
</tr>
</tbody>
</table>

7-1 Segments, Angles, and Inequalities

Replace \( \cdot \) with \(<\), >, or = to make a true sentence.

1. \( JK \cdot KX \)
2. \( LM \cdot JL \)
3. \( KM \cdot JL \)
4. \( KL \cdot XM \)

5. \( m\angle BCD \cdot m\angle BDE \)
6. \( m\angle CBE \cdot m\angle EDC \)

7-2 Exterior Angle Theorem

7. Name the remote interior angles of \( \triangle ABC \) with respect to \( \angle 5 \).

8. \( BD \perp AC \) and \( m\angle 15 = 139 \).
   What is \( m\angle 10 \)?

9. If \( m\angle 1 = 19x \), \( m\angle 16 = 6x \), and \( m\angle DAB = 91 \), find the value of \( x \).
7-3 Inequalities Within a Triangle

In each triangle, list the angles from least to greatest.

10.  
\[ \angle A \] 9, 14, 8

11.  
\[ \angle Z \] 8.5, 10.1, 7.6

In each triangle, list the sides measuring least to greatest.

12.  
\[ \overline{BC} \] 82, 46, 23

13.  
\[ \overline{XZ} \] 96, 16, 9

7-4 Triangle Inequality Theorem

Determine if the numbers given can be measures of the sides of a triangle.

14. 7.7, 16.8, 11.3

15. 36, 12, 28

16. 7, 9, 16

Find the range of possible values for the third side of the triangle.

17. 16, 7

18. 12, 10

19. 5, 9
Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
   • You are probably ready for the Chapter Test.
   • You may want to take the Chapter 7 Practice Test on page 305 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
   • You should complete the Chapter 7 Study Guide and Review on pages 302–304 of your textbook.
   • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   • You may also want to take the Chapter 7 Practice Test on page 305.

☐ I asked for help from someone else to complete the review of all or most lessons.
   • You should review the examples and concepts in your Study Notebook and Chapter 7 Foldable.
   • Then complete the Chapter 7 Study Guide and Review on pages 302–304 of your textbook.
   • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   • You may also want to take the Chapter 7 Practice Test on page 305.

Student Signature

Parent/Guardian Signature

Teacher Signature
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

**NOTE-TAKING TIP:** When you read and learn new concepts, help yourself remember these concepts by taking notes, writing definitions and explanations, and drawing models as needed.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 8. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>base angles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>bases</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>consecutive</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[con-SEK-yoo-tiv]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>diagonals</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>isosceles trapeziod</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>kite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>legs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>median</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>-----------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>midsegment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>nonconsecutive</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>parallelogram</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>quadrilateral</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rectangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rhombus [ROM-bus]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>square</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>trapezoid [TRAP-a-ZOYD]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Refer to quadrilateral \( \text{DEFG} \).

1. **Name all pairs of consecutive angles.**
   - \( \angle D \) and \( \angle G \), \( \angle G \) and \( \angle F \), \( \angle F \) and \( \angle E \), and \( \angle E \) and \( \angle D \) are consecutive angles.

2. **Name all pairs of nonconsecutive vertices.**
   - \( D \) and \( \square \) are nonconsecutive vertices. \( \square \) and \( G \) are nonconsecutive vertices.

3. **Name all pairs of consecutive sides.**
   - \( \overline{DG} \) and \( \square \), \( \overline{FG} \) and \( \square \), \( \overline{EF} \) and \( \square \), and \( \overline{DE} \) and \( \square \) are pairs of consecutive sides.
Find the missing measure if three of the four angle measures in quadrilateral $ABCD$ are 90, 120, and 40.

$$m\angle A + m\angle B + m\angle C + m\angle D = 360 \quad \text{Theorem 8-1}$$

Substitution

$$250 + m\angle D - 250 = 360 - 250 \quad \text{Subtract.}$$

$$m\angle D =$$

Your Turn  Find the missing measure if three of the four angle measures in quadrilateral $RMSQ$ are 115, 75, and 50.
Parallelograms

SOL G.8.a The student will investigate and identify properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles, and diagonals.

**WHAT YOU’LL LEARN**
- Identify and use the properties of parallelograms.

**BUILD YOUR VOCABULARY** (page 147)

A parallelogram is a \[ \underline{\text{rectangle}} \] with two pairs of \[ \underline{\text{opposite sides}} \].

**THEOREMS**
- **Theorem 8-2**
  Opposite angles of a parallelogram are congruent.
- **Theorem 8-3**
  Opposite sides of a parallelogram are congruent.
- **Theorem 8-4**
  The consecutive angles of a parallelogram are supplementary.

**EXAMPLES**

In parallelogram \( KLMN \), \( KL = 23 \), \( KN = 15 \), and \( m\angle K = 105^\circ \).

1. **Find \( LM \) and \( MN \).**
   
   \[ KL \parallel MN \text{ and } KN \parallel LM \]
   
   \[ KL = \underline{\text{23}} \text{ and } KN = \underline{\text{15}} \]
   
   **Definition of congruent segments**
   
   \[ MN = \underline{\text{23}} \text{ and } LM = \underline{\text{15}} \]
   
   **Replace \( KL \) with \( LM \) and \( KN \) with \( MN \).**

2. **Find \( m\angle M \).**

   \[ \angle M \equiv \angle K \]
   
   **Theorem 8-2**
   
   \[ m\angle M = \underline{\text{m}\angle K} \]
   
   **Definition of congruent angles**
   
   \[ m\angle M = \underline{\text{105}} \]
   
   **Replace \( m\angle K \) with \( m\angle M \).**
3 Find \( m\angle L \).

\[
m\angle L + m\angle K = 180 \quad \text{Theorem 8-4}
\]

\[
m\angle L + \underline{105} = 180 \quad \text{Replace } m\angle K \text{ with } \underline{105}.
\]

\[
m\angle L + 105 - 105 = 180 - 105 \quad \text{Subtract.}
\]

\[
m\angle L = \underline{75}.
\]

**Your Turn** In parallelogram \( ABCD \), \( AB = 8 \), \( BC = 3 \), and \( m\angle C = 115 \).

a. Find \( AD \) and \( CD \).

b. Find \( m\angle A \).

c. Find \( m\angle B \).

**Theorem 8-5**
The diagonals of a parallelogram bisect each other.

**Theorem 8-6**
A diagonal of a parallelogram separates it into two congruent triangles.

4 In parallelogram \( PQRS \), if \( PR = 32 \), find \( PL \).

Theorem 8-5 states that the diagonals of a parallelogram bisect each other. Therefore, \( PL \cong LR \) or \( PL = \frac{1}{2}(PR) \).

\[
PL = \frac{1}{2}(PR)
\]

\[
PL = \frac{1}{2}(32) \quad \text{Replace } PR \text{ with } 32.
\]
Tests for Parallelograms

**Theorem 8-7**
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

**Example 1**
In quadrilateral $WXYZ$, if $\triangle WYZ \cong \triangle YWX$, how could you prove that $WXYZ$ is a parallelogram?

Show that both pairs of opposite sides are congruent.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle WYX \cong \triangle YWX$</td>
<td>1. Given</td>
</tr>
<tr>
<td>$WX \cong YZ$</td>
<td>2.</td>
</tr>
<tr>
<td>$WZ \cong YX$</td>
<td>3. CPCTC</td>
</tr>
<tr>
<td>$WXYZ$ is a parallelogram.</td>
<td>4.</td>
</tr>
</tbody>
</table>

**Your Turn**
In quadrilateral $ABCD$, $\angle CAB \cong \angle ACD$ and $AB \cong CD$. Show that $ABCD$ is a parallelogram by providing a reason for each step.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\angle CAB \cong \angle ACD$</td>
<td>1. Given</td>
</tr>
<tr>
<td>$AB \cong CD$</td>
<td>2. Given</td>
</tr>
<tr>
<td>$AC \cong AC$</td>
<td>3.</td>
</tr>
<tr>
<td>$\triangle CAB \cong \triangle ACD$</td>
<td>4. SAS</td>
</tr>
<tr>
<td>$AD \cong BC$</td>
<td>5.</td>
</tr>
<tr>
<td>$ABCD$ is a parallelogram.</td>
<td>6.</td>
</tr>
</tbody>
</table>
Determine whether each quadrilateral is a parallelogram. If the figure is a parallelogram, give a reason for your answer.

2. The figure has one pair of opposite sides that are and congruent. Therefore, the quadrilateral is a parallelogram by Theorem 8-8.

3. One pair of opposite sides is congruent but . The other pair of opposite sides is but not congruent. Therefore, the quadrilateral is not a parallelogram.

Your Turn Determine whether the figure is a parallelogram. Justify your answer.

a. 

b. 

Theorem 8-8
If one pair of opposite sides of a quadrilateral is parallel and congruent, then the quadrilateral is a parallelogram.

Theorem 8-9
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Write It Explain how alternate interior angles could be used to show that opposite angles in a parallelogram are congruent.

Remember It There is usually more than one way to prove a statement.
Identify the parallelogram shown.
The parallelogram has four sides and right angles. It is a.

Your Turn Identify the parallelogram shown.

Theorem 8-10
The diagonals of a rectangle are congruent.
Theorem 8-11
The diagonals of a rhombus are perpendicular.
Theorem 8-12
Each diagonal of a rhombus bisects a pair of opposite angles.
Refer to rhombus ABCD.

2 Which angles are congruent to $\angle 1$?
Theorem 8-12 states the diagonals of a rhombus opposite $\angle 1$. Therefore, $\angle 2$, $\angle 3$, and $\angle 6$.

3 If $m\angle 7 = 35$, find $m\angle ADC$.
Theorem 8-12 states the diagonals of a rhombus bisect angles. Therefore, $m\angle 7 = \frac{1}{2}(m\angle ADC)$.

$\frac{1}{2}(m\angle ADC) \cdot \frac{1}{2} = m\angle ADC$.
Multiply each side.

Write It
Explain how squares can be rhombi, rectangles, and parallelograms.

Your Turn Refer to the figure.

a. Which angles are congruent to $\angle PSQ$ in square $PQRS$?

b. If $AP = 7$, find $QS$.

Homework Assignment
Page(s):
Exercises:
In trapezoid $ABCD$, name the bases, legs, and the base angles.

**Bases:** $\overline{AB}$ and $\overline{DC}$ are parallel segments.

**Legs:** $\overline{AD}$ and $\overline{BC}$ are nonparallel segments.

**Base Angles:** $\angle A$ and $\angle D$ form one pair of base angles, while $\angle C$ and $\angle B$ are the other pair of base angles.

**Your Turn** In trapezoid $WXYZ$, name the bases, legs, and the base angles.
The median of a trapezoid is the segment that joins the bases of the legs.

Another name for the median is the midsegment.

If the legs of the trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**.

**Theorem 8-13**
The median of a trapezoid is parallel to the bases, and the length of the median equals one-half the sum of the lengths of the bases.

**Example 2**
Find the length of the median $KL$ in trapezoid $EFGH$ if $EF = 35$ and $GH = 40$.

$$KL = \frac{1}{2}(EF + GH)$$  \hspace{1cm} \text{Theorem 8-13}$$KL = \frac{1}{2}(35 + 40)$$  \hspace{1cm} \text{Replace } EF \text{ and } GH.$$KL = \frac{1}{2}(75)$$ or $$KL = 37.5$$

**Your Turn**
Find the length of the median $NO$ in trapezoid $JKLM$ if $JK = 22$ and $LM = 26$.

The **Theorem 8-14**
Each pair of base angles in an isosceles trapezoid is congruent.
The measure of one angle in an isosceles trapezoid is 55. Find the measures of the other three angles.

Let \( \angle 1 \) be the given angle, and let \( \angle 2 \) be the base angle congruent to \( \angle 1 \).

\[ \angle 2 \cong \angle 1 \]  
**Theorem 8-14**

\[ m\angle 2 = \]  

\[ m\angle 2 = \]  
Replace \( m\angle 1 \) by 55.

Let \( \angle 3 \) and \( \angle 4 \) be the other pair of base angles, with \( \angle 3 \) adjacent to \( \angle 1 \).

\[ m\angle 3 + m\angle 1 = \]  
Consecutive interior angles are supplementary.

\[ m\angle 3 + \]  
Replace \( m\angle 1 \) with .

\[ m\angle 3 + 55 - \]  
Subtract 55 from each side.

\[ m\angle 3 = \]  

Since \( \angle 3 \) and \( \angle 4 \) are congruent base angles, \( m\angle 4 = \).

The measures of the three missing angles are , , and .

**Your Turn** The measure of one angle in an isosceles trapezoid is 76. Find the measures of the other three angles.
**8-1 Quadrilaterals**

1. Name the side opposite \( WZ \).  

2. Name two diagonals.  

3. Name the vertex opposite \( Z \).  

4. Name all consecutive sides.  

5. Find \( m \angle X \) and \( m \angle Y \).  

**8-2 Parallelograms**

Given that \( JKLM \) is a parallelogram, find the missing measures.

6. \( m \angle L \)  
7. \( m \angle J \)  
8. \( LM \)  
9. \( m \angle K \)  
10. If the measure of one angle of parallelogram \( PQRS \) is 79, what are the measures of the other three interior angles?
State whether each figure is a parallelogram. Justify your reason.

11. [Diagram of a square]

12. [Diagram of a trapezoid]

13. Explain why quadrilateral $ABCD$ is a parallelogram.

Underline the best term to complete the statement.

14. A parallelogram with four congruent sides is a [rhombus/rectangle].

Identify each figure with as many terms as possible. Indicate if no term applies.

**Quadrilateral** | **Parallelogram** | **Square** | **Rhombus** | **Rectangle**

15. [Diagram of a trapezoid]

16. [Diagram of a square]
8-5 Trapezoids

Complete each statement.

17. The segment that joins the midpoints of each leg of a trapezoid is the ______.

18. A ______ is a quadrilateral with exactly one pair of parallel sides.

19. The nonparallel sides of a trapezoid are its ______.

20. The parallel sides of a trapezoid are its ______.

Refer to trapezoid $ABCD$ with median $JK$. Name each of the following.

21. bases

22. legs

23. base angle pairs

24. If $AB = 29$ and $DC = 23$, what is $JK$?

25. If $AD = 18$, find $JD$.

26. If $WXYZ$ is an isosceles trapezoid and one base angle measures 66, what are the remaining angle measures?
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
   • You are probably ready for the Chapter Test.
   • You may want to take the Chapter 8 Practice Test on page 345 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
   • You should complete the Chapter 8 Study Guide and Review on pages 342–344 of your textbook.
   • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   • You may also want to take the Chapter 8 Practice Test on page 345.

☐ I asked for help from someone else to complete the review of all or most lessons.
   • You should review the examples and concepts in your Study Notebook and Chapter 8 Foldable.
   • Then complete the Chapter 8 Study Guide and Review on pages 342–344 of your textbook.
   • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   • You may also want to take the Chapter 8 Practice Test on page 345.

Student Signature  Parent/Guardian Signature

Teacher Signature
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

**Begin with a sheet of notebook paper.**

**STEP 1**  
Fold  
Fold lengthwise to the holes.

**STEP 2**  
Cut  
Cut along the top line and then cut 10 tabs.

**STEP 3**  
Label  
Label each tab with important terms.  
Store the Foldable in a 3-ring binder.

**NOTE-TAKING TIP:** You can design visuals such as graphs, diagrams, pictures, charts, and concept maps to help you organize information so that you can remember what you are learning.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 9. 
As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>cross products</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>extremes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>golden ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>means</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>polygon [PA-lee-gon]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>--------------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>proportion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[pro-POR-shun]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[RAY-she-oh]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>scale drawing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>scale factor</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sides</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>similar polygons</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A comparison of \( \frac{75}{400} \) numbers by division is called a ratio.

Write each ratio in simplest form.

1. \( \frac{75}{400} \)

   \[
   \frac{75}{400} = \frac{75 \div \square}{400 \div \square} = \square
   \]

   Divide the numerator and denominator by \( \square \).

2. 24 inches to 3 feet

   The units of measure must be the same in a ratio. There are \( \square \) inches in one foot, so 24 inches equals \( \square \) feet.

   The ratio is \( \square \).

Your Turn

Write each ratio in simplest form.

a. \( \frac{169}{39} \)

b. 30 minutes to \( 2 \frac{1}{2} \) hours
An equation that shows two equivalent ratios is a proportion.

The cross products are the product of the and the product of the.

In a proportion, the of the first ratio and the of the second ratio are the extremes.

In a proportion, the of the first ratio and the of the second ratio are the means.

**Theorem 9-1 Property of Proportions**
For numbers $a$ and $c$ and any nonzero numbers $b$ and $d$, if \( \frac{a}{b} = \frac{c}{d} \), then $ad = bc$. Conversely, if $ad = bc$, then \( \frac{a}{b} = \frac{c}{d} \).

**EXAMPLE**

3 Solve \( \frac{24}{30} = \frac{6x + 4}{35} \).

\[
\frac{24}{30} = \frac{6x + 4}{35}
\]

\[
(35) = (6x + 4)
\]

\[
840 = 180x + 120
\]

Subtract 120 from each side.

\[
720 = 180x
\]

Divide each side by 180.

\[
\frac{720}{180} = \frac{180x}{180}
\]

\[
4 = x
\]
**Remember It**
The denominator can never equal zero.

**Your Turn** Solve \( \frac{15}{x - 1} = \frac{4}{5} \).

**Example**

The ratio of children to adults at a holiday parade is 2.5 to 1. If there are 1440 adults at the parade, how many children are there?

\[
\begin{align*}
\text{children} : 2.5 & : x \\
1 : 1440 & : \text{adults}
\end{align*}
\]

\[
2.5(1440) = x
\]

**Your Turn** The ratio of Republicans to Democrats casting their votes in the local election was 73 to 27. If 135 Democrats voted, how many Republicans cast their votes?
Similar Polygons

**SOL G.14.a** The student will use proportional reasoning to solve practical problems, given similar geometric objects.

**WHAT YOU’LL LEARN**
- Identify similar polygons.

**BUILD YOUR VOCABULARY** (pages 164–165)

A polygon is a figure in a plane formed by segments called sides.

Similar polygons are the same but not necessarily the same.

**KEY CONCEPT**

Similar Polygons: Two polygons are similar if and only if their corresponding angles are congruent and the measures of their corresponding sides are proportional.

**EXAMPLE**

1. Determine if the polygons are similar. Justify your answer.

The polygons are congruent. The corresponding angles are congruent and

\[
\frac{2}{3} = \frac{3}{3} = \frac{2}{3} = \frac{7.5}{7.5}.
\]

**Your Turn** Determine if the polygons are similar. Justify your answer.

\[
\frac{30}{40} = \frac{40}{50} = \frac{20}{40} \quad \text{and} \quad \frac{40}{40} = \frac{40}{40}.
\]
Find the values of $x$ and $y$ if $\triangle ABC \sim \triangle FED$.

