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Overview
Algebra 2 Multimedia Applications

Engaging computer simulations, plus videos of real people using mathematics, bring the world into your classroom. These highly interactive multimedia materials are carefully correlated to the Algebra 2 textbook chapters.

Choose some or all of the applications to extend and reinforce concepts.

Involving your students in applying mathematics and solving problems in realistic situations.

Data collection and analysis form the basis of many explorations. Students are actively engaged in the interactions. They make decisions, operate simulated devices, collect data, record data, and interpret results.

Complete onscreen instructions guide students through the interactions. Hot links explain new terminology. Explore buttons lead to animated in-depth explorations of selected topics.

Real-World Activities
Correlated to Curriculum

Each chapter engages students in a compelling scenario.

- Graphing and mapping the ocean floor.
- Calculating fuel efficiency on a car trip.
- Finding lowest truck-rental costs.
- Programming a jet plane simulator with matrices.
- Maximizing profit for a T-shirt shop.
- Calculating an asteroid’s orbit.
- Designing safe tracks for thrill rides.
- Focusing photo images in cameras.
- Modeling the growth of plankton.
- Exploring fractal trees as models of lungs.
- Analyzing games of chance.
- Testing audio speaker quality.
Video of people who use mathematics in their careers provide genuine, credible evidence that mathematics is vital to students’ future employment.

- Navy technician, Greg Moiles
- CAD technician, Doug Baillie
- Seismologist, Hiroko Shike
- CAD technician, Mona Ellis
- Civil engineer, Ngyra Stebbins
- Business owner, Susie Harris
- Biochemist, Michael Hickey
- Epidemiologist, Lynne Turner
- Harpist, Felipe G. Villalobos

**Easy to Use**
Each chapter is packed with activities and is easy to use. Just click to move to a page and then follow the onscreen instructions.

**English and Spanish Spoken Text**
You can hear the text spoken in either English or Spanish.

**Classroom Settings**
Ideal for student partners in a lab setting, these materials are also appropriate for individual or whole class use.

**Worksheets**
Accompanying worksheets ensure that students record their results and reflect on their work at the computer.

**Homework**
Student worksheets for each chapter include an After the Computer page, which can be assigned as homework.

**Quiz**
Each chapter ends with an informal quiz for students to check their understanding.
About this Teacher’s Guide

Read the Getting Started section before you begin. After you start the program, refer to the Using the CD-ROM section as needed.

For information on using the applications with your curriculum, see the Content Chart and the Teaching Suggestions sections.

As you make plans to use these activities with your students, refer to the Teacher Notes on each chapter. Student worksheets and answer keys are included.

Getting Started
• Installing Algebra 2 Multimedia Applications on a Macintosh or Windows system (p. 4)

Using the CD-ROM
• Main Menu (p. 6)
• Navigation Buttons (p. 6)
• Help Button (p. 7)
• Interactions (p. 7)
• Special Features: Video, Spoken Text, English/Spanish, Quizzes, and Student Worksheets (p. 8)

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• Each chapter’s content (p. 10)
• Learning objectives

Teaching Suggestions
• Introducing Students to the Program (p. 13)
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Student Worksheets
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Getting Started
Algebra 2 Multimedia Applications

Windows® Operating System

Minimum System Requirements
- 486SX/33MHz processor or higher; 486DX/66MHz recommended
- 8 MB RAM, 16 MB recommended
- Windows-compatible 16-bit sound card
- 2X CD-ROM drive or higher; 4X CD-ROM recommended
- 256 color VGA monitor
- Microsoft-compatible mouse
- Keyboard
- Microsoft Windows 3.1 operating system or higher; Windows 95 recommended
- No hard disk space needed, except for installation of QuickTime™

Installation Instructions
1. Insert the *Algebra 2 Multimedia Applications* CD-ROM disk into the CD-ROM drive.

2. From Windows 3.1, open Program Manager and choose Run from the File menu. From Windows 95, click the Start button and choose Run.