Use the corresponding order of the vertices to write proportions.

\[
\frac{AB}{FE} = \frac{BC}{FD} \quad \text{Definition of similar polygons}
\]

\[
\frac{5}{y} = \frac{x}{8} \quad \text{Substitution}
\]

Write the proportion to solve for $x$.

\[
\frac{x}{15} = \frac{12}{\text{(8)}} \quad \text{(8)}
\]

\[
\text{(8)} = \frac{120}{12x} \quad \text{Divide each side by } \text{(8)}
\]

\[
x = \frac{12}{12}x = \frac{120}{12}x = \frac{120}{12} = \frac{10}{1}
\]

Now write the proportion that can be solved for $y$.

\[
\frac{5}{y} = \frac{8}{\text{(12)}} \quad \text{(12)} = y(8) \quad \text{Cross products}
\]

\[
60 = 8y \quad \text{(8)} = \frac{8y}{8} \quad \text{Divide each side by } \text{(8)}
\]

\[
\frac{60}{8} = \frac{8y}{8} \quad \text{(8)} = y
\]

So, $x = \frac{10}{1}$ and $y = \frac{60}{8}$.
The triangles are similar. Find the values of \( x \) and \( y \).

In the blueprint, 1 inch represents an actual length of 16 feet. Use the blueprint to find the actual dimensions of the dining room.

\[
\frac{1\text{ in.}}{16\text{ ft}} = \frac{\text{blueprint}}{\text{actual}}
\]

\[
x = \frac{1\text{ in.}}{16\text{ ft}} \times \text{blueprint} = \frac{1\text{ in.}}{16\text{ ft}} \times \frac{\text{blueprint}}{\text{actual}}
\]

\[
y = \frac{1\text{ in.}}{16\text{ ft}} \times \text{blueprint} = \frac{1\text{ in.}}{16\text{ ft}} \times \frac{\text{blueprint}}{\text{actual}}
\]

Cross products

The dimensions of the dining room are \( x \) ft by \( y \) ft.

Your Turn Refer to Example 3. Find the dimensions of the kitchen.
Similar Triangles

Postulate 9-1 AA Similarity
If two angles of one triangle are congruent to two corresponding angles of another triangle, then the two triangles are similar.

Theorem 9-2 SSS Similarity
If the measures of the sides of a triangle are proportional to the measures of the corresponding sides of another triangle, then the triangles are congruent.

Theorem 9-3 SAS Similarity
If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and their included angles are congruent, then the triangles are similar.

EXAMPLE

Determine whether the triangles are similar. If so, tell which similarity test is used and write a similarity statement.

Since \( m \angle F = \) \( \boxed{ \quad } \) and \( m \angle H = \) \( \boxed{ \quad } \), \( \triangle FGH \sim \triangle MLK \) by \( \boxed{ \quad } \).

WRITE IT

Why must only two pairs of corresponding angles be congruent for two triangles to be similar rather than three?

Your Turn

Determine whether the triangles are similar. If so, tell which similarity test is used and write a similarity statement.
2 Find the value of $x$.

Since \( \frac{8}{12} = \frac{12}{18} \), the triangles are similar by SAS similarity.

\[
\frac{8}{12} = \frac{14}{x}
\]

Definition of similar polygons

\[
x = (12)(14)
\]

Cross products

\[
8x = 168
\]

Divide each side by

\[
\frac{8x}{8} = \frac{168}{8}
\]

\[
x = 21
\]

Your Turn Find the value of $x$.

\[
\frac{5}{7} = \frac{x}{21}
\]
Example 3 The shadow of a flagpole is 2 meters long at the same time that a person’s shadow is 0.4 meters long. If the person is 1.5 meters tall, how tall is the flagpole?

\[
\frac{\text{flagpole’s shadow}}{\text{person’s shadow}} = \frac{2}{0.4} = \frac{x}{\text{flagpole’s height}}
\]

Cross products

\[
0.4x = 3
\]

Divide.

\[
x = \frac{3}{0.4} = 7.5
\]

The flagpole is 7.5 meters tall.

Your Turn A diseased tree must be cut down before it falls. Which direction the fall is directed depends on the height of the tree. The man who will cut the tree down is 74-in. tall and casts a shadow 60-in. long. If the tree’s shadow measures 20 feet from its base, how tall is the tree?
**Proportional Parts and Triangles**

**SOL G.5.a** The student will investigate and identify congruence and similarity relationships between triangles.

**WHAT YOU’LL LEARN**
- Identify and use the relationships between proportional parts of triangles.

**Theorem 9-4**
If a line is parallel to one side of a triangle and intersects the other two sides, then the triangle formed is similar to the original triangle.

**Example 1**
Using the figure, complete the proportion \( \frac{?}{VW} = \frac{ST}{SW} \).

Since \( VW \parallel RT \), \( \triangle SVW \sim \triangle SRT \).

Therefore, \( \frac{VW}{SW} = \frac{ST}{SW} \).

**Your Turn**
Use the figure to complete the proportion \( \frac{XY}{YA} = \frac{?}{YB} \).

**Example 2**
In the figure, \( MN \parallel KL \). Find the value of \( x \).

\( \triangle JMN \sim \triangle JKL \)

\[
\frac{MN}{KL} = \frac{JN}{JL} \]

**Definition of similar polygons**
Substitution

\[
x = \frac{6}{9x} \]

**Cross products**
Divide each side by 9.
**Your Turn** Find the value of $b$.

Find the value of $x$.

**Theorem 9-5**
If a line is parallel to one side of a triangle and intersects the other two sides, then it separates the sides into segments of proportional lengths.

**EXAMPLE**

In the figure, $\overline{AB} \parallel \overline{DE}$.
Find the value of $x$.

\[
\frac{CE}{EB} = \frac{CD}{DA}
\]

Theorem 9.5

\[
x = \frac{x + 5}{6}
\]

$CE = x$, $EB = 6$,

$CD = x + 5$, $DA = 8$

Cross products

Distributive Property

Subtract $6x$ from each side.

Divide each side by 2.

**Your Turn** Find the value of $a$. 

---

**Homework Assignment**

Page(s):
Exercises:
**WHAT YOU’LL LEARN**

- Use proportions to determine whether lines are parallel to sides of triangles.

**Theorem 9-6**

If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side.

**Example 1**

Determine whether \( \overline{DE} \parallel \overline{BC} \).

Determine whether \( \frac{BD}{DA} \) and \( \frac{CE}{EA} \) form a proportion.

\[
\begin{align*}
\frac{BD}{DA} & \quad ? \quad \frac{CE}{EA} \\
6 & \quad ? \quad 8 \\
(8) & \quad ? \quad (4) 
\end{align*}
\]

Cross products

\[24 = \checkmark\]

Therefore, \( \overline{DE} \parallel \overline{BC} \) by Theorem 9-6.

**Your Turn**

Determine whether \( \overline{HJ} \parallel \overline{KM} \).
Theorem 9-7
If a segment joins the midpoint of two sides of a triangle, then it is parallel to the third side, and its measure equals one-half the measure of the third side.

For Examples 2 and 3, refer to the figure shown.

2. In the figure, $X$, $Y$, and $Z$ are midpoints of the sides of $\triangle UVW$. If $XZ = 7c$, then what does $UW$ equal?

\[
\begin{align*}
XZ &= \frac{1}{2} \text{X}\text{Z} \\
\text{Theorem 9-7} \\
\frac{1}{2} \text{UV} &= \frac{1}{2} \text{UW} \\
\text{Replace XZ with } \text{X}\text{Z}. \\
7c &= \frac{1}{2} \text{UW} \\
\text{Multiply each side by } \frac{1}{2}. \\
\text{UW} &= \text{UW}
\end{align*}
\]

3. In the figure, if $m\angle UYX = d$, then what is $m\angle YWZ$?

By Theorem 9-6, $XY \parallel VW$. Since $XY$ and $VW$ are parallel segments cut by transversal $UW$, $\angle UYX$ and $\angle VW$ are congruent angles.

Therefore, $m\angle YWZ = d$.

Your Turn
$N$, $O$, and $P$ are the midpoints of the sides of $\triangle EFG$.

a. If $EF = 25$, then what does $NP$ equal?

b. If $m\angle EGF = 85$, then what is $m\angle ENO$?
9–6 Proportional Parts and Parallel Lines

**SOL G.14.a** The student will use proportional reasoning to solve practical problems, given similar geometric objects.

**WHAT YOU’LL LEARN**
- Identify and use the relationships between parallel lines and proportional parts.

**THEOREM 9-8**
If three or more parallel lines intersect two transversals, the lines divide the transversals proportionally.

**EXAMPLES**

1. Complete the proportion \( \frac{ST}{RT} = \frac{NP}{?} \).

   Since \( \square \parallel \overline{NS} \parallel \overline{PT} \), the transversals are divided ______________________. Therefore, \( \frac{ST}{RT} = \frac{NP}{?} \).

2. In the figure, \( a \parallel b \parallel c \). Find the value of \( x \).

   \[
   \frac{TS}{SR} = \frac{NM}{x}
   \]

   \[
   \frac{9}{15} = \frac{x}{x}
   \]

   \[
   9(x) = 15\left( \phantom{x} \right)
   \]

   \[
   9x = \phantom{x}
   \]

   \[
   x = \phantom{x}
   \]

   Cross products

   Divide each side by \( \phantom{x} \).

**REVIEW IT**
What is the definition of a transversal? (Lesson 4-2)
Your Turn

a. Complete the proportion $\frac{ZP}{QP} = \frac{NB}{?}$.

b. In the figure, $a \parallel b \parallel c$. Find the value of $x$.

**Theorem 9-9**

If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.
The perimeter of \( \triangle DEF \) is 90 units, and \( \triangle ABC \sim \triangle DEF \). Find the value of each variable.

\[ \frac{DE}{AB} = \frac{\text{perimeter of } \triangle DEF}{\text{perimeter of } \triangle ABC} \]

Theorem 9-10

\[ \frac{x}{26} = \frac{90}{60} \]

\[ 26 + 10 + 24 = \]

Cross products

\[ 60x = 2340 \]

Divide.

\[ x = \]

Because the triangles are similar, find \( y \) and \( z \).

\[ \frac{DF}{DE} = \frac{AC}{AB} \quad \frac{EF}{DE} = \frac{BC}{AB} \]

\[ \frac{y}{39} = \frac{26}{26} \quad \frac{z}{24} = \frac{24}{26} \]

\[ 26y = 390 \quad 26z = \]

\[ y = \] \[ z = \]
Your Turn  The perimeter of \( \triangle ABC \) is 20 units, and \( \triangle ABC \sim \triangle XYZ \). Find the value of each variable.

The scale factor, also known as the constant of , is the found by comparing the measures of corresponding sides of similar triangles.

**EXAMPLE**

2 Determine the scale factor of \( \triangle ABC \) to \( \triangle DEF \).

\[
\frac{AB}{DE} = \frac{24}{3} = \frac{8}{1}
\]

\[
\frac{BC}{EF} = \frac{30}{2} = \frac{15}{1}
\]

\[
\frac{AC}{DF} = \frac{24}{3} = \frac{8}{1}
\]

The scale factor is .

Your Turn  Determine the scale factor of \( \triangle RST \) to \( \triangle XYZ \).
## Indicate whether the statement is true or false.

1. Every proportion has two cross products.  
   Answer: True

2. A ratio is a comparison of two numbers by division.  
   Answer: True

3. The two cross products of a ratio are the extremes and the means.  
   Answer: True

4. Cross products are always equal in a proportion.  
   Answer: True

5. Simplify $\frac{220}{70}$.  
   Answer: $\frac{220}{70} = 3.14285714286$

6. Solve: $\frac{84}{63} = \frac{12}{11 - x}$  
   Answer: $x = 8$

## Complete the sentence.

7. In measures of corresponding sides are proportional, and corresponding angles are congruent.

8. represent something either too large or too small to be drawn at actual size.
9. Given that the rectangles are similar, find the values of $x$ and $y$ to show similarity.

- $x = 7$
- $y = 4$

9-3
Similar Triangles

Determine whether the pair of triangles is similar. Justify your reasons.

10.

11.

9-4
Proportional Parts and Triangles

Complete the proportions.

12. $\frac{AD}{DE} = \frac{CB}{AE}$

13. $\frac{AE}{EB} = \frac{AD}{CE}$

9-5
Triangles and Parallel Lines

Vertices $A$, $B$, and $C$ are midpoints.

14. $\overline{AC} \parallel \overline{BD}$.

15. If $BC = 6$, then $RT = \overline{AC}$.

16. If $SB = 4$, $AC = \overline{RT}$. 

184  Geometry: Concepts and Applications
9-6

Proportional Parts and Parallel Lines

A tract of land bordering school property was divided into sections for five biology classes to plant gardens. The fences separating the plots are parallel, and the plots’ front measures are shown. The entire back border measures 254 feet. What are the individual border lengths, to the nearest tenth of a foot?

17. $A =$

18. $D =$

19. $E =$

9-7

Perimeters and Similarity

Complete the sentence.

20. The scale factor is also called the constant of _________.

21. Find the scale factor.

$\triangle JKL \sim \triangle MNO$. The perimeter of $\triangle JKL$ is 54. What are the values for the variables?

22. $a =$

23. $b =$

24. $c =$
Check the one that applies. Suggestions to help you study are given with each item.

I completed the review of all or most lessons without using my notes or asking for help.

• You are probably ready for the Chapter Test.
• You may want to take the Chapter 9 Practice Test on page 397 of your textbook as a final check.

I used my Foldable or Study Notebook to complete the review of all or most lessons.

• You should complete the Chapter 9 Study Guide and Review on pages 394–396 of your textbook.
• If you are unsure of any concepts or skills, refer back to the specific lesson(s).
• You may also want to take the Chapter 9 Practice Test on page 397.

I asked for help from someone else to complete the review of all or most lessons.

• You should review the examples and concepts in your Study Notebook and Chapter 9 Foldable.
• Then complete the Chapter 9 Study Guide and Review on pages 394–396 of your textbook.
• If you are unsure of any concepts or skills, refer back to the specific lesson(s).
• You may also want to take the Chapter 9 Practice Test on page 397.
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

**Foldable**

Begin with a sheet of \(8\frac{1}{2}'' \times 11''\) paper.

**STEP 1** Fold
Fold the short side in fourths.

**STEP 2** Draw
Draw lines along the folds and label each column:
- Prefix
- Number of Sides
- Polygon Name
- Figure

**NOTE-TAKING TIP:** When you take notes, it is important to record major concepts and ideas. Refer to your journal when reviewing for tests.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 10. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>altitude</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>apothem</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>center</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>composite figure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>concave</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>convex</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>irregular figure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>line of symmetry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>------------------------------</td>
<td>---------------</td>
<td>------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>line symmetry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>polygonal region</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>regular polygon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>regular tessellation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rotational symmetry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>semi-regular tessellation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>significant digits</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>symmetry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tessellation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tessellation [tes-a-LAY-shun]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>turn symmetry</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Refer to the figure for Examples 1–2.

1. Identify polygon VWXYZ.
   - The polygon has ___ sides. It is a ___.

2. Determine whether the polygon VWXYZ appears to be regular or not regular. If not regular, explain why.
   - The ___ appear to be the same length, and the ___ appear to have the same measure. The polygon is regular.

2. Name two nonconsecutive vertices of polygon VWXYZ.
   - W and Z, W and Y, V and X, V and Y, X and Z are examples of ___ vertices.

Your Turn Refer to the figure for parts a, b, and c.

a. Identify polygon DEFGHIJ by its sides.

b. Determine whether the polygon DEFGHIJ appears to be regular or not regular. If not regular, explain why.

C. Name two nonconsecutive vertices of polygon DEFGHIJ.
Classify each polygon as convex or concave.

a. When all the diagonals are drawn, points lie outside of the polygon. So polygon ABCDEF is __________.

b. Diagonal QS lies outside the polygon, so PQRSTU is __________.

**Your Turn** Classify each polygon as convex or concave.

a. 

b. 

**Answer**

a. Convex

b. Concave
Diagonals and Angle Measure

Refer to the regular pentagon for Examples 1–2.

Find the sum of the measures of the interior angles.

The sum of the measures of the interior angles of a pentagon is $\frac{(n-2)\cdot 180}{n}$.

Find the measure of one interior angle.

Each interior angle of a regular polygon has the same measure. Divide the measure of the measures by the number of angles.

The measure of one interior angle of a regular pentagon is $\frac{180}{5}$.
a. Find the sum of the measures of the interior angles of a regular 15-sided polygon.

b. Find the measure of one interior angle of a regular 15-sided polygon.

Theorem 10-2
In any convex polygon, the sum of the measures of the exterior angles, one at each vertex, is 360.

Find the measure of one exterior angle of a regular octagon.

By Theorem 10-2, the sum of the measures of exterior angles is ___. An octagon has _____ exterior angles.

measure of one exterior angle = \( \frac{360}{8} = \) ___

Your Turn: Find the measure of one exterior angle of a regular 15-sided polygon.
Areas of Polygons

**Postulate 10-1 Area Postulate**
For any polygon and a given unit of measure, there is a unique number $A$ called the measure of the area of the polygon.

**Postulate 10-2**
Congruent polygons have equal areas.

**Postulate 10-3 Area Addition Postulate**
The area of a given polygon equals the sum of the areas of the nonoverlapping polygons that form the given polygon.

**Build Your Vocabulary** (pages 188–189)

Any polygon and its are called a polygonal region.

A composite figure is a figure made from that have been placed together.

**Example**

Find the area of the polygon. Each square represents 1 square centimeter.

Since the area of each square represents one square centimeter, the area of each triangular half square represents 0.5 square centimeter. There are 8 squares and 4 half squares.

$A = 8(1) \text{ cm}^2 + 4(0.5) \text{ cm}^2$

$= \square \text{ cm}^2 + \square \text{ cm}^2$

$= \square \text{ cm}^2$

**Your Turn** Find the area of the polygon. Each square represents 1 square inch.
Estimate the area of the polygon. Each square represents 20 square miles.

Count each square as one unit and each partial square as a half unit regardless of size. There are \( \square \) whole squares and \( \square \) partial squares.

\[
\text{number of squares} = (1) + (0.5)
\]
\[
= \square + \square
\]
\[
= \square
\]

Area \( \approx 20 \times \square \) square miles, or

The area of the polygon is about \( \square \) square miles, or

Your Turn A swimming pool at a resort is shaped as shown on the grid. Each square on the grid represents 16 square meters. Estimate the area of the pool.
Areas of Triangles and Trapezoids

**Theorem 10-3 Area of a Triangle**
If a triangle has an area of \( A \) square units, a base of \( b \) units, and a corresponding altitude of \( h \) units, then \( A = \frac{1}{2}bh \).

**Examples**

Find the area of each triangle.

1. 
   ![Diagram](6 ft 15 ft)
   \[ A = \frac{1}{2}bh \quad \text{Theorem 10-3} \]
   \[ = \frac{1}{2}(\ BOX ) (\ BOX ) \quad \text{Replace } b \text{ with } \ BOX \text{ and } h \text{ with } \ BOX \]
   \[ = \frac{1}{2} \]
   \[ = \]

2. 
   ![Diagram](8 cm 9 cm)
   \[ A = \frac{1}{2}bh \quad \text{Theorem 10-3} \]
   \[ = \frac{1}{2}(\ BOX ) (\ BOX ) \quad \text{Replace } b \text{ with } \ BOX \text{ and } h \text{ with } \ BOX \]
   \[ = \frac{1}{2} \]
   \[ = \]

**Your Turn** Find the area of each triangle.

a. 
   ![Diagram](4 11)
**ORGANIZE IT**
Draw and label the base and altitude for each triangle on your Foldable. In addition, draw a trapezoid as an example of a quadrilateral. Draw and label the bases and altitude for the trapezoid.

**BUILD YOUR VOCABULARY** (page 188)

The altitude of a trapezoid is a segment perpendicular to each base.

**Theorem 10-4 Area of a Trapezoid**
If a trapezoid has an area of $A$ square units, bases of $b_1$ and $b_2$ units, and an altitude of $h$ units, then $A = \frac{1}{2}h(b_1 + b_2)$.

**EXAMPLE**

Find the area of the trapezoid.

$$A = \frac{1}{2}h(b_1 + b_2)$$

Replace $h$ with 6, $b_1$ with 4, and $b_2$ with 18.

**Your Turn** Find the area of the trapezoid.

$$A = \frac{1}{2} \times 6 \times (4 + 18)$$

$$A = 60$$
Areas of Regular Polygons

What You’ll Learn

• Find the areas of regular polygons.

Remember It

Only regular polygons have apothems.

Build Your Vocabulary (page 188)

The center of a regular polygon is an interior point that is equidistant from all sides. The segment drawn from the center and to a side of a regular polygon is an apothem.

Theorem 10-5 Area of a Regular Polygon
If a regular polygon has an area of \( A \) square units, an apothem of \( a \) units, and a perimeter of \( P \) units, then 
\[
A = \frac{1}{2}ap.
\]

Example

A regular octagon has a side length of 9 inches and an apothem that is about 10.9 inches long. Find the area of the octagon.

First, find the perimeter of the octagon.

\[ P = 8s \]

All sides of a regular octagon are congruent.

\[ = 8(9) \text{ or } 72 \]

Replace \( s \) with 9.

Now find the area.

\[ A = \frac{1}{2}ap \]

Theorem 10-5

\[ = \frac{1}{2}(10.9)(72) \]

Replace \( a \) with 10.9 and \( P \) with 72.

The area of the octagon is about \( \text{in}^2 \).

Your Turn

A regular pentagon has a side length of 8 inches and an apothem that is about 5.5 inches long. Find the area of the pentagon.
A regular octagon has a side length of 12 inches and an apothem that is about 14.5 inches long. Find the area of the shaded region of the octagon.

Find the area of the octagon minus the area of the unshaded region.

Area of an octagon:

\[ A = \frac{1}{2}aP \]

Theorem 10-5

\[ = \frac{1}{2}(14.5)(96) \]

Replace \( a \) with 14.5 and \( P \) with 96.

\[ = \text{in}^2 \]

Area of a Triangle:

\[ A = \frac{1}{2}bh \]

Theorem 10-3

\[ = \frac{1}{2}(12)(14.5) \]

Replace \( b \) with 12 and \( h \) with 14.5.

\[ = \text{in}^2 \]

The area of one triangular section is 87 in\(^2\). There are 5 triangular sections in the unshaded region.

The area of the unshaded region is \( 5(87) = \text{in}^2 \).

Subtract the area of the unshaded region from the area of the octagon.

Area of shaded region = \( \text{in}^2 \) or \( \text{in}^2 \)

Your Turn Find the area of the shaded region of the regular hexagon.
Symmetry

Find all lines of symmetry for equilateral triangle $ABC$.

Fold along all possible lines to see if the sides match. There are lines of symmetry along the lines shown in the figure.

Draw all lines of symmetry for regular pentagon $JKXYZ$.

Remember It
Figures that have rotational symmetry do not necessarily have line symmetry.
Which of the figures have rotational symmetry?

a.

The figure can be turned 120° and 240° to look like the original. The figure has symmetry.

b.

The figure must be turned 360° about its center to look like the original. Therefore, it have rotational symmetry.

Your Turn

Which of the figures has rotational symmetry?

a.

b.
Tessellations

Identify the figures used to create each tessellation. Then identify the tessellation as regular, semi-regular, or neither.

1. Only squares are used. A square is a regular polygon. The tessellation is regular.

2. Hexagons are used and there are no gaps in the pattern, but the hexagons are not the same order at every vertex to form a pattern, it is a semi-regular tessellation.

Your Turn Identify the tessellation as regular, semi-regular, or neither.

a. 

b. 

Homework Assignment

Page(s): 
Exercises: 

© Glencoe/McGraw-Hill
Indicate whether the statement is true or false.

1. All the diagonals of a concave polygon lie on the interior. [ ]
2. A regular polygon is both equilateral and equiangular. [ ]

Identify each figure by its sides. Indicate if the polygon appears to be regular or not regular. If not regular, justify your reason.

3. [ ]
4. [ ]

Find the sum of the measures of the interior angles.

5. [ ]
6. [ ]
Find the measure of one interior angle and one exterior angle of the regular polygon.

7. dodecagon

8. decagon

9. The sum of the measures of four exterior angles of a pentagon is 280. What is the measure of the fifth exterior angle?

Indicate whether the statement is true or false.

10. A polygon and its interior are known as a polygonal region.

Find the area of the polygon in square units.

11. 12.

Indicate whether the statement is true or false.

13. The segment perpendicular to the parallel bases of a trapezoid is a median.

Find the area of the triangle or trapezoid.

14. 15.
16. Find the area of a trapezoid whose altitude measures 4.5 cm and has bases measuring 6.2 and 8.8 cm.

17. What is the area of a triangle with base length $6\frac{1}{3}$ in. and height 2 in.?

18. Find the area of a regular 11-sided polygon with each side measuring 7 cm and an apothem length of 11.9 cm.

19. Find the area of the shaded region.

20. When a line is drawn through a figure and makes each half a mirror image of the other, the figure has [line/rotational] symmetry.

21. When a figure looks exactly as it does in its original position after being turned less than 360º around a fixed point, it has [line/rotational] symmetry.

Determine whether the figure has line symmetry, rotational symmetry, both, or neither.

22. 23.

24. 25.

Identify the tessellation as regular, semi-regular, or neither.
Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
   • You are probably ready for the Chapter Test.
   • You may want to take the Chapter 10 Practice Test on page 449 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
   • You should complete the Chapter 10 Study Guide and Review on pages 446–448 of your textbook.
   • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   • You may also want to take the Chapter 10 Practice Test on page 449 of your textbook.

☐ I asked for help from someone else to complete the review of all or most lessons.
   • You should review the examples and concepts in your Study Notebook and Chapter 10 Foldable.
   • Then complete the Chapter 10 Study Guide and Review on pages 446–448 of your textbook.
   • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   • You may also want to take the Chapter 10 Practice Test on page 449 of your textbook.

Visit geomconcepts.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 10.

---

Student Signature  
Parent/Guardian Signature  
Teacher Signature
Circles

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

**Begin with seven sheets of plain paper.**

**STEP 1**
**Draw**
Draw and cut a circle from each sheet. Use a small plate or a CD to outline the circle.

**STEP 2**
**Staple**
Staple the circles together to form a booklet.

**STEP 3**
**Label**
Label the chapter name on the front. Label the inside six pages with the lesson titles.

**NOTE-TAKING TIP:** When you take notes, write concise definitions in your own words. Add examples that illustrate the concepts.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 11. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>adjacent arcs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>arcs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>center</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>central angle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>chord</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>circle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>circumference</td>
<td>[sir-KUM-fur-ents]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>circumscribed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>concentric</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>diameter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>---------------------------------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>experimental probability</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>inscribed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>loci</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>locus</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>major arc</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>minor arc</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pi ($\pi$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>radius</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sector</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>semicircle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>theoretical probability</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Parts of a Circle

**SOL G.10** The student will investigate and solve practical problems involving circles, using properties of . . . , chords, . . . . Problems will include finding arc length and the area of a sector, and may be drawn from applications of architecture, art, and construction.

**WHAT YOU’LL LEARN**

- Identify and use parts of circles.

**BUILD YOUR VOCABULARY** (pages 208–209)

A **circle** is the set of all points in a plane that are a given distance from a given point in the plane, called the **radius** of the circle.

In a circle, all points are **equidistant** from the **center**.

A **radius** is a segment whose endpoints are the **endpoints** of the circle and a **point on the circle**.

A **chord** is a segment whose **endpoints** are on the circle.

A **diameter** is a **segment** that contains the **endpoints** of the circle.

Two circles are **concentric** if they lie in the same plane, have the same **radius**, and have **radii** of different lengths.

**ORGANIZE IT**

Under the tab for Lesson 11-1, draw a circle with a radius, a chord and a diameter. Label each special segment.

**EXAMPLES**

Use circle $P$ to determine whether each statement is true or false.

1. **$RT$** is a diameter of circle $P$.
   
   $\overline{RT}$; $\overline{RT}$ **go through the center** $P$. Therefore, $\overline{RT}$ is not a diameter.

2. **$PS$** is a radius of circle $P$.
   
   $\overline{PS}$; the endpoints of $\overline{PS}$ are on the **circle** $P$ and a point on the circle $S$. Therefore, $\overline{PS}$ is a radius.
**WRITE IT**

Describe the differences between a radius, a diameter, and a chord.

---

**Your Turn** Use circle $T$ to determine whether each statement is true or false.

**a.** $\overline{AB}$ is not a diameter.

---

**b.** $\overline{TD}$ is not a radius.

---

**Theorem 11-1**
All radii of a circle are congruent.

**Theorem 11-2**
The measure of the diameter $d$ of a circle is twice the measure of the radius $r$ of the circle.

---

**EXAMPLE**

3 In circle $R$, $\overline{QT}$ is a diameter. If $QR = 7$, find $QT$.

$QR$ is a radius, and $d = 2r$.

\[
QT = 2(QR) \quad \text{Replace } d \text{ and } r.
\]

\[
QT = 2(7) \quad \text{Replace } QR \text{ with } 7.
\]

\[
QT = 14
\]

**Your Turn** In circle $A$, $\overline{FC}$ is a diameter. If $FC = 25$, find $AB$.

---

**HOMEWORK ASSIGNMENT**

Page(s): 
Exercises:
Arcs and Central Angles

When two sides of an angle meet at the center of a circle, a central angle is formed.

Each side of the central angle intersects a point on the circle, dividing it into lines called arcs.

A minor arc is formed by the intersection of the circle and sides of a central angle with interior degree measure less than 180.

A major arc is the part of the circle in the of the central angle that measures greater than 180.

Semicircles are arcs whose endpoints lie on the diameter of the circle.

Adjacent arcs are arcs of a circle with exactly one point in common.

In circle J, find \( m\widehat{LM} \), \( m\angle KJM \), and \( m\angle LKM \).

\[
\begin{align*}
\widehat{LM} &= m\angle LJM & \text{Measure of minor arc} \\
\angle LJM &= 125 \\
\angle KJM &= m\angle KM & \text{Measure of central angle} \\
m\angle KJM &= 130 & \text{Substitution} \\
m\angle LKM &= 360 - m\angle LJM - m\angle KJM & \text{Measure of major arc} \\
m\angle LKM &= 360 - 125 - 130 \\
m\angle LKM &= 105 & \text{Substitution}
\end{align*}
\]

The degree measure of a minor arc is the degree measure of its central angle.

The degree measure of a major arc is 360 minus the degree measure of its central angle.

The degree measure of a semicircle is 180.

WHAT YOU’LL LEARN

- Identify major arcs, minor arcs, and semicircles and find the measures of arcs and central angles.

BUILD YOUR VOCABULARY

(pages 208–209)

What You’ll Learn

- Identify major arcs, minor arcs, and semicircles and find the measures of arcs and central angles.

Key Concepts

The degree measure of a minor arc is the degree measure of its central angle.

The degree measure of a major arc is 360 minus the degree measure of its central angle.

The degree measure of a semicircle is 180.

Example

In circle J, find \( m\widehat{LM} \), \( m\angle KJM \), and \( m\angle LKM \).

\[
\begin{align*}
\widehat{LM} &= m\angle LJM & \text{Measure of minor arc} \\
\angle LJM &= 125 \\
\angle KJM &= m\angle KM & \text{Measure of central angle} \\
m\angle KJM &= 130 & \text{Substitution} \\
m\angle LKM &= 360 - m\angle LJM - m\angle KJM & \text{Measure of major arc} \\
m\angle LKM &= 360 - 125 - 130 \\
m\angle LKM &= 105 & \text{Substitution}
\end{align*}
\]

Under the tab for Lesson 11-2, draw a circle with a central angle. Label the central angle, the major and minor arcs and give examples of degree measurements for each.
In circle \( A \), \( \overline{CE} \) is a diameter. Find \( m_{BC} \), \( m_{BE} \), and \( m_{BDE} \).

\[
\begin{align*}
\overline{BC} &= m\angle BAC \\
\overline{BC} &= m_{BC} \\
\overline{BE} + \overline{BC} &= m_{EBC} \\
\overline{BE} + m_{BC} &= 180 \\
m_{BE} &= 132 \\
m_{BDE} &= m_{BE} - m_{BE} \\
m_{BDE} &= 132 - 132 \\
m_{BDE} &= 0
\end{align*}
\]

Your Turn In circle \( X \), \( m\angle AXB = 70 \), \( m\overline{DC} = 45 \), and \( \overline{BE} \) and \( \overline{AD} \) are diameters.

a. Find \( m_{EA} \), \( m\angle BXC \), and \( m_{ED} \).

b. Find \( m_{AC} \), \( m_{DAE} \), and \( m_{ABE} \).

Theorem 11-3 In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.
In circle $R$, if $PR \perp QT$, find $PQ$.

$\triangle PSQ$ is a right angle.

Definition of perpendicular

$\triangle PSQ$ is a right triangle.

Definition of right triangle

$(SQ)^2 + (PQ)^2 = (PQ)^2$

Pythagorean Theorem

$SQ = 24$

Theorem 11-5

$PQ^2 + 24^2 = (PQ)^2$

Replace $PS$ with $10$ and $SQ$ with $24$.

$100 + 576 = (PQ)^2$

Take the square root of each side.

$\sqrt{676} = \sqrt{(PQ)^2}

= PQ$

**Your Turn**

In circle $P$, $AB = 8$ and $PD = 3$.

Find $PC$. 
In circle $W$, find $XV$ if $UW \perp XV$, $VW = 35$, and $WY = 21$.

1. $\triangle VYW$ is a right triangle.  
2. $(WY)^2 + (YV)^2 = (XY)^2$  
3. $21^2 + (YV)^2 = 35^2$  
4. $+ (YV)^2 = 1225$  
5. $(YV)^2 = \sqrt{1225}$  
6. $\sqrt{(YV)^2} = \sqrt{25}$  
7. $YV = 5 = XY$  
8. $XV = YV + XY$  
9. $XV = 5 + 28$  
10. $XV = 33$

Your Turn: In circle $G$, if $CG \perp AE$, $EG = 20$, $CG = 12$, find $AE$. 

**Homework Assignment**

Page(s):  
Exercises: 

---

© Glencoe/McGraw-Hill
11-4  Inscribed Polygons

**WHAT YOU’LL LEARN**
- Inscribe regular polygons in circles and explore the relationship between the length of a chord and its distance from the center of the circle.

**EXAMPLE**

**Build Your Vocabulary** (pages 208–209)

A polygon is **inscribed** in a circle if and only if every vertex of the polygon lies on the circle.

A **circumscribed** polygon is a polygon with each side tangent to a circle.

**Construct a regular octagon.**

Construct a quadrilateral by connecting the consecutive vertices of two diameters.

Bisect adjacent angles. Extend the bisectors through the diameters of the circle to the edges of the circle. The other four are where the other two perpendicular bisectors intersect the circle. Connect all of the consecutive vertices to form the regular octagon.

**Your Turn**

Construct a regular hexagon.
Theorem 11-6
In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

Example 2
In circle $O$, point $O$ is the midpoint of $AB$. If $CR = 2x - 1$ and $ST = x + 10$, find $x$.

\[
OA = \quad = TS
\]

Definition of midpoint

Theorem 11-6

Substitution

Add to each side.

Subtract from each side.

Your Turn
In circle $Y$, $NY = YO$. If $AX = 2x + 15$ and $BZ = 3x + 6$, what is the value of $x$?
### 11–5 Circumference of a Circle

**WHAT YOU’LL LEARN**
- Solve problems involving circumference of circles.

**BUILD YOUR VOCABULARY** (pages 208–209)

The perimeter of a circle is known as the **circumference**. It is the **perimeter** around the circle.

The ratio of the **diameter** of a circle to its **radius** is always equal to the irrational number **π**.

**Theorem 11-7  Circumference of a Circle**
If a circle has a circumference of $C$ units and a radius of $r$ units, then $C = 2\pi r$ or $C = \pi d$.

**EXAMPLES**

1. **The radius of a circle is 8 feet. Find the circumference of the circle to the nearest tenth.**

   $C = 2\pi r$

   Theorem 11-7

   $C = 2\pi (8)$

   Replace $r$ with 8.

   $C = 16\pi \approx 50.3$ feet

2. **The diameter of a plastic pipe is 5 cm. Find the circumference of the pipe to the nearest centimeter.**

   $C = \pi d$

   Theorem 11-7

   $C = \pi (5)$

   Substitution

   $C = 5\pi \approx 15.7$ cm
a. Find the circumference of circle \( A \) to the nearest tenth.

b. The diameter of a \( CD \) is 4.5 inches. Find its circumference to the nearest tenth.

A circular garden has a radius of 20 feet. There is a path around the garden that is 3 feet wide. Jasmine stands on the inside edge of the path, and Hitesh stands on the outside edge. They each walk around the garden exactly once while staying along their edge of the path. To the nearest foot, how much farther does Hitesh walk than Jasmine?

Jasmine:

\[ C = 2\pi r \]

Hitesh:

\[ C = 2\pi r \]

\[ C = 2\pi \left( \text{radius of garden} \right) \]

\[ C = 2\pi \left( 20 + 3 \right) \]

\[ C = 2\pi \left( 23 \right) \]

So, Hitesh walked \( \text{radius of garden} + 3 \) or approximately \( \text{radius of garden} + 3 \) feet more than Jasmine.

Your Turn: A circle has a circumference of 20.5 meters. Find the radius of the circle to the nearest tenth.
Theorem 11-8 Area of a Circle
If a circle has an area of \( A \) square units and a radius of \( r \) units, then \( A = \pi r^2 \).

**Example**

1. Find the area of circle \( G \).

   \[
   A = \pi r^2 \quad \text{Theorem 11-8}
   \]

   \[
   A = \pi \underline{\_\_\_\_\_\_\_}^2 \quad \text{Replace } r.
   \]

   \[
   A = 100\pi \approx \underline{\_\_\_\_\_\_\_} \text{ cm}^2
   \]

   **Your Turn** Find the area of a circle to the nearest tenth whose diameter is 10 cm.

2. If circle \( S \) has a circumference of \( 16\pi \) inches, find the area of the circle to the nearest hundredth.

   \[
   C = 2\pi r \quad \text{Theorem 11-7}
   \]

   \[
   = 2\pi r \quad \text{Replace } C \text{ with } \underline{\_\_\_\_\_\_\_}.\]

   \[
   \frac{16\pi}{2\pi} = \frac{2\pi r}{2\pi}
   \]

   \[
   = r
   \]

   \[
   A = \pi r^2 \quad \text{Theorem 11-8}
   \]

   \[
   A = \pi \underline{\_\_\_\_\_\_\_}^2 \quad \text{Replace } r \text{ with } \underline{\_\_\_\_\_\_\_}.\]

   \[
   A = 64\pi \approx \underline{\_\_\_\_\_\_\_} \text{ in}^2
   \]
Your Turn  Find the area of the circle to the nearest hundredth whose circumference is \(84\pi\) cm.