3. In the Command Line text box, type the drive letter for your CD-ROM drive, a colon, and then the word *setup*. (Example: d:setup) Press Enter or click OK.

4. If your computer does not already have QuickTime for Windows, you must install it. The Windows 95 version of Algebra requires the 32-bit version of QuickTime for Windows. Repeat steps 2 and 3, substituting *QTSetup* for *setup*. Follow the setup instructions.

5. To use *Algebra 2 Multimedia Applications* from Windows 3.1, open the Glencoe program group and double-click the Glencoe Algebra 2 icon. From Windows 95, click the Start button, select Glencoe, and select Glencoe Algebra 2.

*Note:* The CD-ROM disk must remain in the drive while the program is in use.
**Macintosh® Operating System**

**Minimum System Requirements**
- 68030 processor or higher required; 68040 or PowerPC processor recommended
- 8 MB RAM, 16 MB recommended
- 3.6 MB available space on startup hard disk
- 2X CD-ROM drive or higher, 4X CD-ROM recommended
- 256 color monitor
- Mouse
- Keyboard
- Macintosh System 7.0 or higher

**Installation Instructions**
1. Insert the *Algebra 2 Multimedia Applications* CD-ROM disk into the CD-ROM drive. The CD-ROM will open a window on the desktop.
2. In the Glencoe Algebra 2 window, double-click the Read Me file for more information on installing software for video.
3. If your system does not have current versions of QuickTime and Indeo® Video Codec, you will need to install them. They are both included on the Glencoe CD-ROM disk.
   - Double-click the Install QuickTime icon. The installer will automatically restart your computer when it is finished.
   - Double-click the Install Indeo Video icon. Your computer will automatically restart again when the installation is finished.
4. To use *Algebra 2 Multimedia Applications*, double-click the *Start Algebra 2* icon.
   - *Note:* The CD-ROM disk must remain in the drive while the program is in use.
Using the CD-ROM

Main Menu
The main menu appears automatically after the title screen. The titles are shown. Choose one by clicking the red button.

If you want to switch from English to Spanish spoken text, click the button at the bottom of the menu screen.

Navigation Buttons
Red buttons at the bottom left of each screen enable you to move from page to page, return to the main menu, or quit the program.

Click the forward arrow button to move to page 1.
**Using the CD-ROM**

Click the Jump button on the title screen or the page number button to show the Jump Bar.

Click on a page number to move directly to that page in the chapter.

**Help Button**
Click the Help button to see more information on pointers (cursors), navigation buttons, activity instructions, activities, video, speech, “hot links,” and Explore features.

**Interactions**
Each chapter consists of five or six pages. The pages and their content are varied to fit each specific chapter, but the following general format is used.

Title

1. Page 1 Introduction and goals
2. Page 2 An exploratory interaction
3. Page 3 Explanation or further explorations
4. Page 4 A major investigation
5. Page 5 Real-world applications
6. Page 6 Quiz

Students follow the step-by-step instructions on the left side of the interaction pages. Each set of instructions begins with an overview of the activity. Click each numbered step in sequence. Go back to reread previous screens if necessary.

Students complete worksheets as they work through the chapter. (See the section on Student Worksheets in this guide.)
**Special Features**

**Video**
Video controls are located under the video window. They simulate the standard control buttons and slider used on video equipment. Click the forward button to begin play. The sound control button on the far left adjusts the sound level.

The slider lets you replay part of the video. The pause button lets you stop the action.

---

**Spoken Text**
The printed information and instructions on all pages can be heard by clicking the speaker button at the beginning of each text block.

Set the spoken text to English or Spanish on the main menu screen.
English or Spanish
The spoken text can be heard in English or Spanish. Click the corresponding button at the bottom of the main menu. The printed text is always in English; the recorded voice provides the Spanish translation.

Quiz
Each chapter concludes with a quiz of four to six questions. This enables students to check that they have mastered the mathematics used in the activities. A score is reported at the end of the quiz, but it is not recorded. Students can click the Reset button to try the quiz again or let their partner take the quiz.