BUILD YOUR VOCABULARY  

**Theoretical probability** is the chance for a successful outcome based on ______.  

**Experimental probability** is calculated from actual observations and recording ______. It is the chance for a successful outcome based on observing patterns of occurrences.

**Example**

A pond has a radius of 10 meters. In the center of the pond is a square island with a side length of 5 meters. The seeds of a nearby maple tree float down randomly over the pond. What is the probability that a randomly-chosen seed will land in the water rather than on the island? Assume that the seed will land somewhere within the circular edge of the pond.

\[
A_{\text{of pond}} = \pi \cdot r^2
\]

\[
= \text{ } \approx \text{ } \text{m}^2
\]

\[
A_{\text{of island}} = s^2 = \text{ } \text{m}^2
\]

\[
P(\text{landing in pond}) = \frac{A_{\text{of pond}} - A_{\text{of island}}}{A_{\text{of pond}}}
\]

\[
= \frac{314.2 - \text{ }}{314.2} = \text{ }
\]

\[
\approx \text{ }
\]
Assume that all darts will land on the dartboard. Find the probability that a randomly-thrown dart will land in the shaded region.

Find the area of a 45° sector of a circle whose radius is 8 in. Round to the nearest hundredth.

\[
A = \left( \frac{N}{360} \right) \pi r^2
\]

\[
A = \left( \frac{45}{360} \right) \pi (8)^2
\]

\[
A = (0.125)(64)\pi
\]

\[
A = \boxed{25.13} \approx \boxed{25.1} \text{ in}^2
\]

Find the area of a 30° sector of a circle whose radius is 7.75 feet. Round to the nearest hundredth.

A sector of a circle is a region bounded by a central angle and its corresponding arc.

**Theorem 11-9  Area of a Sector of a Circle**
If a sector of a circle has an area of \( A \) square units, a central angle measurement of \( N \) degrees, and a radius of \( r \) units, then

\[
A = \left( \frac{N}{360} \right) \pi r^2.
\]

**Build Your Vocabulary** (page 209)

A sector of a circle is a region bounded by a central angle and its corresponding arc.

**Example**

Find the area of a 45° sector of a circle whose radius is 8 in. Round to the nearest hundredth.

\[
A = \left( \frac{45}{360} \right) \pi (8)^2
\]

\[
A = (0.125)(64)\pi
\]

\[
A = \boxed{25.13} \approx \boxed{25.1} \text{ in}^2
\]

**Homework Assignment**

Page(s):
Exercises:
### Study Guide

**Foldables**
Use your Chapter 11 Foldable to help you study for your chapter test.

**Vocabulary Puzzlemaker**
To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 11, go to:
www.glencoe.com/sec/math/t_resources/free/index.php

**Build your Vocabulary**
You can use your completed Vocabulary Builder (pages 208–209) to help you solve the puzzle.

---

**11-1 Parts of a Circle**

Underline the term that best completes the statement.

1. A chord that contains the center of the circle is the [diameter/radius].
2. A [chord/radius] is a segment with endpoints of the circle.
3. Two circles are [circumscribed/concentric] if they lie on the same plane, have the same center, and have radii of different lengths.

**11-2 Arcs and Central Angles**

In circle $C$, $BD$ is a diameter and $m\angle GCF = 63$. Find each measure.

4. $mFG$

5. $mAD$

6. $mAB$

7. $mGEF$
Complete each statement.

8. If two chords are congruent in the same circle, the intercepted ___ are also congruent.

9. When the diameter of the circle bisects a chord of the circle, then it is ___ to the chord and ___ the corresponding arc.

10. In a circle, if two arcs are ___ , their ___ are congruent.

11. Construct an equilateral triangle inscribed in a circle with radius 1 inch.

12. Draw a circle inscribed in the triangle from the previous problem. Which segment of the triangle equals the radius of the inscribed circle?

13. What is the approximate length of the segment in Exercise 12?
Find the circumference of each circle.

14. \( r = \frac{1}{2} \text{ yd} \) 

15. \( d = 4.2 \text{ in.} \)

Find the radius of the circle whose circumference is given.

16. \( 47 \text{ ft} \)

17. \( 22.7 \text{ in.} \)

Underline the term that best completes the statement.

18. A region of a circle bounded by a central angle and its corresponding arc is a(n) [arc/sector].

19. The segment with endpoints at the center and on the circle is a [sector/radius].

20. Find the area of the shaded region in circle \( B \) to the nearest hundredth.
Are You Ready for the Chapter Test?

Check the one that applies. Suggestions to help you study are given with each item.

- I completed the review of all or most lessons without using my notes or asking for help.
  - You are probably ready for the Chapter Test.
  - You may want to take the Chapter 11 Practice Test on page 491 of your textbook as a final check.

- I used my Foldable or Study Notebook to complete the review of all or most lessons.
  - You should complete the Chapter 11 Study Guide and Review on pages 488–490 of your textbook.
  - If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  - You may also want to take the Chapter 11 Practice Test on page 491.

- I asked for help from someone else to complete the review of all or most lessons.
  - You should review the examples and concepts in your Study Notebook and Chapter 11 Foldable.
  - Then complete the Chapter 11 Study Guide and Review on pages 488–490 of your textbook.
  - If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  - You may also want to take the Chapter 11 Practice Test on page 491.

Visit geomconcepts.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 11.

Student Signature

Parent/Guardian Signature

Teacher Signature
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with a plain piece of 11" × 17" paper.

**STEP 1**
Fold
Fold the paper in thirds lengthwise.

**STEP 2**
Open
Open and fold a 2" tab along the short side. Then fold the rest in fifths.

**STEP 3**
Draw
Draw lines along folds and label as shown.

**NOTE-TAKING TIP:** When taking notes, explain each new idea or concept in words and give one or more examples.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 12. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>axis</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>composite solid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cone</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cube</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cylinder [SIL-in-dur]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>edge</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>face</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>lateral area [LAT-er-ul]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>lateral edge</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>lateral face</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>net</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>oblique cone [oh-BLEEK]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>oblique cylinder</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>oblique prism</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>----------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>oblique pyramid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Platonic solid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>polyhedron</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[pa-lee-HEE-drun]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>prism</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[PRIZ-um]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pyramid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[PEER-a-MID]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>regular pyramid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>right cone</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>right cylinder</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>right prism</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>right pyramid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>similar solids</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>slant height</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>solid figures</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sphere</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[SFEER]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>surface area</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tetrahedron</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>volume</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Solid Figures

**Build Your Vocabulary**

**Solid figures** enclose a part of space.

Solids with flat surfaces that are **polyhedrons** are known as **polyhedrons**.

The two-dimensional polygonal surfaces of a polyhedron are its **faces**.

Two faces of a polyhedron in a segment called an **edge**.

A **prism** is a **prism** with two faces, called bases, which are formed by congruent polygons that lie in parallel planes.

Faces in a prism that are not bases are parallelograms and are called **lateral faces**.

The intersection of two lateral faces in a prism are called **lateral edges** and are parallel segments.

A **pyramid** is a solid with all faces but one intersecting at a common point called the vertex. The face not intersecting at the vertex is the base. The base of a pyramid is a polygon. The faces meeting at the vertex are lateral faces and are triangles.

**Example 1**

**Name the faces, edges, and vertices of the polyhedron.**

The faces are quadrilaterals **ABCD**, **DCGH**, **ADHE**, **ABFE**, **EFGH**, **AEF**, **BHF**.

The edges are **BC**, **CD**, **EF**, **FG**, **GH**, **EH**.

The vertices are **A**, **B**, **D**, **E**, **F**, **H**.

**Remember It**

Euclidean solids are also called solid figures.
Your Turn
Name the faces, edges, and vertices of the polyhedron.

WRITE IT
Give three real-world examples of polyhedrons.

BUILD YOUR VOCABULARY (pages 228–229)

A Platonic solid is a _______ polyhedron.

A cube is a special rectangular prism where all the faces are _______.

A triangular pyramid is known as a tetrahedron because all of its faces are _______.

A cylinder is a solid that is not a _______. Its bases are two congruent _______ in parallel planes, and its lateral surface is curved.

A cone is a solid that is not a _______. Its base is a _______, and the lateral surface is curved.

A composite solid is a solid made by _______ two or more solids.
Is the pyramid in the figure a tetrahedron or a rectangular pyramid?

The pyramid has a base and lateral faces. It is a pyramid.

Your Turn Describe the Washington Monument in terms of solid figures.
**Surface Areas of Prisms and Cylinders**

**Build Your Vocabulary** (pages 228–229)

In a **right prism**, a lateral edge is also an altitude.

In an **oblique prism**, a lateral edge is not an altitude.

The **lateral area** of a solid figure is the **sum** of all the areas of its lateral faces.

The **surface area** of a solid figure is the **sum** of the areas of all its surfaces.

A **net** is a two-dimensional figure that **folds** to form a solid.

---

**Theorem 12-1  Lateral Area of a Prism**

If a prism has a lateral area of $L$ square units and a height of $h$ units and each base has a perimeter of $P$ units, then $L = Ph$.

**Theorem 12-2  Surface Area of a Prism**

If a prism has a surface area of $S$ square units and a height of $h$ units and each base has a perimeter of $P$ units and an area of $B$ square units, then $S = Ph + 2B$.

---

**Example**

Find the lateral area and total surface area of a cube with side length 6 inches.

<table>
<thead>
<tr>
<th>Perimeter of Base</th>
<th>Area of Base</th>
<th>Lateral Area</th>
<th>Surface Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P = 4s$</td>
<td>$B = s^2$</td>
<td>$(24)(6)$</td>
<td>$144 + 72$</td>
</tr>
<tr>
<td>$= 4(6)$</td>
<td>$= 6^2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The lateral area of the cube is $144 \text{ in}^2$, and the surface area is $216 \text{ in}^2$. 

---

**WHAT YOU’LL LEARN**

- Find the lateral areas and surface areas of prisms and cylinders.

---

**Foldables**

In the box for **Surface Area of Prisms**, make a sketch of a prism. Then write the formula for finding the surface area of a prism.
What is the difference between lateral area and surface area?

Find the lateral area and the surface area of the rectangular prism.

Find the lateral area and the surface area of the triangular prism.

Use the Pythagorean Theorem to find the length of side $b$.

Perimeter of Base

$P = 10 + 6 + b$

Area of Base

$B = \frac{1}{2}bh$

Find the lateral and surface areas.

$L = Ph$

$S = L + 2B$

Your Turn

Find the lateral area and the surface area of the triangular prism.
The axis of a cylinder is the segment whose centers of the circular bases.

In a right cylinder, the axis is also an altitude.

In an oblique cylinder, the axis is not an altitude.

**Theorem 12-3  Lateral Area of a Cylinder**
If a cylinder has a lateral area of $L$ square units and a height of $h$ units and the bases have radii of $r$ units, then $L = 2\pi rh$.

**Theorem 12-4  Surface Area of a Cylinder**
If a cylinder has a surface area of $S$ square units and a height of $h$ units and the bases have radii of $r$ units, then $S = 2\pi rh + 2\pi r^2$.

**Example**

3. Find the lateral area and surface area of the cylinder to the nearest hundredth.

$L = 2\pi rh$

$S = 2\pi rh + 2\pi r^2$

$L = 2\pi (8)(14)$

$= 2\pi (8)(14)$

$= 703.72 + 2\pi (8)^2$

$\approx \boxed{530.15}$

$\approx \boxed{530.15} + \boxed{3.14 \times 64}$

$\approx \boxed{530.15} + \boxed{200.96}$

$\approx \boxed{731.11}$

The lateral area is about $\boxed{703.72}$ in$^2$, and the surface area is about $\boxed{731.11}$ in$^2$.

**Your Turn**

Find the lateral area and surface area of the cylinder to the nearest hundredth.
Volumes of Prisms and Cylinders

**SOL G.13** The student will use formulas for surface area and volume of three-dimensional objects to solve practical problems. Calculators will be used to find decimal approximations for results.

**Build Your Vocabulary** (page 229)

**Volume** measures the space contained within a solid.

**Theorem 12-5 Volume of a Prism**
If a prism has a volume of \( V \) cubic units, a base with an area of \( B \) square units, and a height of \( h \) units, then \( V = Bh \).

**Examples**

1. **Find the volume of the triangular prism.**

   Area of triangular base
   \[ B = \frac{1}{2}(10)(24) \]
   \[ V = Bh \]
   \[ = \left( \frac{1}{2}(10)(24) \right) \times \left( \frac{24}{10} \right) \]
   \[ = \frac{600}{10} \times \frac{24}{10} \]
   \[ = 60 \times 2.4 \]
   \[ = 144 \text{ m}^3 \]

2. **Find the volume of the rectangular prism.**

   Area of base \( B = (2)(5) \)
   \[ V = Bh \]
   \[ = (2)(5) \times (20) \]
   \[ = 10 \times 20 \]
   \[ = 200 \text{ ft}^3 \]

**Your Turn**

a. **Find the volume of the triangular prism.**

b. **Find the volume of a rectangular prism with base dimensions of 8 cm by 9 cm and height 4.1 cm.**
**Theorem 12-6  Volume of a Cylinder**
If a cylinder has a volume of $V$ cubic units, a radius of $r$ units, and a height of $h$ units, then $V = \pi r^2 h$.

---

**EXAMPLE**

3  Find the volume of the cylinder to the nearest hundredth.

$V = \pi r^2 h$

$= \pi (5)^2 (12)$

$= 300\pi$

$\approx \boxed{942.48} \text{ cm}^3$

---

**EXAMPLE**

4  Leticia is making a sand sculpture by filling a glass tube with layers of different-colored sand. The tube is 24 inches high and 1 inch in diameter. How many cubic inches of sand will Leticia use to fill the tube?

$V = \pi r^2 h$

$= \pi (0.5)^2 (24)$

$= (0.25)(24)\pi$

$= 6\pi$

$\approx \boxed{18.85} \text{ in}^3$

Leticia will need about 18.85 in$^3$ of sand.

---

**Your Turn**

Find the volume of the cylinder to the nearest hundredth.

---

**Your Turn**

Sam fills the cylindrical coffee grind containers. One bag has $32\pi$ cubic inches of grinds. How many cylindrical containers can Sam fill with two bags of grinds if each cylinder is 4 inches wide and 4 inches high?
Find the lateral area and the surface area of the square pyramid.

First, find the perimeter and the area of the base.

\[ P = 4s \]
\[ = 4(15) \text{ or } 60 \]

\[ B = s^2 \]
\[ = 15^2 \text{ or } 225 \]

In a right pyramid or a right cone, the altitude is perpendicular to the base at its center.

In an oblique pyramid or an oblique cone, the altitude is perpendicular to the base at a point other than its center.

A pyramid is a regular pyramid if and only if it is a pyramid and its base is a polygon.

The height of each face of a regular pyramid is called the slant height of the pyramid.
### Lateral Area and Surface Area of a Square Pyramid

**Lateral Area**

\[ L = \frac{1}{2}Pl \]

\[ = \frac{1}{2}(60)(25) \]

\[ = \text{cm}^2 \]

**Surface Area**

\[ S = L + B \]

\[ = 750 + 225 \]

\[ = \text{cm}^2 \]

### Example

2. Find the lateral area and the surface area of a regular triangular pyramid with a base perimeter of 24 inches, a base area of 27.7 square inches, and a slant height of 8 inches.

\[ L = \frac{1}{2}Pl \]

\[ = \frac{1}{2}(24)(8) \]

\[ = \text{in}^2 \]

\[ S = L + B \]

\[ = 96 + 27.7 \]

\[ = \text{in}^2 \]

### Your Turn

Find the lateral area and the surface area of the square pyramid.

### Your Turn

Find the lateral area and the surface area of a regular triangular pyramid with a base perimeter of 18 inches, a base area of 15.6 square inches, and a slant height of 11 inches.
The lateral area is about \( \approx \) cm\(^2\), and the surface area is about \( \approx \) cm\(^2\).

**Your Turn** Find the lateral area and the surface area of the cone to the nearest hundredth.
Volumes of Pyramids and Cones

**WHAT YOU’LL LEARN**
- Find the volumes of pyramids and cones.

**EXAMPLES**

1. Find the volume of the rectangular pyramid.

   \[ B = lw \]
   \[ = (10)(4) \text{ or } \]
   \[ V = \frac{1}{3}Bh \]
   \[ = \frac{1}{3}(40)(12) \]
   \[ = \text{ cm}^3 \]

2. Find the volume of the cone to the nearest hundredth.

   Find the height \( h \)
   \[ h^2 + 21^2 = 35^2 \]
   \[ h^2 + 441 = 1225 \]
   \[ h^2 = 784 \]
   \[ \sqrt{h^2} = \sqrt{784} \]
   \[ h = \text{ in} \]

   \[ V = \frac{1}{3} \pi r^2h \]
   \[ = \frac{1}{3} \pi (21)^2(28) \]
   \[ \approx \text{ in}^3 \]

**Theorem 12-11 Volume of a Pyramid**
If a pyramid has a volume of \( V \) cubic units and a height of \( h \) units and the area of the base is \( B \) square units, then
\[ V = \frac{1}{3}Bh. \]

**Theorem 12-12 Volume of a Cone**
If a cone has a volume of \( V \) cubic units, a radius of \( r \) units, and a height of \( h \) units, then
\[ V = \frac{1}{3} \pi r^2h. \]
Your Turn

a. Find the volume of the triangular pyramid.

b. Find the volume of the cone to the nearest hundredth.

Example

The sand in a cone with radius 3 cm and height 10 cm is poured into a square prism with height of 29.5 cm and base area of 4 cm². How far up the side of the prism will the sand reach when leveled?

Volume of Cone

\[ V = \frac{1}{3} \pi r^2 h \]

\[ = \frac{1}{3} \pi (3)^2 (10) \]

\[ = 30\pi \]

\[ \approx 94.25 \]

Volume of Prism

\[ V = Bh \]

\[ 94.25 = 4h \]

\[ h \approx 23.56 \]

The sand will level off at a height of about 23.56 cm in the prism.

Your Turn

The salt in a cone with radius 6 cm and height 8 cm is poured into a square prism with height of 20 cm and base area of 12 cm². Will the prism be able to hold all of the salt?
Find the surface area and volume of a sphere with radius 5 cm.

Surface Area

\[ S = 4\pi r^2 \]
\[ = 4\pi (5)^2 \]
\[ = 100\pi \]
\[ \approx \text{cm}^2 \]

Volume

\[ V = \left(\frac{4}{3}\right)\pi r^3 \]
\[ = \left(\frac{4}{3}\right)\pi (5)^3 \]
\[ = \left(\frac{500}{3}\right)\pi \]
\[ \approx \text{cm}^3 \]

Your Turn

Find the surface area and volume of a sphere with diameter 15 in.
Some students build a snow sculpture from a cylinder and a sphere of snow. Both the sphere and the cylinder have a radius of 1 ft. and the height of the cylinder is 4 ft. Find the volume of the snow used to build the sculpture.