Student Worksheets
A three-page worksheet accompanies each chapter on the CD-ROM. Reproducible worksheets are included in this guide. An answer key for each is also provided. Each worksheet consists of two parts:

At the Computer
Students should complete this section at the computer to record their results and to further focus their thinking.

After the Computer
You may choose to use this page as homework. It does not require that the student have access to the CD-ROM, but reinforces and extends the computer activity. It includes a Mathematics Journal writing exercise.
## Content Chart

This chart provides an overview of the applications. For more detailed information, refer to the Teacher Notes for each chapter, which are included in this guide.

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Goals</th>
<th>Activities</th>
<th>Videos</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Analyzing Equations and Inequalities</td>
<td>The Mathematics of Sonar: A Deep Subject</td>
<td>1. Collect and analyze data.&lt;br&gt;2. Interpret graphs.&lt;br&gt;3. Find equations to fit data.&lt;br&gt;4. Relate graphs to maps.</td>
<td>• Graph ocean depth using sonar echoes.&lt;br&gt;• Map ocean floor using graphs.</td>
<td>A sailor explains how sonar is used to determine ocean depth.&lt;br&gt;A Navy technician explains how torpedoes use sonar.</td>
</tr>
<tr>
<td>2. Graphing Linear Relations and Functions</td>
<td>Driving Across America</td>
<td>1. Collect and graph data.&lt;br&gt;2. Fit lines to graphed points.&lt;br&gt;3. Interpret the slope and intercept of lines.&lt;br&gt;4. Use linear functions to model events.</td>
<td>• Graph fuel efficiency and interpret slope.&lt;br&gt;• Model fuel efficiency for a trip.</td>
<td>A CAD technician describes the use of slope in bridge design.</td>
</tr>
<tr>
<td>5. Exploring Polynomials and Radical Expressions</td>
<td>Complex Numbers and Fractals</td>
<td>1. Graph complex numbers.&lt;br&gt;2. Multiply complex numbers.&lt;br&gt;3. Use complex numbers to create Julia set fractals.&lt;br&gt;4. Apply iteration.</td>
<td>• Iterate functions using complex numbers.&lt;br&gt;• Create and compare Julia set fractals.</td>
<td>An electrical engineer shows how complex numbers are used to represent impedance.</td>
</tr>
<tr>
<td>Chapter</td>
<td>Title</td>
<td>Goals</td>
<td>Activities</td>
<td>Videos</td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
<td>-------</td>
<td>------------</td>
<td>--------</td>
</tr>
</tbody>
</table>
| 6. Exploring Quadratic Functions and Inequalities | Maximize Your Profits—It’s the Quadratic Way! | 1. Record, plot, and analyze data.  
2. Express the number sold as a function of price.  
3. Fit linear functions to data.  
4. Use quadratic functions to find maximum profit. | • Set the selling price for T-shirts and observe resulting sales and profit.  
• Create a quadratic equation for profit and find the maximum point. | A video describes how city planners use quadratic functions to model traffic flow. |
2. Explore polynomial functions.  
3. Predict the degree of polynomial from its graph.  
4. Use polynomials as models. | • Create tracks and graph time, distance, and speed.  
• Model tracks with polynomial functions.  
• Evaluate tracks for safe speed. | A civil engineer describes how she uses polynomial functions in designing tunnels. |
| 8. Analyzing Conic Sections | Orbits and Intersecting Paths | 1. Explore circles and ellipses.  
2. Use ellipses to represent orbits.  
3. Determine the intersection of two orbits. | • Explore circles and ellipses by manipulating their graphs.  
• Use an ellipse and a circle to model orbits and find intersection points. | A video describes astronomers’ predictions that a comet would collide with Jupiter. |
2. Interpret graphs.  
3. Solve rational equations.  
4. Apply rational equations to photography. | • Move a camera lens and film to focus pictures and graph data.  
• Interpret a rational function graph to focus hummingbird pictures. | An electrical engineer describes how rational functions model resistors. |
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Goals</th>
<th>Activities</th>
<th>Videos</th>
</tr>
</thead>
<tbody>
<tr>
<td>10. Exploring Exponential and Logarithmic Functions</td>
<td>Exponential Growth in a Drop of Water!</td>
<td>1. Compare exponential and other growth. 2. Collect, graph, and analyze data. 3. Fit an exponential function to data. 4. Apply exponential models to biological processes.</td>
<td>• Graph three growth models and compare plankton growth. • Collect and graph plankton growth data and fit an exponential function.</td>
<td>A campground owner describes how her invested money grows exponentially.</td>
</tr>
<tr>
<td>12. Investigating Discrete Mathematics and Probability</td>
<td>Calculating Probability</td>
<td>1. Collect and analyze data. 2. Calculate probabilities. 3. Compare exponential and theoretical probabilities.</td>
<td>• Investigate probabilities in a game show. • Experiment and analyze probabilities in a game of chance.</td>
<td>An epidemiologist describes how probability is used to study diseases by analyzing samples from populations.</td>
</tr>
<tr>
<td>13. Exploring Trigonometric Functions</td>
<td>Mathematics and Forest Fires: A Hot Combination</td>
<td>1. Measure angles in degrees and radians. 2. Construct a protractor. 3. Apply trigonometric functions to locating fires. 4. Solve problems involving right triangles.</td>
<td>• Explore angle measurement in radians and degrees. • Apply trigonometry to locating forest fires.</td>
<td>A civil engineer explains how she uses trigonometry to calculate angles for drainage pipes.</td>
</tr>
<tr>
<td>14. Using Trigonometric Graphs and Identities</td>
<td>Trigonometric Functions: A Sound Tool</td>
<td>1. Explore the sine function and its graph. 2. Identify amplitude and cycles. 3. Find sine functions to model sounds.</td>
<td>• Explore frequency and amplitude of sine functions. • Apply sine functions to testing audio equipment.</td>
<td>A harpist describes how he plucks harp strings to vary pitch and volume.</td>
</tr>
</tbody>
</table>
Teaching Suggestions

Introducing Students to the Program
These materials are extremely easy to use. Students do not require training to operate this program. A brief demonstration will help them use their time most efficiently. You may want to show how to move from page to page, access the help feature, and follow the steps for an activity. A demonstration computer system with either a large monitor or projection device works well for whole-class demonstrations.

Emphasize that students will explore real-world situations. Many activities involve collecting and analyzing data. The video segments describe how people use mathematics in their careers.

Be sure to explain how the worksheets are used to record students’ results and help clarify their thinking. Students should read, view, or explore a page on the screen and then do the worksheet exercises that correspond to that page. Then they are ready to move to the next page on the computer.

Explain how you want your students to use the quiz at the end of the chapter, individually or with a partner, and whether you will assign the After the Computer worksheet as homework.

Students in a Lab Setting
A lab setting with two students at a computer is ideal for using these materials. Pairing students with partners is highly recommended, even if there are enough computers for every student.

Working with Partners
Benefits of working with a partner include increased communication, sharing of ideas, and more thoughtful work. Also, partners can check results and keep track of worksheet exercises. Be sure to have partners share responsibilities equally.

In the Lab
After a brief demonstration of the program, students can easily work independently in a lab setting. Briefly describe how the computer activities relate to recent class activities. Distribute worksheets, and direct students to the chapter they will use. Clarify how much time is available and how they should use the worksheets and the computer quiz.

After the lab, have students compare their results using the worksheet exercises as a guide.
Whole Class
If only one computer is available, use it with one or more large monitors or a projection device for the whole class. Test your system to be sure that the screens are clearly visible to your students and that the audio levels enable all students to hear the video segments.

Using Worksheets
Distribute the worksheets for the chapter. Briefly describe how the computer activities relate to recent class activities.

Either operate the program yourself or have a student operate it. Pause at each page to allow time for students to complete the corresponding worksheet exercises. As time permits, have students discuss their responses to the computer activities as you go.