Volume of Cylinder

\[ V = \pi r^2 h \]

\[ = \pi (1)^2 (4) \]

\[ = 4\pi \]

\[ \approx 12.57 \]

Volume of Sphere

\[ V = \frac{4}{3}\pi r^3 \]

\[ = \frac{4}{3}\pi (1)^3 \]

\[ = \frac{4}{3}\pi \]

\[ \approx 4.19 \]

The volume of the snow used for the sculpture is about 12.57 + 4.19, or \( \approx 16.76 \) ft\(^3\).

**Your Turn**

Felix and Brenda want to share an ice cream cone. Brenda wants half the scoop of ice cream on top, while Felix wants the ice cream inside the cone. Assuming the half scoop of ice cream on top is a perfect sphere, who will have more ice cream? The cone and scoop both have radii of 1.5 inch; the cone is 3.25 inches long.
**Similarity of Solid Figures**

**SOL G.14.a** The student will use proportional reasoning to solve practical problems, given similar geometric objects.

**WHAT YOU’LL LEARN**
- Identify and use the relationship between similar solid figures.

**BUILD YOUR VOCABULARY** (page 229)

**Similar solids** are solids that have the same _shape_ but not necessarily the same _size_.

**EXAMPLE**

Determine whether the pair of solids is similar.

![Diagram of solids with measurements]

\[
\frac{9}{27} = \frac{18}{54}
\]

\[
(9)(54) \neq (27)(18)
\]

The corresponding lengths are in _proportion_, so the solids are **similar**.

**Your Turn** Determine whether each pair of solids is similar.

**a.**

![Diagram of solids with measurements]

**b.**

![Diagram of solids with measurements]

**Theorem 12-15**

If two solids are similar with a scale factor of \(a:b\), then the surface areas have a ratio of \(a^2:b^2\) and the volumes have a ratio of \(a^3:b^3\).
2 For the similar prisms, find the scale factor of the prism on the left to the prism on the right. Then find the ratios of the surface areas and the volumes.

The scale factor is \( \frac{21}{7} = \frac{30}{10} \) or \( \frac{3}{1} \).

The ratio of the surface areas is \( \frac{3^2}{1^2} \) or \( \frac{9}{1} \).

The ratio of the volumes is \( \frac{3^3}{1^3} \) or \( \frac{27}{1} \).

**Your Turn** Find the scale factor of the prism on the left to the prism on the right. Then find the ratios of the surface areas and the volumes.

3 Sara made a scale model of the Great American Pyramid in Memphis, Tennessee, which has a base side length of 544 ft and a lateral area of 456,960 ft². If the scale factor of the model to the original is 1:136, what will be the lateral area of the model?

\[
\frac{\text{surface area of the model}}{\text{surface area of Great Amer. Pyr.}} = \frac{1^2}{136^2}
\]

\[
L = \frac{1}{136^2}
\]

\[
18,496L = 456,960
\]

\[
L = \frac{456,960}{18,496} = 25 \text{ ft}^2
\]

**Your Turn** A scale model of a house is made using a scale factor of \( \frac{1}{112} \). What fraction of the actual house material would the dollhouse need to cover all of its floors?
### 12-1 Solid Figures

Complete each sentence.

1. Two faces of a polyhedron intersect at a(n) ________________.

2. A triangular pyramid is called a ________________.

3. A ________________ is a figure that encloses a part of space.

4. Three faces of a polyhedron intersect at a point called a(n) ________________.

### 12-2 Surface Areas of Prisms and Cylinders

Find the lateral area and surface area of each solid to the nearest hundredth.

5. a regular pentagonal prism with apothem \( a = 4 \), side length \( s = 6 \), and height \( h = 12 \)

   a. \( L = \) ________________

   b. \( S = \) ________________

6. a cylinder with radius \( r = 42 \) and height \( h = 10 \)

   a. \( L \approx \) ________________

   b. \( S \approx \) ________________
Find the volume of each solid and round to the nearest hundredth.

7. the regular pentagonal prism from Exercise #5

8. How much water will a 24 in. by 15 in. by 10 in. fish tank hold?

Find the lateral and surface areas of each solid.

9. a rectangular pyramid with base dimensions 2 ft by 3 ft and lateral height \( h = 1 \) ft
   a. \( L = \) 
   b. \( S = \)

10. a cone with diameter 3.6 cm and lateral height 2.4 cm
    a. \( L = \) 
    b. \( S = \)

Find the volume of each solid rounded to the nearest hundredth, if necessary.

11. a cone with its height as three times the radius

12. the cone in Exercise #10
Complete the sentence.

13. The set of all points a given distance from the center is a \underline{circle}.

A beach ball will have a diameter of 30 in.

14. How much material will be used to make the beach ball?

15. How much air will be needed to fill it?

Solids having the same shape but not always the same size are \underline{similar}.

If the radius of a sphere is doubled:

17. How does the surface area change?

18. How does the volume change?

The diameter of the moon is about 2160 miles. The diameter of the Earth is about 7900 miles.

19. Assuming both are spheres, what is the scale factor of the Earth to the moon?

20. Are they similar solid figures?
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

I completed the review of all or most lessons without using my notes or asking for help.
- You are probably ready for the Chapter Test.
- You may want to take the Chapter 12 Practice Test on page 543 of your textbook as a final check.

I used my Foldable or Study Notebook to complete the review of all or most lessons.
- You should complete the Chapter 12 Study Guide and Review on pages 540–542 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 12 Practice Test on page 543.

I asked for help from someone else to complete the review of all or most lessons.
- You should review the examples and concepts in your Study Notebook and Chapter 12 Foldable.
- Then complete the Chapter 12 Study Guide and Review on pages 540–542 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 12 Practice Test on page 543.

Student Signature  Parent/Guardian Signature  Teacher Signature
Right Triangles and Trigonometry

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

**Foldables®**

**STEP 1**
**Stack**
Stack sheets of paper with edges $\frac{1}{4}$ inch apart.

**STEP 2**
**Fold**
Fold up bottom edges. All tabs should be the same size.

**STEP 3**
**Crease**
Crease and staple along fold.

**STEP 4**
**Turn**
Turn and label the tabs with the lesson names.

**NOTE-TAKING TIP:** When taking notes, it is often helpful to remember what you’ve learned if you can paraphrase or summarize key terms and concepts in your own words.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 13. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$30^\circ-60^\circ-90^\circ$ triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$45^\circ-45^\circ-90^\circ$ triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>angle of depression</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>angle of elevation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cosine</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hypsometer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>perfect square</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>radical expression</td>
<td></td>
<td></td>
<td>[RAD-ik-ul]</td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>---------------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>radical sign</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>radicand [RAD-i-KAND]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>simplest form</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sine</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>square root</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tangent [TAN-junt]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>trigonometric identity [TRIG-guh-no-MET-rik]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>trigonometric ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>trigonometry</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
What You’ll Learn

• Multiply, divide, and simplify radical expressions.

Build Your Vocabulary (pages 252–253)

Perfect squares are products of two factors, or when a number multiplies itself.

The square root, therefore, is one of equal factors.

A number has both positive (+) and negative (−) square roots, indicated by the radical sign \(\sqrt{}\).

A radical expression is an expression that contains a .

The number under the radical sign \(\sqrt{}\) is the radicand.

Write It

What are the next three perfect squares after 16?

EXAMPLES

Simplify each expression.

1. \(\sqrt{36}\)

\(\sqrt{36} = \underline{6}\), because \(6^2 = 36\).

2. \(\sqrt{81}\)

\(\sqrt{81} = \underline{9}\), because \(9^2 = 81\).

3. \(\sqrt{24}\)

\(\sqrt{24} = \sqrt{2 \cdot 2 \cdot 2 \cdot 3}\)

Prime factorization

\(= \sqrt{2 \cdot 2} \cdot \sqrt{2 \cdot 3}\)

Product Property of Square Roots

\(= 2 \cdot \sqrt{6}\)

\(\sqrt{2 \cdot 2} = 2\)

= \underline{}
4 \sqrt{6} \cdot \sqrt{30}
\begin{align*}
\sqrt{6} \cdot \sqrt{30} &= \sqrt{6 \cdot \phantom{00}^{\phantom{00}}} \\
&= \sqrt{6 \cdot 6 \cdot 5} \\
&= \phantom{00} \cdot \sqrt{5} \\
&= \phantom{00}
\end{align*}

Prime factorization

Product Property of Square Roots

Product Property of Square Roots

\sqrt{6 \cdot 6} = 6

Your Turn

Simplify each expression.

a. \sqrt{25}

b. \sqrt{121}

c. \sqrt{18}

d. \sqrt{3 \cdot \sqrt{12}}

Examples

Simplify each expression.

5 \frac{\sqrt{16}}{\sqrt{8}}
\begin{align*}
\frac{\sqrt{16}}{\sqrt{8}} &= \frac{\sqrt{16}}{\sqrt{8}} \\
&= \phantom{00}
\end{align*}

Quotient Property

\frac{\sqrt{121}}{\sqrt{49}}
\begin{align*}
\frac{\sqrt{121}}{\sqrt{49}} &= \frac{\sqrt{121}}{\sqrt{49}} \\
&= \phantom{00}
\end{align*}

Quotient Property

Your Turn

Simplify each expression.

a. \frac{\sqrt{20}}{\sqrt{4}}

b. \frac{\sqrt{144}}{\sqrt{25}}
### EXAMPLES

7. Simplify \( \frac{\sqrt{10}}{\sqrt{7}} \).

\[
\frac{\sqrt{10}}{\sqrt{7}} = \frac{\sqrt{10}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{10 \cdot 7}}{\sqrt{7 \cdot 7}} = \frac{\sqrt{70}}{7}
\]

We used the Identity Property and the Product Property of Square Roots to simplify the above radical expression. The denominator does not have a radical sign.

8. Simplify \( \frac{16}{\sqrt{6}} \).

\[
\frac{16}{\sqrt{6}} = \frac{16}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{16 \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}} = \frac{16 \sqrt{6}}{6}
\]

We used the Identity Property and the Product Property of Square Roots to simplify the above expression and eliminate the radical in the denominator.

### Key Concept

Rules for Simplifying Radical Expressions

1. There are no perfect square factors other than 1 in the radicand.
2. The radicand is not a fraction.
3. The denominator does not contain a radical expression.

### Your Turn

Simplify.

a. \( \frac{\sqrt{7}}{\sqrt{2}} \)

b. \( \frac{4}{\sqrt{5}} \)
In a scale model of a town, a baseball diamond has sides 36 inches long. What is the distance from first base to third base on the model? Round to the nearest tenth.

Theorem 13-1 45°-45°-90° Triangle Theorem
In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times the length of a leg.

In a scale model of a town, a baseball diamond has sides 36 inches long. What is the distance from first base to third base on the model? Round to the nearest tenth.

\[ h = s\sqrt{2} \]

The distance from first to third base on the scale model is about ___ inches.

Your Turn  Find the length of the diagonal of a square whose side measures 22 inches.
EXAMPLE

If \( \triangle DJT \) is an isosceles right triangle and the measure of the hypotenuse is \( \sqrt{200} \), find the measure of either leg.

\[
h = s\sqrt{2} \\
\]

Theorem 13-1

\[
\frac{h}{s\sqrt{2}} = s\sqrt{2} \\
\]

Substitution

\[
\frac{h}{s} = s \\
\]

The length of each leg measures \( s \).

Your Turn

If \( \triangle XYZ \) is an isosceles right triangle and the measure of the hypotenuse is 25, find the measure of either leg.
**30°-60°-90° Triangles**

**SOL G.7** The student will solve practical problems involving right triangles by using the Pythagorean Theorem, properties of special right triangles, and right triangle trigonometry. Solutions will be expressed in radical form or as decimal approximations.

**What You’ll Learn**
- Use the properties of 30°-60°-90° triangles.

**Remember It**
The shorter leg is always opposite the 30° angle, and the longer leg is always opposite the 60° angle.

**Theorem 13-2 30°-60°-90° Triangle Theorem**
In a 30°-60°-90° triangle, the hypotenuse is twice the length of the shorter leg, and the longer leg is $\sqrt{3}$ times the length of the shorter leg.

**Example**

1. In $\triangle ABC$, $a = 12$. Find $b$ and $c$.

   \[ a = b\sqrt{3} \]
   
   The longer leg is $\sqrt{3}$ times the length of the shorter leg.
   
   Replace $a$ with $b\sqrt{3}$.

   \[ b\sqrt{3} = b \sqrt{3} \]
   
   Replace $b\sqrt{3}$ with $b$.

   \[ c = 2b \]
   
   The hypotenuse is twice the shorter leg.
   
   Replace $b$ with $\frac{1}{2}c$.

   \[ c = 2\left(\frac{1}{2}c\right) \]
   
   \[ c = c \]

**Your Turn** Refer to Example 1.

a. If $b = 3.5$, find $a$ and $c$.

b. If $b = \frac{1}{3}$, find $a$ and $c$. 
2. In \( \triangle DEF \), \( DE = 18 \). Find \( EF \) and \( DF \).

Use Theorem 13–2.

\[ DE = EF \sqrt{3} \quad \text{The longer leg is } \sqrt{3} \text{ times the shorter leg.} \]

Replace \( DE \) with ______.

\[ = EF \sqrt{3} \quad \text{Divide each side by } \sqrt{3}. \]

\[ = EF \quad \text{Replace } EF \text{ with ______.} \]

\[ DF = 2(\sqrt{3}EF) \quad \text{The hypotenuse is twice the shorter leg.} \]

Replace \( EF \) with ______.

\[ DF = 2\sqrt{3} \]

\[ DF = \quad \text{Associative Property} \]

Your Turn Refer to Example 2. If \( DE = 11 \), find \( EF \) and \( DF \).

3. Find the length, to the nearest tenth, of the median in the equilateral triangle.

The median bisects one side into two 5-meter segments and is opposite the 60° angle.

\[ x = 5\sqrt{3} \quad \text{Theorem 13-2} \]

\[ \approx \quad \text{meters} \]

Your Turn Find the length, to the nearest tenth, of the median in the equilateral triangle.
Find \( \tan K \) and \( \tan M \).

\[
\tan K = \frac{ML}{KL} \quad \text{opposite} \quad \frac{K}{L} \quad \text{adjacent} \\
= \frac{21}{28} \quad \text{or} \\
\text{Substitution}
\]

\[
\tan M = \frac{KL}{ML} \quad \text{opposite} \quad \frac{K}{L} \quad \text{adjacent} \\
= \frac{28}{21} \quad \text{or} \\
\text{Substitution}
\]

**Your Turn** Find \( \tan 30^\circ \), \( \tan 45^\circ \), \( \tan 60^\circ \).
Find QR to the nearest tenth of a meter.

\[ \tan \angle Q = \frac{QR}{PQ} \quad \text{opposite adjacent} \]

\[ \tan \angle P = \frac{QR}{PR} \quad \text{Substitution} \]

\[ QR \cdot \frac{PQ}{PR} = QR \quad \text{Multiplication Property of Equality} \]

A ranger sights the top of a tree at a 40° angle of elevation. Find the height of the tree if it is 80 feet from where the ranger is standing.

\[ \tan \angle Q = \frac{\text{height of tree}}{QR} \quad \text{opposite adjacent} \]

\[ \tan \angle Q = \text{height of tree} \quad \text{Multiplication Property of Equality} \]

The height of the tree is about feet.

**Build Your Vocabulary** (page 252)

The line of sight and a horizontal line when looking up form the **angle of elevation**.

Angles of elevation can be measured using a **hypsometer**.

The line of sight and a horizontal line when looking form the **angle of depression**.
a. Find $YZ$ to the nearest tenth of a foot.

b. The ranger sights the top of another tree at a $52^\circ$ angle of elevation. Find the height of the tree if it is 20 feet from where he stands.

**EXAMPLE**

Find $m\angle 1$ to the nearest tenth.

\[
\tan(\angle 1) = \frac{\text{opposite}}{\text{adjacent}}
\]

\[
\tan^{-1}\left(\frac{22}{47}\right) = \text{Definition of arctangent}
\]

The measure of $\angle 1$ is about .

**Your Turn** Find $m\angle 2$ to the nearest tenth.
Sine and Cosine Ratios

SOL G.7 The student will solve practical problems involving right triangles by using the Pythagorean Theorem, properties of special right triangles, and right triangle trigonometry. Solutions will be expressed in radical form or as decimal approximations.

Both the sine and the cosine ratios relate an angle measure to the ratio of the measures of a triangle’s sides to its hypotenuse.

If \( \angle A \) is an acute angle of a right triangle,

\[
\sin A = \frac{\text{measure of leg opposite } \angle A}{\text{measure of hypotenuse}}, \quad \text{and} \\
\cos A = \frac{\text{measure of leg adjacent } \angle A}{\text{measure of hypotenuse}}.
\]

**Find \( \sin K, \cos K, \sin M, \) and \( \cos M \).**

\[
\sin K = \frac{LM}{KM} = \text{Substitution} = \text{Substitution} \\
\cos K = \frac{KL}{KM} = \text{Substitution} = \text{Substitution}
\]

\[
\sin M = \frac{KL}{KM} = \text{Substitution} = \text{Substitution} \\
\cos M = \frac{LM}{KM} = \text{Substitution} = \text{Substitution}
\]

**Your Turn** Find \( \sin 30^\circ, \cos 30^\circ, \sin 45^\circ, \cos 45^\circ, \sin 60^\circ, \) and \( \cos 60^\circ \).
**Example 2**

Find the value of $x$ to the nearest tenth.

\[
\sin 26 = \frac{x}{200}
\]

\[
200 \sin 26 = x
\]

\[
\approx x
\]

**Example 3**

Find the measure of $\angle K$ to the nearest degree.

\[
\sin K = \frac{LM}{KM}
\]

\[
\sin K = \frac{68}{82}
\]

\[
m \angle K = \sin^{-1} \left( \frac{68}{82} \right)
\]

\[
m \angle K \approx
\]

**Your Turn**

a. Find the value of $x$ to the nearest tenth.

b. Find the measure of $\angle A$ to the nearest degree.

**Homework Assignment**

**Theorem 13-3**

If $x$ is a measure of an acute angle of a right triangle, then $\frac{\sin x}{\cos x} = \tan x.$
**13-1**

**Simplifying Square Roots**

Simplify.

1. \( \sqrt{63} \)
2. \( \frac{1}{\sqrt{3}} \)
3. \( \sqrt{10} \cdot \sqrt{8} \)

4. Find the value of \( x \) if \( \frac{2}{\sqrt{x}} = \frac{2\sqrt{x}}{3} \).

**13-2**

**45°-45°-90° Triangles**

A fabric square is cut on the diagonal for a quilt. The perimeter of the square is 116 in.

5. What is the length of each leg/side?

6. What is the length of the hypotenuse/diagonal?

7. What is the measure of each leg of an isosceles right triangle if its hypotenuse measures 10?
8. The Gothic arch, similar to the figure, is based on an equilateral triangle. Find the width of the base of the triangle if the median is 4 ft long.