In activities that involve data gathering, have one or more students operate the program to decide what data to gather. Then the rest of the class will record and use these same data.

Individual Students
Students can also use the program individually (or with partners) at a computer located in a classroom or media center. If possible, provide a brief demonstration to all students before they begin using the program independently. You may want to designate a few experienced students as “computer consultants” to assist other students if the need arises.

Using Worksheets
Make copies of the worksheets available. Although the CD-ROM is effective on its own, the worksheets provide a record of student work and focus student thinking on the mathematics in the activity. This can be especially valuable when students work with the program individually.

Follow up with discussions of the activity, either in small groups or as a whole class.
Assessment

A major assessment tool for these applications is teacher observations of students at work at the computer. Are students able to follow the instructions to complete the activities? Do students make choices in the program that demonstrate their understanding of the mathematics involved?

Listen to the discussions between student pairs to assess how well the concepts are understood. Do students discuss, explain, and defend their ideas and conclusions with their partners? Do students record appropriate responses to worksheet exercises?

After the computer activity, discuss the activity and have student groups share their results.

The quiz at the end of each chapter enables students to assess their understanding of concepts and skills in the chapter. You may want students to take the quiz individually and record their score on their worksheet. You may encourage students to retake the quiz to try to make a “perfect score.” You may choose to have students who are working with partners take the quiz jointly.

The worksheet exercises provide a written record of student work. You may choose to assign the After the Computer worksheet page as homework.
The Mathematics of Sonar: A Deep Subject

Interactive Activities
- Graph ocean depth using sonar echoes.
- Map ocean floor using graphs.

Video
- A sailor explains how sonar is used to determine ocean depth.
- A Navy technician explains how torpedoes use sonar.

Prerequisites
- Read graphs.
- Evaluate equations.
- Graph linear equations.

Students move a simulated ship and take sonar readings along its path. They interpret the graph of distance along the path and ocean depth. They graph the ping time of the sonar echo and the depth reading. Then they observe that this graph is linear, because the speed of sound in water is constant in a specific portion of the ocean.

They write equations for the distance of the ship from a buoy and the depth of the ocean, based on the ping time.

Students fit a line to collected data points to determine the speed of sound in water. They interpret graphs of distance and ocean depth along various paths and use this information to create a color contour map of a portion of the ocean floor.

Sample Screen
Goals
1. Collect and analyze data.
2. Interpret graphs.
3. Find equations to fit data.
4. Relate graphs to maps.

Content
Page 1 Introduction and Goals
Page 2 Exploration: Graph Ocean Floor
Page 3 Video: Sonar
Page 4 Application: Map Ocean Floor
Page 5 Video: Torpedoes and Sonar
Page 6 Quiz

Objectives
Page 2 Exploration
Measure and graph ocean depth using sonar.

Page 3 Application
Use equations and graphed data to map the ocean floor.

Tips & Suggestions
• You may want to relate this activity to students’ experiences in boats or canoes on lakes and rivers.
• On page 4, the speed of sound is 4800 feet per second.
• On page 4, the depth tiles that students place on the map are considered correctly placed if they are “reasonable” based on the average depth in that section of the simulated ocean.
• Students need to understand the following terms: sonar, ping time, and buoy.

Extensions
• Compare the speed of sound in water with the speed of sound in air and other substances.
• Research the need for sonar in both military and civilian uses.
Driving Across America

Students collect data on fuel efficiency (miles per gallon) during several segments of a simulated car trip on varying terrain. They interpret a graph of distance traveled and gallons of gasoline used. They move a line to fit the data and observe the slope of the line. They relate the slope of the best fit line with the average number of miles per gallon for the trip.

Students review the concept of slope and how to calculate it from the coordinates of two points on a line.

On a simulated road trip across the U.S., students collect data on distance and gallons of gasoline used. They graph the data and fit a linear equation to each segment of the trip. They find a linear equation to model the entire trip and interpret the slope of the line as the average fuel efficiency.

Sample Screen
Goals
1. Collect and graph data.
2. Fit lines to graphed points.
3. Interpret the slope and intercept of lines.
4. Use linear functions to model events.

Content
Page 1 Introduction and Goals
Page 2 Exploration: Slope and Fuel Efficiency
Page 3 Information: Calculating Slope
Page 4 Application: Model Fuel Efficiency
Page 5 Quiz

Objectives
Page 2 Exploration
Relate slope to fuel efficiency (MPG).

Page 4 Application
Use linear functions to predict.

Tips & Suggestions
• You may want to relate this activity to the terrain in your area, to car trips students have taken, or to the costs of operating a car.

• On page 2, be sure that students notice that it is the x-intercept, not the more familiar y-intercept, that is changed by moving the slider.

• Students need to understand the following terms: terrain, fuel efficiency, miles per gallon, and slope.

Extensions
• Compare gas mileage for various similar cars, vans, or trucks. Graph the data and investigate the variation. Explain the differences between city and highway MPG rates. Compare these to the terrain differences in this activity.
Mathematics: A Moving Experience

Students explore points and planes in three dimensions. They move a point in simulated space, observing the point’s coordinates. They explore how a plane intersects the three axes.

Students create an equation in three variables to represent the cost of renting a truck.

\[
\text{Total Cost} = \text{Per Day Fee} \times \text{Number of Days} + \text{Per Mile Fee} \times \text{Number of Miles} + \text{Fixed Fee}
\]

From data on truck rental fees for two companies, students write equations for their total costs. These equations are graphed in three dimensions. Students manipulate a point on the \(x-y\) plane and observe the two corresponding points on the planes that represent the two companies. They determine the conditions under which one company’s rental cost is lower.

Sample Screen
Goals
1. Visualize planes, lines, and points in three dimensions.
2. Graph linear equations in three variables.

Content
Page 1  Introduction, Goals, and Video: CAD Technician
Page 2  Exploration: Graph Points and Planes
Page 3  Activity: Equations with Three Variables
Page 4  Application: Solve Equations Graphically
Page 5  Information: Biology Researchers; Pilots
Page 6  Quiz

Objectives
Page 2 Exploration
Explore three-dimensional graphs.

Page 3 Application
Graphically solve systems of equations in three variables.

Tips & Suggestions
• On page 2, three-dimensional space is displayed on a two-dimensional computer screen. Many points \((x, y, z)\) map onto the same blue point \((x, y)\). The gray “shadow” point identifies the specific point that is displayed.

• On page 4, the truck rental fee data change each time this activity is begun.

• On page 4, there is a range of tolerance on the green line of intersection of the planes. This can make the two values for \(z\) slightly different, even though the gray dot appears to be on the green line.

• On page 4, be sure that students use the Explore sequence located in the Instructions, Step 3 Explore the Graphs. This provides an explanation of the green line of intersection of the two planes.

• The profit equation is central to this interaction. Be sure students complete and understand the equation activity on page 3 before they begin page 4.

• Students need to understand the following terms: octant, intercepts, and plane.

Extensions
• Create cardboard models of intersecting planes.

• Research CAD (Computer Aided Design), including the features of CAD software and the career fields where it is used.
Students explore matrices that model rotations and apply them to rotations of a jet plane. They experiment with multiplication of matrices.

Students relate clockwise and counterclockwise rotations.

They apply rotation matrices to programming a jet plane’s attitude for a flight simulator. After selecting appropriate matrices, students test their program and observe whether the jet crashes or runs the course successfully.

Sample Screen
Goals
1. Apply matrix multiplication.
2. Relate matrices to rotations.
3. Model aircraft attitude with matrices.

Content
Page 1 Introduction and Goals Activity: Matrices and Rotation
Page 2 Exploration: Modeling Rotations
Page 3 Activity: Angles of Rotation
Page 4 Application: Jet Simulator
Page 5 Video: Seismologist
Page 6 Quiz

Objectives
Page 2 Exploration
Multiply matrices to model rotations.