Find the missing measure. Simplify all radicals.

9. \[ \begin{align*} \text{Given:} & \quad 60^\circ, 11, 5.5 \\ \text{Find:} & \quad b \\ \text{Solution:} & \quad b = \ldots \end{align*} \]

10. \[ \begin{align*} \text{Given:} & \quad 60^\circ, 6, 6\sqrt{3} \\ \text{Find:} & \quad c \\ \text{Solution:} & \quad c = \ldots \end{align*} \]

11. You spot a cat on the roof of a house 80 feet away from where you’re standing. Your eye level is 5 feet above ground level, and the angle of elevation from eye level is 33°. How tall is the house?

12. If \( y = 20 \), find \( x \) and \( z \).

13. If \( z = 2.3 \), find \( x \) and \( y \).

14. If \( x = 9 \), find \( y \) and \( z \).
Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
  • You are probably ready for the Chapter Test.
  • You may want to take the Chapter 13 Practice Test on page 581 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
  • You should complete the Chapter 13 Study Guide and Review on pages 578–580 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 13 Practice Test on page 581.

☐ I asked for help from someone else to complete the review of all or most lessons.
  • You should review the examples and concepts in your Study Notebook and Chapter 13 Foldable.
  • Then complete the Chapter 13 Study Guide and Review on pages 578–580 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 13 Practice Test on page 581.

Visit geomconcepts.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 13.

Student Signature

Parent/Guardian Signature

Teacher Signature
Circle Relationships

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

**NOTE-TAKING TIP:** When taking notes, define new terms and write about the new ideas and concepts you are learning in your own words. Write your own examples that use the new terms and concepts.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 14. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>external secant segment [SEE-kant]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>externally tangent [TAN-junt]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>inscribed angle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>intercepted arc</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>internally tangent</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>---------------</td>
<td>------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>point of tangency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>secant angle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>secant-tangent angle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>secant segment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tangent</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tangent-tangent angle</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Inscribed Angles

Determine whether \( \angle ABC \) is an inscribed angle. Name the intercepted arc for the angle.

The vertex of \( \angle ABC \), point \( B \), is on circle \( Q \). Therefore, \( \angle ABC \) is an inscribed angle. The intercepted arc is \( AC \).

Your Turn Determine whether \( \angle JKL \) is an inscribed angle. Name the intercepted arc for the angle.

Theorem 14-1
The degree measure of an inscribed angle equals one-half the degree measure of its intercepted arc.
Refer to the figure.

2 If $m\overarc{AB} = 76$, find $m\angle ADB$.

$$m\angle ADB = \frac{1}{2}(m\overarc{AB})$$

Theorem 14-1

$$m\angle ADB = \frac{1}{2}(76)$$

Replace $m\overarc{AB}$.

$$m\angle ADB = 38$$

3 If $m\angle BDC = 40$, find $m\overarc{BC}$.

$$m\angle BDC = \frac{1}{2}(m\overarc{AB})$$

Theorem 14-1

$$40 = \frac{1}{2}(m\overarc{BC})$$

Replace $m\angle BDC$.

$$2 \cdot 40 = 2 \cdot \frac{1}{2}(m\overarc{BC})$$

Multiply each side by 2.

$$m\overarc{BC} = 40$$

Your Turn Refer to the figure.

a. If $m\overarc{ZW} = 124$, find $m\angle WXZ$.

b. If $m\angle YXZ = 49$, find $mYZ$.

Theorem 14-2

If inscribed angles intercept the same arc or congruent arcs, then the angles are congruent.

In circle $A$, suppose $m\angle TLN = 6y + 7$ and $m\angle TWN = 7y$. Find the value of $y$.

$\angle TLN$ and $\angle TWN$ both intercept $TN$.

$$\angle TLN \cong \angle TWN$$

Theorem 14-2

$$m\angle TLN = m\angle TWN$$

Definition of congruent angles

$$6y + 7 = 7y$$

Replace $m\angle TLN$ and $m\angle TWN$.

Subtract 6$y$ from each side.

$$y = 7$$
In the circle, if \( m\angle AHM = 10x \) and \( m\angle ATM = 20x - 30 \), find the value of \( x \).

\[
\begin{align*}
m\angle AHM &= 10x \\
m\angle ATM &= 20x - 30
\end{align*}
\]

Theorem 14-3
If an inscribed angle of a circle intercepts a semicircle, then the angle is a right angle.

Your Turn
In circle \( G \), \( m\angle 1 = 6x - 5 \) and \( m\angle 2 = 3x - 4 \). Find the value of \( x \).

Inscribed angle \( DEF \) intercepts semicircle \( DF \). \( \angle DEF \) is a right angle by Theorem 14-3.
Therefore, \( \angle 1 \) and \( \angle 2 \) are complementary.

\[
\begin{align*}
m\angle 1 + m\angle 2 &= 90 \\
(6x - 5) + (3x - 4) &= 90
\end{align*}
\]

\[
\begin{align*}
6x - 5 + 3x - 4 &= 90 \\
9x - 9 + 9x &= 90 + 9 \\
9x &= 99 \\
x &= 11
\end{align*}
\]

Your Turn
In circle \( W \), \( m\angle KRP = \frac{1}{2}x \) and \( m\angle KRP = \frac{1}{3}x + 5 \). Find the value of \( x \).
Tangents to a Circle

Under the tab for Tangents to a Circle, write the definition of tangent and draw a picture to illustrate the concept. Record the theorems and other important information from this lesson.

**WHAT YOU’LL LEARN**

• Identify and apply properties of tangents to circles.

**BUILD YOUR VOCABULARY** (page 271)

In a plane, a line is a **tangent** if and only if it intersects a circle in exactly **one** point.

The point of intersection is the **point of tangency**.

**Theorem 14-4**

In a plane, if a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

**Theorem 14-5**

In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is a tangent.

**EXAMPLE**

1. **AB** is tangent to circle **C** at **B**. Find **BC**.

   \( \overline{AB} \perp \overline{CB} \) by Theorem 14-4, making \( \angle CBA \) a right angle by definition. Therefore, \( \triangle ABC \) is a right triangle.

   \[
   (BC)^2 + (AB)^2 = (AC)^2
   \]

   Replace \( AB \) and \( AC \).

   \[
   (BC)^2 + 784 = 1225
   \]

   Square \( BC \) and \( AC \).

   \[
   (BC)^2 + 784 - 784 = 1225 - 784
   \]

   Subtract 784 from each side.

   \[
   (BC)^2 = 441
   \]

   Take the square root of each side.

   \[
   BC = 21
   \]
**Theorem 14-6**

If two segments from the same exterior point are tangent to a circle, then they are congruent.

**Example 2**

$EF$ and $EG$ are tangent to circle $H$. Find the value of $x$.

\[
3x + 10 - \boxed{} = 43 - \boxed{}
\]

Subtract 10 from each side.

\[
3x = 33
\]

Divide each side by $3$.

\[
x = \boxed{}
\]

**Your Turn**

$AD$, $AC$, and $AB$ are tangents to circles $Q$ and $R$, respectively. Find the value of $x$.

\[
6x + 5 = -2x + 37
\]

If two circles are tangent and one circle is inside the other, the circles are internally tangent. If two circles are tangent and circle is inside the other, the circles are externally tangent.
Secant Angles

**WHAT YOU’LL LEARN**
- Find measures of arcs and angles formed by secants.

**BUILD YOUR VOCABULARY** (page 271)

A secant segment is a segment that contains an arc of a circle.

A secant angle is the angle formed when two secant segments intersect.

**Theorem 14-7**
A line or line segment is a secant to a circle if and only if it intersects the circle in two points.

**Theorem 14-8**
If a secant angle has its vertex inside a circle, then its degree measure is one-half the sum of the degree measures of the arcs intercepted by the angle and its vertical angle.

**Theorem 14-9**
If a secant angle has its vertex outside a circle, then its degree measure is one-half the difference of the degree measures of the intercepted arcs.

**EXAMPLE**

1. Find \( m \angle 1 \).

The vertex of \( \angle 1 \) is inside circle \( P \).

\[
m \angle 1 = \frac{1}{2}(m\overline{AB} + m\overline{CD})
\]

**Theorem 14-8**

\[
m \angle 1 = \frac{1}{2}\left( \right)
\]

Replace \( m\overline{AB} \) and \( m\overline{CD} \).

\[
m \angle 1 = \frac{1}{2}\left( \right)
\]

or

**Your Turn**
If \( m\overline{MA} = 40 \) and \( m\overline{HT} = 50 \), find \( m \angle 1 \).
**EXAMPLE**

2 Find \( m \angle J \).

The vertex of \( \angle J \) is outside circle \( Q \).

\[
m \angle J = \frac{1}{2}(m \overline{MN} - m \overline{KL})
\]

Theorem 14-9

\[
m \angle J = \frac{1}{2} \left( \begin{array}{c} \square \\ \square \end{array} \right) \quad \text{or} \quad \frac{1}{2} \left( \begin{array}{c} \square \\ \square \end{array} \right)
\]

**EXAMPLE**

3 Find the value of \( x \). Then find \( m \overline{CD} \).

The vertex lies inside circle \( P \).

\[
57 = \frac{1}{2}(m \overline{AB} + m \overline{CD})
\]

\[
57 = \frac{1}{2} \left( \begin{array}{c} \square \\ \square \end{array} \right) + \left( \begin{array}{c} \square \\ \square \end{array} \right)
\]

\[
57 = \frac{1}{2}(9x + 6)
\]

Combine like terms.

\[
2 \cdot 57 = 2 \cdot \frac{1}{2}(9x + 6)
\]

Multiply each side by 2.

\[
114 = 9x + 6 - 6
\]

Subtract 6 from each side.

\[
114 = 9x
\]

Subtraction Property

\[
x = 12
\]

Division Property

\[
m \overline{CD} = 6x + 7 = 6 \left( \begin{array}{c} \square \\ \square \end{array} \right) + 7 = \left( \begin{array}{c} \square \\ \square \end{array} \right) + 7 = \left( \begin{array}{c} \square \\ \square \end{array} \right)
\]

**Your Turn**

a. If \( m \overline{CE} = 85 \) and \( m \overline{BD} = 40 \), find \( m \angle A \).

b. Find the value of \( x \). Then find \( m \overline{TH} \).
In the figure, $\overline{AD}$ is tangent to circle $K$ at $A$.

Find $m\angle 1$.

Vertex $D$ of the secant-tangent angle is outside circle $K$. Apply Theorem 14-10.

The degree measure of the whole circle is $360^\circ$. So, the measure of $AC$ is $360^\circ - 160^\circ - 110^\circ = 90^\circ$.

$$m\angle 1 = \frac{1}{2}(m\overline{AB} - m\overline{AC})$$

Substitution

$$m\angle 1 = \frac{1}{2}\left(\begin{array}{c}
160^\circ \\
110^\circ
\end{array}\right)$$

or

$$m\angle 1 = \frac{1}{2}\left(\begin{array}{c}
160^\circ \\
110^\circ
\end{array}\right)$$

Find $m\angle 2$.

Vertex $A$ of the secant-tangent angle is on circle $K$.

$$m\angle 2 = \frac{1}{2}(m\overline{ABC})$$

Substitution

$$m\angle 2 = \frac{1}{2}\left(\begin{array}{c}
160^\circ \\
110^\circ
\end{array}\right)$$

or

$$m\angle 2 = \frac{1}{2}\left(\begin{array}{c}
160^\circ \\
110^\circ
\end{array}\right)$$
Your Turn

a. \( \overline{AZ} \) is tangent to circle \( D \) at \( A \). If \( m\angle AB = 150 \), find \( m\angle Z \).

b. \( \overline{EF} \) is tangent to circle \( D \) at \( E \). If \( m\angle EGC = 230 \), find \( m\angle FEC \).

Build Your Vocabulary (page 271)

A tangent-tangent angle is formed by two of a circle. Its vertex is always outside the circle.

Theorem 14-12

The degree measure of a tangent-tangent angle is one-half the difference of the degree measures of the intercepted arcs.

Example

3 Find \( m\angle G \).

\( \angle G \) is a tangent-tangent angle. Apply Theorem 14-12.

By definition of a right angle, \( m\angle FOH = 90 \). So, \( m\angle FH = 90 \), because a minor arc is congruent to its central angle.

Since the sum of the measures of a minor arc and its major arc is 360°, major arc \( FH \) is 360° – 90° = 270°.

\[
m\angle G = \frac{1}{2} (\text{major arc } FH - \text{minor arc } FH)
\]

\[
m\angle G = \frac{1}{2} (270 - 90)
\]

\[
m\angle G = \frac{1}{2} (180) \text{ or } \frac{180}{2} \]

Your Turn  

Find \( m\angle B \).
In circle $A$, find the value of $x$.

$PT \cdot TR = QT \cdot TS$

$6 \cdot ____ = ____ \cdot 12$

$48 = 12x$

$\frac{48}{4} = \frac{12x}{4}$

$\frac{x}{4} = x$

Your Turn Find the value of $x$ in the circle.
2. Find the value of $x$ to the nearest tenth.

$(x + 6) \cdot 6 = (5 + 7) \cdot 5$

Theorem 14-14

Distributive Property

$6x + 36 - 36 = 60 - 36$

Subtract 36 from each side.

$6x = 24$

Divide each side by 6.

$x = \frac{24}{6}$

3. Use the value of $x$ to find the value of $y$.

$y^2 = (x + 5) \cdot 5$

Theorem 14-15

Substitution

$y^2 = 4 \cdot 5$

$y^2 = 20$

Take the square root.

$y = \sqrt{20}$

Your Turn

a. Find the value of $x$.

b. Find the value of $x$. 
Write the equation of a circle with center at \((-4, 0)\) and a radius of 5 units.

Equation of a Circle

\[(x - h)^2 + (y - k)^2 = r^2\]

\[(x - (-4))^2 + (y - 0)^2 = 5^2\]

\[\text{Equation for the circle is } \boxed{\ \boxed{\ \boxed{\ \boxed{}}}}\]

Find the coordinates of the center and the measure of the radius of a circle whose equation is

\[
\left(x + \frac{3}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}.
\]

Rewrite the equation.

\[
\left(x - h\right)^2 + \left(y - k\right)^2 = r^2
\]

\[
\left(x - \left(-\frac{3}{2}\right)\right)^2 + \left(y - \frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2
\]

Since \(h = \boxed{\ \boxed{}}\), \(k = \boxed{\ \boxed{}}\), and \(r = \boxed{\ \boxed{}}\), the center of the circle is at \(\boxed{\ \boxed{\ \boxed{}}}\). Its radius is \(\boxed{\ \boxed{}}\).
Your Turn

a. Write the equation of a circle with center $C(5, -3)$ and a radius of 6 units.

b. Find the coordinates of the center and the measure of the radius of a circle whose equation is $(x + 2)^2 + (y + 7)^2 = 81.$
**14-1 Inscribed Angles**

In circle $P$, $\overline{AC}$ is a diameter; $m\angle CD = 68$ and $m\angle BE = 96$. Find each of the following.

1. $m\angle ABC$

2. $m\angle CED$

3. $m\overline{AD}$

In circle $A$, $\overline{HE}$ is a diameter.

4. If $m\angle HTC = 52$, find $m\overline{CH}$.

5. Find $m\overline{HCE}$.

6. If $m\angle HTC = 52$, find $m\overline{CEH}$.
7. If a line is tangent to a circle, then it is perpendicular to the [point of tangency/vertex].

8. $AB$ is tangent to circle $C$. Find the value of $x$.

9. Circle $P$ is inscribed in right $\triangle CTA$. Find the perimeter of $\triangle CTA$ if the radius of circle $P$ is 5, $CT = 18$, and $JT = 11$.

10. A [radius/secant segment] is a line segment that intersects a circle in exactly two points.

Find the value of $x$.

11. $\phantom{12}$

12. $\phantom{12}$
Underline the best term to complete the statement.

13. The measure of a(n) [tangent-tangent/inscribed] angle is always one-half the difference of the measures of the intercepted arcs.

Find the value of \( x \). Assume that segments that appear to be tangent are tangent.

14.

15.

14-5

Segment Measures

Find the value of \( x \).

16.

17.

14-6

Equations of Circles

18. Write the equation of the circle with center \((-5, 9)\) and radius \(2\sqrt{5}\).

19. What are the coordinates of the center and length of the radius for the circle \((x + 4)^2 + y^2 = 121\).
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
   • You are probably ready for the Chapter Test.
   • You may want to take the Chapter 14 Practice Test on page 627 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
   • You should complete the Chapter 14 Study Guide and Review on pages 624–626 of your textbook.
   • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   • You may also want to take the Chapter 14 Practice Test on page 627.

☐ I asked for help from someone else to complete the review of all or most lessons.
   • You should review the examples and concepts in your Study Notebook and Chapter 14 Foldable.
   • Then complete the Chapter 14 Study Guide and Review on pages 624–626 of your textbook.
   • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   • You may also want to take the Chapter 14 Practice Test on page 627.

Student Signature

Parent/Guardian Signature

Teacher Signature
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

**Begin with four sheets of \(8\frac{1}{2}'' \times 11''\) grid paper.**

**STEP 1**
Fold
Fold each sheet of paper in half along the width. Then cut along the crease.

**STEP 2**
Staple
Staple the eight half-sheets together to form a booklet.

**STEP 3**
Cut
Cut seven lines from the bottom of the top sheet, six lines from the second sheet, and so on.

**STEP 4**
Label
Label each tab with a lesson number. The last tab is for vocabulary.

**NOTE-TAKING TIP:** To help you organize data, create a study guide or study cards when taking notes, solving equations, defining vocabulary words and explaining concepts.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 15. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>compound statement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>conjunction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>contrapositive</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>coordinate proof</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>deductive reasoning</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>disjunction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>indirect proof</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>indirect reasoning</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>inverse</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Law of Detachment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>-------------------------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>Law of Syllogism [SIL-oh-jiz-um]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>logically equivalent</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>negation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>paragraph proof</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>proof</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>proof by contradiction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>statement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>truth table</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>truth value</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>two-column proof</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Let $p$ represent “An octagon has eight sides” and $q$ represent “Water does not boil at $90^\circ C$.”

1. Write the negation of statement $p$.

   $\sim p$: An octagon have eight sides.

2. Write the negation of statement $q$.

   $\sim q$: Water boil at $90^\circ C$. 

A statement is any sentence that is either true or false, but not both.

Every has a truth value, true (T) or false (F).

If a statement is represented by $p$, then $p$ is the negation of the statement.

The relationship between the of a statement are organized on a truth table.

When two statements are , they form a compound statement.

A conjunction is a statement formed by joining two statements with the word .

A disjunction is a statement formed by joining two statements with the word .

Build Your Vocabulary (pages 290–291)

A statement is any sentence that is either true or false, but not both.

Every has a truth value, true (T) or false (F).

If a statement is represented by $p$, then $p$ is the negation of the statement.

The relationship between the of a statement are organized on a truth table.

When two statements are , they form a compound statement.

A conjunction is a statement formed by joining two statements with the word .

A disjunction is a statement formed by joining two statements with the word .

Examples

Let $p$ represent “An octagon has eight sides” and $q$ represent “Water does not boil at $90^\circ C$.”

1. Write the negation of statement $p$.

   $\sim p$: An octagon have eight sides.

2. Write the negation of statement $q$.

   $\sim q$: Water boil at $90^\circ C$. 

### Your Turn

Let $p$ represent “Tofu is a protein source” and $q$ represent “$\pi$ is not a rational number.”

**a.** Write the negation of statement $p$.

**b.** Write the negation of statement $q$.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\sim p$</th>
<th>$q$</th>
<th>$\sim q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True</td>
<td>False</td>
<td>True</td>
<td>False</td>
</tr>
</tbody>
</table>

### Examples

Let $p$ represent “$9^2 = 99$”, $q$ represent “An equilateral triangle is equiangular”, and $r$ represent “A rectangular prism has six faces.” Write the statement for each conjunction or disjunction. Then find the truth value.

1. $\sim p \land q$
   - $9^2 \neq 99$ and an equilateral triangle is equiangular. Because $p$ is True, $\sim p$ is False. Therefore, $\sim p \land q$ is False because both $\sim p$ and $q$ are False.

2. $p \lor \sim r$
   - $9^2 = 99$ or a rectangular prism does not have six faces. Because $r$ is True, $\sim r$ is False. Therefore, $p \lor \sim r$ is True because both $p$ and $\sim r$ are True.

3. $\sim q \land \sim r$
   - An equilateral triangle is not equiangular and a rectangular prism does not have six faces. Because $q$ is False, $\sim q$ is True; and because $r$ is True, $\sim r$ is False. Therefore, $\sim q \land \sim r$ is False because both $\sim q$ and $\sim r$ are False.
Let $p$ represent "0.5 is an integer", $q$ represent "A rhombus has four congruent sides", and $r$ represent "A parallelogram has congruent diagonals." Write the statement for each conjunction or disjunction. Then find the truth value.

a. $\sim p \land q$

b. $\sim p \lor r$

c. $q \land \sim r$

Construct a truth table for the conjunction $\sim (p \land q)$.

Make columns with the headings $p$, $q$, and $\sim (p \land q)$. Then, list all possible combinations of truth values for $p$ and $q$. Use these truth values to complete the last two columns of the and its .

Your Turn Construct a truth table for the disjunction $\sim (p \lor q)$.

REMEMBER IT A disjunction is false only when both statements are false. The converse of a conditional is false when $p$ is false and $q$ is true. A conditional is false only when $p$ is true and $q$ is false.

BUILD YOUR VOCABULARY (pages 290–291)

The inverse of a conditional is formed by both $p$ and $q$. The contrapositive of a conditional statement is formed by negating the of the statement. Two statements are logically equivalent if their truth tables are the .

HOMEWORK ASSIGNMENT

Page(s):  
Exercises:  

Your Turn Let $p$ represent “0.5 is an integer”, $q$ represent “A rhombus has four congruent sides”, and $r$ represent “A parallelogram has congruent diagonals.” Write the statement for each conjunction or disjunction. Then find the truth value.

a. $\sim p \land q$

b. $\sim p \lor r$

c. $q \land \sim r$

Construct a truth table for the conjunction $\sim (p \land q)$.

Make columns with the headings $p$, $q$, and $\sim (p \land q)$. Then, list all possible combinations of truth values for $p$ and $q$. Use these truth values to complete the last two columns of the and its .

Your Turn Construct a truth table for the disjunction $\sim (p \lor q)$.

REMEMBER IT A disjunction is false only when both statements are false. The converse of a conditional is false when $p$ is false and $q$ is true. A conditional is false only when $p$ is true and $q$ is false.

BUILD YOUR VOCABULARY (pages 290–291)

The inverse of a conditional is formed by both $p$ and $q$. The contrapositive of a conditional statement is formed by negating the of the statement. Two statements are logically equivalent if their truth tables are the .

HOMEWORK ASSIGNMENT

Page(s):  
Exercises:  

Your Turn Let $p$ represent “0.5 is an integer”, $q$ represent “A rhombus has four congruent sides”, and $r$ represent “A parallelogram has congruent diagonals.” Write the statement for each conjunction or disjunction. Then find the truth value.

a. $\sim p \land q$

b. $\sim p \lor r$

c. $q \land \sim r$

Construct a truth table for the conjunction $\sim (p \land q)$.

Make columns with the headings $p$, $q$, and $\sim (p \land q)$. Then, list all possible combinations of truth values for $p$ and $q$. Use these truth values to complete the last two columns of the and its .

Your Turn Construct a truth table for the disjunction $\sim (p \lor q)$.

REMEMBER IT A disjunction is false only when both statements are false. The converse of a conditional is false when $p$ is false and $q$ is true. A conditional is false only when $p$ is true and $q$ is false.

BUILD YOUR VOCABULARY (pages 290–291)

The inverse of a conditional is formed by both $p$ and $q$. The contrapositive of a conditional statement is formed by negating the of the statement. Two statements are logically equivalent if their truth tables are the .
Deductive Reasoning

SOL G.1.d The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include using deductive reasoning, including the law of syllogism.

**WHAT YOU’LL LEARN**

**Build Your Vocabulary** (pages 290–291)

**Deductive reasoning** is the process of using facts, rules, definitions, and properties in a logical order.

The **Law of Detachment** allows us to reach logical statements.

The **Law of Syllogism** is similar to the Transitive Property of Equality.

**Examples**

Use the Law of Detachment to determine a conclusion that follows from statements (1) and (2). If a valid conclusion does not follow, then write no valid conclusion.

1. **(1) In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other line.**

   (2) \( AB \parallel CD \) and \( EF \perp AB \).

   \[ p: AB \parallel CD \] and \( EF \perp AB \).

   Statement (1) indicates that \( p \rightarrow q \) is \( \boxed{\text{true}} \), and statement (2) indicates that \( p \) is \( \boxed{\text{true}} \). So, \( EF \perp CD \).

2. **(1) Two nonvertical lines have the same slope if and only if they are parallel.**

   **(2) \( AB \) is a vertical line.**

   \[ p: \text{Two lines are nonvertical and} \] \( \boxed{\text{true}} \).

   \[ q: \text{Two lines have the same} \] \( \boxed{\text{false}} \).

   Statement (2) indicates that \( p \) is \( \boxed{\text{false}} \). Therefore, there is no valid conclusion.
Use the Law of Detachment to determine a conclusion that follows from statements (1) and (2). If a valid conclusion does not follow, then write no valid conclusion.

a. (1) If a figure is an isosceles triangle, then it has two congruent angles.
(2) A figure is an isosceles triangle.

b. (1) If a hexagon is regular, each interior angle measures 120°.
(2) The hexagon is regular.

Use the Law of Syllogism to determine a conclusion that follows from statements (1) and (2).

3. Use the Law of Syllogism to determine a conclusion that follows from statements (1) and (2).
   (1) If \( m\angle K = 90 \), then \( \angle K \) is a right angle.
   (2) If \( \angle K \) is a right angle, then \( \triangle JKL \) is a right triangle.

   \( p: m\angle K = 90 \)
   \( q: \angle K \) is a right angle.
   \( r: \triangle JKL \) is a triangle.

Use the Law of Syllogism to conclude \( p \rightarrow r \).

Therefore, if \( \) , then \( \triangle JKL \) is a triangle.

Your Turn Use the Law of Syllogism to determine a conclusion that follows from statements (1) and (2).
(1) If it is rainy tomorrow, then Alan cannot play golf.
(2) If Alan cannot play golf, then he will watch television.
Write a paragraph proof for the conjecture.

In $\triangle RST$, if $TX \perp RS$ and $TX$ bisects $\angle RTS$, then $RX \cong XS$.

Given: $TX \perp RS$; $TX$ bisects $\angle RTS$.
Prove: $RX \cong XS$

Proof: If $TX \perp RS$, then $\angle RXT$ and $\angle TXS$ are right angles and $\triangle RXT$ and $\triangle TXS$ are right triangles.

If $TX$ bisects $\angle RTS$, then $\angle RTX \cong \angle STX$ by the definition of angle bisector. Also, $TX \cong TX$ since congruence is reflexive. So, $\triangle RTX \cong \triangle STX$ by the Reflexive Property. Therefore, $RX \cong XS$ because parts of congruent triangles are congruent (CPCTC).
2 If $\angle 1$ and $\angle 2$ are congruent, then $\ell$ is parallel to $m$.

**Given:** $\angle 1 \cong \angle 2$

**Prove:** $\ell \parallel \text{ }$

**Proof:** Vertical angles are congruent so $\angle 2 \cong \angle 3$. Since $\angle 1 \cong \angle 2$, $\angle 1 \cong \angle 3$ by substitution. If two lines in a are cut by a so that corresponding angles are , then the lines are . Therefore, .

**Your Turn** Write a paragraph proof for each conjecture.

**a.** If $A$ is the midpoint of $DC$ and $EB$, then $\triangle DAE \cong \triangle CAB$.

**Given:** $A$ is the midpoint of $DC$ and $EB$

**Prove:** $\triangle DAE \cong \triangle CAB$

**b.** If $\angle 3 \cong \angle 4$, then $\angle 5 \cong \angle 6$. 

*REMEMBER IT* There is more than one way to plan a proof.
Preparing for Two-Column Proofs

SOL G.1.b The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include translating a short verbal argument into symbolic form.

**WHAT YOU’LL LEARN**

- Use properties of equality in algebraic and geometric proofs.

**BUILD YOUR VOCABULARY** (page 291)

A two-column proof is a deductive argument with _______ and _______ organized in two columns.

**EXAMPLE**

Justify the steps for the proof of the conditional. If \( \angle XWY \cong \angle XYW \), then \( \angle AWX \cong \angle BYX \).

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle XWY = m\angle XYW )</td>
<td>2.</td>
</tr>
<tr>
<td>3. ( m\angle AWX + m\angle XWY = 180; m\angle BYX + m\angle XYW = 180 )</td>
<td>3.</td>
</tr>
<tr>
<td>4. ( m\angle AWX + m\angle XWY = )</td>
<td>4. Substitution</td>
</tr>
<tr>
<td>5. ( m\angle AWX + m\angle XWY = )</td>
<td>5. Substitution</td>
</tr>
<tr>
<td>6. ( m\angle AWX = )</td>
<td>6. Subtraction property</td>
</tr>
<tr>
<td>7.</td>
<td>7. Definition of congruent angles</td>
</tr>
</tbody>
</table>
Justify the steps for the proof of the conditional. If $m\angle AOC = m\angle BOD$, then $m\angle AOB = m\angle COD$.

**Given:** $m\angle AOC = m\angle BOD$
**Prove:** $m\angle AOB = m\angle COD$

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $m\angle AOC = m\angle BOD$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $bh$</td>
<td>2. Multiplication property</td>
</tr>
<tr>
<td>3. $\frac{2A}{h} = b$</td>
<td>3.</td>
</tr>
<tr>
<td>4. $b = \ldots$</td>
<td>4. Symmetric property</td>
</tr>
</tbody>
</table>

**REMEMBER IT**
You cannot write a statement unless you give a reason to justify it.

**WRITE IT**
What information is always in the first statement of a proof? What information can always be found in the last statement?

**EXAMPLE**
Show that if $A = \frac{1}{2}bh$, then $b = \frac{2A}{h}$.

**Given:** $A = \frac{1}{2}bh$
**Prove:** $b = \frac{2A}{h}$

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $A = \frac{1}{2}bh$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\ldots = bh$</td>
<td>2. Multiplication property</td>
</tr>
<tr>
<td>3. $\frac{2A}{h} = b$</td>
<td>3.</td>
</tr>
<tr>
<td>4. $b = \ldots$</td>
<td>4. Symmetric property</td>
</tr>
</tbody>
</table>
Your Turn  Show that if $PV = nRT$, then $R = \frac{PV}{nT}$.

Given: 

Prove: 

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1.</td>
</tr>
<tr>
<td>2.</td>
<td>2.</td>
</tr>
<tr>
<td>3.</td>
<td>3.</td>
</tr>
</tbody>
</table>
Write a two-column proof for the conjecture.

If \( \angle 1 = \angle 2 \), then quadrilateral \( ABCD \) is a trapezoid.

Given: \( \angle 1 = \angle 2 \)
Prove: \( \ell \parallel m \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 1 \equiv \angle 2 )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 2 \equiv \angle 3 )</td>
<td>2.</td>
</tr>
<tr>
<td>3. ( \angle 3 \equiv \angle 4 )</td>
<td>3. Substitution</td>
</tr>
<tr>
<td>4. If two lines in a plane are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.</td>
<td>4.</td>
</tr>
<tr>
<td>5. Quadrilateral ( ABCD ) is a trapezoid.</td>
<td>5.</td>
</tr>
</tbody>
</table>

Your Turn

Write a two-column proof. If \( \triangle XYZ \) is isosceles with \( XZ \equiv XY \) and \( OZ \equiv NY \), then \( OY \equiv NZ \).

Given: \( \triangle XYZ \) is isosceles with \( XZ \equiv XY \) and \( OZ \equiv NY \)
Prove: \( OY \equiv NZ \)
2 Write a two-column proof.

**Given:** $X$ is the midpoint of both $BD$ and $AC$.

**Prove:** $\triangle DXC \cong \triangle BXA$

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $X$ is the midpoint of both $BD$ and $AC$.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $DX \cong BX$; $\square \cong \square$</td>
<td>2.</td>
</tr>
<tr>
<td>3. $\square \cong \square$</td>
<td>3. Vertical angles are congruent.</td>
</tr>
<tr>
<td>4. $\square \cong \square$</td>
<td>4.</td>
</tr>
</tbody>
</table>
Your Turn  Write a two-column proof.

Given: $AD$ and $CE$ bisect each other.

Prove: $AE \parallel CD$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1.</td>
</tr>
<tr>
<td>2.</td>
<td>2.</td>
</tr>
<tr>
<td>3.</td>
<td>3.</td>
</tr>
<tr>
<td>4.</td>
<td>4.</td>
</tr>
<tr>
<td>5.</td>
<td>5.</td>
</tr>
<tr>
<td>6.</td>
<td>6.</td>
</tr>
</tbody>
</table>

HW Assignment

Page(s):
Exercises:
Coordinate Proofs

**SOL G.1.b** The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include translating a short verbal argument into symbolic form.

**WHAT YOU’LL LEARN**

- Use coordinate proofs to prove theorems.

**BUILD YOUR VOCABULARY** (page 290)

A proof that uses a geometric figure on a coordinate plane is a coordinate proof.

**EXAMPLE**

1. Position and label a rectangle with length $b$ and height $d$ on a coordinate plane.
   - Use the origin as a vertex or center.
   - Place one side on the $x$-axis and one side on the $y$-axis.
   - Label the vertices $A$, $B$, $C$, and $D$.
   - Label the coordinates $D(0, 0)$, $C(b, 0)$, $B(b, d)$, and $A(0, d)$.

**Guidelines for Placing Figures on a Coordinate Plane**

1. Use the origin as a vertex or center.
2. Place at least one side of a polygon on an axis.
3. Keep the figure within the first quadrant, if possible.
4. Use coordinates that make computations as simple as possible.

Under the tab for Lesson 15-6, summarize the Guidelines for Placing Figures on a Coordinate Plane.

**EXAMPLE**

2. Write a coordinate proof to prove that the opposite sides of a parallelogram are congruent.

**Given:** parallelogram $ABDC$

**Prove:** $AB \cong CD$ and $AC \cong BD$
Proof:
Label the vertices $A(0, 0)$, $B(a, 0)$, $D(a + b, c)$, and $C(b, c)$. Use the Distance Formula to find $AB$, $CD$, $AC$, and $BD$.

$AB = \sqrt{(a - 0)^2 + (0 - 0)^2}$

$= \sqrt{a^2}$ or $a$

$CD = \sqrt{[(a + b) - b]^2 + (c - c)^2} = \Box$ or $a$

$AC = \sqrt{(b - \Box)^2 + (c - \Box)^2} = \sqrt{b^2 + c^2}$

$BD = \sqrt{[(a + b) - \Box]^2 + (c - \Box)^2}$

$= \Box$

So, $AB = CD$ and $AC = BD$.

Therefore, $\Box$ and $\Box$; opposite sides of a parallelogram are $\Box$.

Your Turn Write a coordinate proof to prove that a $WXYZ$ is a rectangle by proving the diagonals are congruent.
Write a coordinate proof to prove that the length of the segment joining the midpoints of two sides of a triangle is one-half the length of the third side.

**Given:** \( \triangle EFG \) with midpoints \( J \) and \( K \), of \( EF \) and \( FG \)

**Prove:** \( JK = \frac{1}{2} EG \)

Label the vertices \( E(0,0), F(2c,0), \) and \( G(2c,0) \).

Use the Midpoint Formula to find the coordinates of \( J \) and \( K \), and the Distance Formula to find \( JK \) and \( EG \).

Coordinates of \( J \): \( J = \left( \frac{0 + 2a}{2}, \frac{0 + 2b}{2} \right) \)

Coordinates of \( K \): \( K = \left( \frac{2a + 2c}{2}, \frac{2b + 0}{2} \right) = (a + c, b) \)

\( JK = \sqrt{[(a + c) - a]^2 + (b - b)^2} = \sqrt{c^2} \) or \( c \)

\( EG = \sqrt{(2c - 0)^2 + (0 - 0)^2} = \sqrt{(2c)^2} \) or \( 2c \)

\( \frac{1}{2} EG = \frac{1}{2} \sqrt{(2c)^2} = \sqrt{c^2} \) or \( c \)

Therefore, \( JK = \frac{1}{2} EG \).

**Your Turn** Write a coordinate proof to prove that the length of a median segment joining the midpoints of two legs of a trapezoid is one-half the sum of the length of the bases.
Indicate whether the statement is true or false.

1. A table that lists all truth values of a statement is a truth table.

2. \( p \rightarrow q \) is an example of a disjunction.

3. \( \sim p \rightarrow \sim q \) is the inverse of a conditional statement.

4. \( p \lor q \) is an example of a conjunction.

5. Complete the truth table.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Deductive Reasoning

Draw a conclusion from statements (1) and (2).

6. (1) All functions are relations.
   (2) $x = y^2$ is a relation.

7. (1) Integers are rational numbers.
   (2) $(-6)$ is an integer.

8. (1) If it is Saturday, I see my friends.
   (2) If I see my friends, we laugh.

Paragraph Proofs

Indicate whether the statement is true or false.

9. A proof is a logical argument where each statement is backed up by a reason accepted as true.

Write a paragraph proof.

10. Given: $m\angle 1 = m\angle 2; m\angle 3 = m\angle 4$
    Prove: $m\angle 1 + m\angle 4 = 90$
Complete the statement.
11. A proof containing statements and reasons and is organized by steps is a ______ proof.

Complete the proof.

12. Given: $\overline{AB}$ and $\overline{AR}$ are tangent to circle $K$.
Prove: $\triangle BAK \cong \triangle RAK$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{AB}$ and $\overline{AR}$ are tangent to circle $K$</td>
<td>1.</td>
</tr>
<tr>
<td>2. $\overline{BK} \cong \overline{RK}$</td>
<td>2. If 2 segments from the same exterior point are tangent to a circle, then they are $\cong$.</td>
</tr>
<tr>
<td>3. $\overline{AC}$</td>
<td>3.</td>
</tr>
<tr>
<td>4. $\overline{AC}$</td>
<td>4.</td>
</tr>
<tr>
<td>5. $\overline{AC}$ $\cong \triangle RAK$</td>
<td>5.</td>
</tr>
<tr>
<td>6. $\overline{AC}$ $\cong \triangle RAK$</td>
<td>6.</td>
</tr>
</tbody>
</table>

13. Write a two-column proof.

Given: $\overline{AB}$ is tangent to circle $X$ at $B$.
$\overline{AC}$ is tangent to circle $X$ at $C$.
Prove: $\overline{AB} \cong \overline{AC}$
**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{AB}$ is tangent to circle $X$ at $B$. $\overline{AC}$ is tangent to circle $X$ at $C$.</td>
<td>1. <strong>Given</strong></td>
</tr>
<tr>
<td>2. Draw $BX$, $CX$, and $AX$.</td>
<td>2. Through any 2 <strong>points</strong> there is 1 <strong>line</strong>.</td>
</tr>
<tr>
<td>3. $\angle ABX$ and $\angle ACX$ are <strong>right angles</strong></td>
<td>3. If a line is tangent to a circle, then it is $\perp$ to the radius drawn to the point of tangency.</td>
</tr>
<tr>
<td>4. $\overline{BX} \cong \overline{CX}$</td>
<td>4. <strong>All radii of a circle are $\cong$.</strong></td>
</tr>
<tr>
<td>5.</td>
<td>5. Reflexive Property</td>
</tr>
<tr>
<td>6. $\triangle AXB \cong \triangle$</td>
<td>6. <strong>HL</strong></td>
</tr>
<tr>
<td>7.</td>
<td>7. <strong>CPCTC</strong></td>
</tr>
</tbody>
</table>

---

**15-6**

**Coordinate Proofs**

**Complete the statement.**

14. The vertex or center of the figure should be placed on the **origin**.

15. Position and label a rhombus on a coordinate plane with base $r$ and height $t$.  

![Diagram of a rhombus on a coordinate plane with vertices labeled (0, 0), (r, 0), (a, t), and (r + a, t)].
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
  • You are probably ready for the Chapter Test.
  • You may want to take the Chapter 15 Practice Test on page 671 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
  • You should complete the Chapter 15 Study Guide and Review on pages 668–670 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 15 Practice Test on page 671.

☐ I asked for help from someone else to complete the review of all or most lessons.
  • You should review the examples and concepts in your Study Notebook and Chapter 15 Foldable.
  • Then complete the Chapter 15 Study Guide and Review on pages 668–670 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 15 Practice Test on page 671.

Visit geomconcepts.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 15.

Student Signature

Parent/Guardian Signature

Teacher Signature
More Coordinate Graphing and Transformations

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with six sheets of graph paper and an $8\frac{1}{2}'' \times 11''$ poster board.

**STEP 1**  
Staple  
Staple the six sheets of graph paper onto the poster board.

**STEP 2**  
Label  
Label the six pages with the lesson titles.

**NOTE-TAKING TIP:** When taking notes, mark anything you do not understand with a question mark. Be sure to ask your instructor to explain the concepts or sections before your next quiz or exam.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 16. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>center of rotation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>composition of transformations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dilation [dye-LAY-shun]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>elimination [ee-LIM-in-AY-shun]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>reflection</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rotation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>substitution [SUB-sti-TOO-shun]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>system of equations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>translation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>turn</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Solve each system of equations by graphing.

1. \[y = x - 1\]
   \[y = -x + 3\]

Find ordered pairs by choosing values for \(x\) and finding the corresponding \(y\)-values.

<table>
<thead>
<tr>
<th>(y = x - 1)</th>
<th>(y = -x + 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>(x - 1)</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Graph the ordered pairs and draw the graphs of the equations. The graphs intersect at the point whose coordinates are \((2, 1)\). Therefore, the solution of the system of equations is \((2, 1)\).

2. \[y = -2x\]
   \[y = -2x + 3\]

Use the slope and \(y\)-intercept to graph each equation.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Slope</th>
<th>(y)-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y = -2x)</td>
<td>(-2)</td>
<td>0</td>
</tr>
<tr>
<td>(y = -2x + 3)</td>
<td>(-2)</td>
<td>3</td>
</tr>
</tbody>
</table>

The slope of each line is \(-2\) so the graphs are parallel and do not intersect. Therefore, there is no solution.
Your Turn  Solve each system of equations by graphing.

a. \[x - 2y = 2\]
   \[3x + y = 6\]

b. \[3x + 2y = 12\]
   \[3x + 2y = 6\]

Example

Toshiro wants a wildflower garden. He wants the length to be 1.5 times the width and he has 100 meters of fencing to put around the garden. If \(w\) represents the width of the garden and \(\ell\) represents the length, solve the system of equations below to find the dimensions of the wildflower garden.

\[
\ell = 1.5w \\
2w + 2\ell = 100
\]

Solve the second equation for \(\ell\).

\[
2w + 2\ell = 100
\]

The perimeter is [ ] meters.

Subtract [ ] from each side.

Divide.

\[
\ell = [ ]
\]
Use a graphing calculator to graph the equations and to find the coordinates of the intersection point. *Note that these equations can be written as* \( y = 1.5x \) and \( y = 50 - x \) *and then graphed.*

Enter: \( \boxed{\text{[y = ] } 1.5 \times \text{ ENTER} 50 - x \text{ } \boxed{\text{GRAPH}}\)

Next, use the intersection tool on \( \boxed{\text{F5}} \) to find the coordinates of the point of intersection.

The solution is . Since \( w = \) and \( l = \), the width of the garden is meters and the length is meters.

Check your answer by examining the original problem. Is the length of the garden 1.5 times the width? ✔ Does the garden have a perimeter of 100 meters? ✔ The solution checks.

**Your Turn** Ruth wants to enclose an area of her yard for her children to play. She has 72 meters of fence. The length of the play area is 4 meters greater than 3 times the width. What are the dimensions of the play area?
Solving Systems of Equations by Using Algebra

**Build Your Vocabulary (page 314)**

One algebraic method for solving a system of equations is called **substitution**.

Another algebraic method for solving systems of equations is called **elimination**.

**Example**

1. Use substitution to solve the system of equations.

   \[ y = x + 4 \]
   \[ 2x + y = 1 \]

   Substitute \( x + 4 \) for \( y \) in the second equation.

   \[ 2x + \boxed{} = 1 \]

   Combine like terms.

   \[ 3x + 4 = 1 \]

   Subtract \( \boxed{} \) from each side.

   \[ 3x = -3 \]

   Divide each side by \( \boxed{} \).

   \[ \frac{3x}{3} = \frac{-3}{3} \]

   \[ x = \boxed{} \]

   Substitute \(-1\) for \( x \) in the first equation and solve for \( y \).

   \[ y = (-1) + 4 = \boxed{} \]

   The solution to this system of equations is \( \boxed{} \).

**Your Turn**

Use substitution to solve \( 2x - y = 4 \) and \( x = y + 5 \).
2 Use elimination to solve the system of equations.

\[
\begin{align*}
3x - 2y &= 4 \\
4x + 2y &= 10
\end{align*}
\]

\[
\begin{align*}
3x - 2y &= 4 \\
(+) 4x + 2y &= 10
\end{align*}
\]

Add the equations to eliminate the \( y \) terms.

\[
7x + 0 = 14
\]

\[
\frac{7x}{7} = \frac{14}{7}
\]

Divide each side by 7.

\[
x = \[
\]

The value of \( x \) in the solution is \[
\]

Now substitute in either equation to find the value of \( y \).

\[
3x - 2y = 4
\]

\[
3(\[
\] - 2y = 4
\]

\[
6 - 2y - 6 = 4 - 6 \quad \text{Subtract} \[
\] \text{from each side.}
\]

\[
\[
\] = \[
\]

\text{Subtraction Property}

\[
\frac{-2y}{-2} = \frac{-2}{-2}
\]

Divide each side by \[
\].

\[
y = \[
\]

The value of \( y \) in the solution is \[
\].

The solution to the system is \[
\].

**Your Turn** Use elimination to solve \( x + y = 7 \) and \( 2x - y = -1 \).
Use elimination to solve the system of equations.

3 \(x + y = 6\)
\(x - 2y = 9\)

\[
\begin{align*}
3x + y &= 6 \quad (\times 2) \\
6x + 2y &= 12 \\
x - 2y &= 9 \\
7x &= 21 \\
\frac{7x}{7} &= \frac{21}{7} \\
x &= \underline{3}
\end{align*}
\]

Combine like terms.

Divide.

Substitute 3 into either equation to solve for \(y\).

\[
\begin{align*}
3x + y &= 6 \\
3(3) + y &= 6 \\
9 + y &= 6 \\
9 + y - 9 &= 6 - 9 \\
y &= \underline{6 - y = 6 - 9} \\
\text{Subtraction Property}
\end{align*}
\]

The solution of this system is \(x = 3, y = -2\).

Your Turn Use elimination to solve \(7x + 3y = -1\) and \(4x + y = 3\).
**Translations**

**SOL G.2.c** The student will use . . . coordinate methods, to solve problems involving symmetry and transformation. This will include determining whether a figure has been translated, reflected, or rotated.

**Build Your Vocabulary** (page 314)

A translation is a slide of a figure from one position to another.

**Example**

1. Graph $\triangle LMN$ with vertices $L(0, 3)$, $M(4, 2)$, and $N(-3, -1)$. Then find the coordinates of its vertices if it is translated by $(5, 0)$. Graph the translation image.

To find the coordinates of the vertices of $\triangle L'M'N'$, add 5 to each $x$-coordinate and add 0 to each $y$-coordinate of $\triangle LMN$: $(x + 5, y + 0)$.

$L(0, 3) + (5, 0) \rightarrow L'(0 + 5, 3 + 0) = L'$

$M(4, 2) + (5, 0) \rightarrow M'(4 + 5, 2 + 0) = M'$

$N(-3, -1) + (5, 0) \rightarrow N'(-3 + 5, -1 + 0) = N'$

**Your Turn** Graph $\triangle ABC$ with vertices $A(1, 2)$, $B(-3, -1)$, and $C(2, 1)$. Then find the coordinates of its vertices if it is translated by $(3, -2)$. Graph the translation image.
Graph \( \triangle ABC \) with vertices \( A(0, 0) \), \( B(4, 1) \), and \( C(1, 5) \). Then find the coordinates of its vertices if it is reflected over the \( x \)-axis and graph its reflection image.

To find the coordinates of the vertices of \( \triangle A'B'C' \), use the definition of reflection over the \( x \)-axis: \( (x, y) \rightarrow (x, -y) \).

\[
\begin{align*}
A(0, 0) & \rightarrow A' \hspace{1cm} \vdots \\
B(4, 1) & \rightarrow B' \hspace{1cm} \vdots \\
C(1, 5) & \rightarrow C' \hspace{1cm} \vdots
\end{align*}
\]

The vertices of \( \triangle A'B'C' \) are \( \hspace{1cm} \), \( \hspace{1cm} \), and \( \hspace{1cm} \).

In the same \( \triangle ABC \), find the coordinates of the vertices of \( \triangle ABC \) after a reflection over the \( y \)-axis. Graph the reflected image.

To find the coordinates of \( A'', B'', \) and \( C'' \), use the definition of reflection over the \( y \)-axis: \( (x, y) \rightarrow (-x, y) \).

\[
\begin{align*}
A(0, 0) & \rightarrow A'' \hspace{1cm} \vdots \\
B(4, 1) & \rightarrow B'' \hspace{1cm} \vdots \\
C(1, 5) & \rightarrow C'' \hspace{1cm} \vdots
\end{align*}
\]

The vertices of \( \triangle A''B''C'' \) are \( \hspace{1cm} \), \( \hspace{1cm} \), and \( \hspace{1cm} \).
Your Turn

a. Graph quadrilateral QUAD with vertices Q(−3, 3), U(3, 2), A(4, −4), and D(−4, −1). Then find the coordinates of its vertices if it is reflected over the y-axis. Graph its reflection image.

b. Graph △STU with vertices S(1, 2), T(4, 4), and U(3, −3). Then find the coordinates of its vertices if it is reflected over the y-axis and graph its reflection image.
**SOL G.2.c** The student will use . . . coordinate methods, to solve problems involving symmetry and transformation. This will include determining whether a figure has been translated, reflected, or rotated.

**WHAT YOU’LL LEARN**
- Investigate and draw rotations on a coordinate plane.

**BUILD YOUR VOCABULARY** (page 314)
A rotation, also called a turn, is a movement of a figure around a point. The fixed point may be in the center of the object or a point the object and is called the center of rotation.

**ORGANIZE IT**
On the page labeled Rotations, sketch graphs of several different rotations. Explain why each rotation produces the result it does.

**EXAMPLE**
1. Rotate $\triangle ABC$ $270^\circ$ clockwise about point $A$.

   - The center of rotation is $A$. Use a protractor to draw an angle of clockwise about point $A$, using $AB$ as a baseline for your protractor.
   - Draw segment $A'B'$ to $AB$.
   - Trace the figure on a piece of paper and rotate the top paper clockwise, until the figure is rotated clockwise.
   - Draw $\triangle A'B'C'$ congruent to $\triangle ABC$.

**Your Turn** Rotate $\triangle XYZ$ $60^\circ$ counterclockwise about point $Y$.
Graph \( \triangle XYZ \) with vertices \( X(-2, 1) \), \( Y(2, -3) \), and \( Z(3, 5) \). Then find the coordinates of the vertices after the triangle is rotated 180° clockwise about the origin. Graph the rotation image.

- Draw a segment from the origin to point \( X \).
- Use a protractor to reproduce \( OX \) at a 180° angle so that \( OX = OX' \).
- Repeat this procedure with points \( Y \) and \( Z \).

The rotation image \( \triangle X'Y'Z' \) has vertices \( X' \), \( Y' \), and \( Z' \).

**Your Turn** Rotate \( \triangle ABC \) 90° counterclockwise around the origin. The vertices are \( A(0, 4) \), \( B(3, 1) \), and \( C(4, 3) \).
**Dilations**

**WHAT YOU’LL LEARN**
- Investigate and draw dilations on a coordinate plane.

**BUILD YOUR VOCABULARY** (page 314)

A dilation is a transformation that alters the size of a figure, but not its shape. It enlarges or reduces a figure by a factor of $k$.

**EXAMPLE**

Graph $\overline{AB}$ with vertices $A(0, 2)$ and $B(2, 1)$. Then find the coordinates of the dilation image of $\overline{AB}$ with a scale factor of 3, and graph its dilation image.

Since $k > 1$, this is an enlargement. To find the dilation image, multiply each coordinate in the ordered pairs by 3.

<table>
<thead>
<tr>
<th>preimage $\rightarrow$ image</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A(0, 2)$ $(\times 3)$ $A'$</td>
</tr>
<tr>
<td>$B(2, 1)$ $(\times 3)$ $B'$</td>
</tr>
</tbody>
</table>

The coordinates of the endpoints of the dilation image are $A'$ and $B'$.

**Your Turn**

Graph $\triangle JKL$ with vertices $J(1, -2)$, $K(4, -3)$, and $L(6, -1)$. Then find the coordinates of the dilation image of $\triangle JKL$ with a scale factor of 2, and graph its dilation.
Graph \( \triangle DEF \) with vertices \( D(3, 3), E(0, -3), \) and \( F(-6, 3) \). Then find the coordinates of the dilation image with a scale factor of \( \frac{1}{3} \) and graph its dilation image.

Since \( k < 1 \), this is a reduction.

The coordinates of the vertices of the dilation image are \( D', E', \) and \( F' \).

**Your Turn**  Graph quadrilateral \( MNOP \) with vertices \( M(1, 2), N(3, 3), O(3, 5), \) and \( P(1, 4) \). Then find the coordinates of the dilation image with a scale factor of \( \frac{2}{3} \) and graph its dilation image.
Solve each system of equations by graphing.

1. \(x - y = 6\)  
   \(y = 9\) 

2. \(x + y = 27\)  
   \(3x - y = 41\) 

3. \(y = 4x + 2\)  
   \(12x - 3y = 9\)

Complete each statement.

4. Substitution and elimination are methods for solving \[\text{________________________}\].

5. A linear system of equations can have at most \(\text{________________________}\) solution.

Solve the system of equations using substitution or elimination.

6. \(3x - y = 4\)  
   \(2x - 3y = -9\) 

7. \(y = 3x - 8\)  
   \(y = 4 - x\) 

8. \(2x + 7y = 3\)  
   \(x = 1 - 4y\) 

9. \(3x - 5y = 11\)  
   \(x - 3y = 1\)
Complete the statement.

10. When a figure is moved from one position to another without turning, it is called a **translation**.

Find the coordinates of the vertices after the translation. Graph each preimage and image.

11. rectangle $WXYZ$ with vertices $W(-2, -10), X(-2, -10), Y(-7, -2),$ and $Z(-7, -10)$ translated $(6, 9)$

12. $\triangle ABC$ with vertices $A(4, 0), B(2, -1),$ and $C(0, 1)$ translated $(0, -4)$

13. $\triangle JKL$ with vertices $J(-5, -2), K(-2, 7),$ and $L(1, -6)$ translated $(6, 2)$
Complete the statement.

14. A __________ is a flip of a figure over a line.

Find the coordinates of the vertices after the reflection. Graph each preimage and image.

15. quadrilateral $ABCD$ with vertices $A(1, 1), B(1, 4), C(6, 4),$ and $D(6, 1)$ flipped over the $x$-axis

16. quadrilateral $JKLM$ with vertices $J(3, 5), K(4, 0), L(0, -3),$ and $M(-1, 2)$ flipped over the $y$-axis

17. $\triangle XYZ$ with vertices $X(1, 1), Y(4, 1),$ and $Z(1, 3)$ flipped over the $x$-axis
16-5 Rotations

Find the coordinates of the vertices after a rotation about the origin. Graph the preimage and image.

18. $\triangle RST$ with vertices $R(-4, 1)$, $S(-1, 5)$, and $T(-6, 9)$ rotated 90° counterclockwise.

16-6 Dilations

Underline the best term to complete the statement.

19. A [dilation/rotation] alters the size of a figure but does not change its shape.

20. A figure is [reduced/enlarged] in a dilation if the scale factor is between 0 and 1.

Find the coordinates of the dilation image for the given scale factor. Graph the preimage and image.

21. quadrilateral $STUV$ with vertices $S(2, 1)$, $T(0, 2)$, $U(-2, 0)$, and $V(0, 0)$ and scale factor 3.
Are You Ready for the Chapter Test?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
  • You are probably ready for the Chapter Test.
  • You may want to take the Chapter 16 Practice Test on page 713 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
  • You should complete the Chapter 16 Study Guide and Review on pages 710–712 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 16 Practice Test on page 713 of your textbook.

☐ I asked for help from someone else to complete the review of all or most lessons.
  • You should review the examples and concepts in your Study Notebook and Chapter 16 Foldable.
  • Then complete the Chapter 16 Study Guide and Review on pages 710–712 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 16 Practice Test on page 713 of your textbook.

Visit geomconcepts.net to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 16.

Student Signature

Parent/Guardian Signature

Teacher Signature