Page 4 Application
Multiply matrices to program a jet simulation.

Tips & Suggestions
• Students need to understand the following terms: matrix, matrices, attitude, simulator, counterclockwise, and rotation.

Extensions
• Flight simulators can be found in some aviation museums and amusement parks. Flight-simulator software games are available. Students might report on their experiences with these simulators.
• Research other applications of matrices.
Complex Numbers and Fractals

Students review the process of graphing points on the complex plane. They explore iterated functions using complex numbers.

\[ f(x) = x^2 + c, \text{ where } x \text{ and } c \text{ are complex numbers} \]

They graph the iterated values and test for escaping points and prisoner points.

Students review the multiplication of complex numbers.

They perform the first iterations for a function, observe whether the point is an escaping point or a prisoner point, and see the corresponding fractal image generated by the computer. They compare three Julia sets, with varying values of \( c \).

Sample Screen
Goals
1. Graph complex numbers.
2. Multiply complex numbers.
3. Use complex numbers to create Julia set fractals.
4. Apply iteration.

Content
Page 1 Introduction and Goals
   Activity: Graphing Complex Numbers
Page 2 Exploration: Escaping and Prisoner Points
Page 3 Activity: Multiplying Complex Numbers
Page 4 Application: Julia Set Fractals
Page 5 Video: Electrical Engineer
Page 6 Quiz

Objectives
Page 2 Exploration
Graph iterated functions of complex numbers.

Page 4 Application
Apply complex numbers to Julia set fractals.

Tips & Suggestions
- You may want to point out that there are many types of fractal images and many ways to generate fractals. These activities use iteration of a quadratic function in the complex plane.
- On page 2, students click on the graph to graph each complex number. The + cursor must be placed within 0.1 of the actual point in both the x and y directions in order to be considered correct.
- On page 2, if students are not familiar with iteration, be sure they use the Explore activity included in Step 3 Escaping Point.
- On page 4, the fractals are not actually being calculated as they are displayed, because this would take too long on slower computers. The fractal images are stored on the CD-ROM and displayed one line at a time.
- Students need to understand the following terms: iteration, complex numbers, imaginary, escaping point, prisoner point, and fractal.

Extensions
- Research other types of fractal images.
- Research Julia sets and their connection to the Mandelbrot set.
Maximize Your Profits—It’s the Quadratic Way!

Students take the role of a T-shirt shop owner. They set the selling price for the T-shirts, given the fixed expenses and variable expenses for their business. Then they observe the results, the number of shirts sold, and the profit. They look for patterns in the data.

Students fit a line to the data for the number of shirts sold and profit gained and use this linear function to model the number sold.

Then they use this linear function to write a quadratic function to represent profit. They graph the profit function and find the maximum point by observing the graph. They interpret this point in terms of the selling price, profit, and number of shirts sold.

\[
\text{Profit} = (\text{Number Sold} \times \text{Price}) - (\text{Number Sold} \times \text{Fixed Expenses}) - \text{Variable Expenses}.
\]

Sample Screen

Interactive Activities
- Set the selling price for T-shirts and observe resulting sales and profit.
- Create a quadratic equation for profit and find the maximum point.

Video
- A video describes how city planners use quadratic functions to model traffic flow.

Prerequisites
- Graph quadratic equations.
- Read and interpret linear and quadratic graphs.
- Fit a linear function to data points.
Goals
1. Record, plot, and analyze data.
2. Express the number sold as a function of price.
3. Fit linear functions to data.
4. Use quadratic functions to find maximum profit.

Content
Page 1 Introduction and Goals
Page 2 Exploration: Selling Price and Profit
Page 3 Information: Setting Prices
Page 4 Application: Maximum Profit
Page 5 Video: Traffic Flow Models
Page 6 Quiz

Objectives
Page 2 Exploration
Use data and graphs to explore selling price.

Page 4 Application
Use quadratic equations to maximize profit.

Tips & Suggestions
- On page 2, the variable expense (wholesale cost of each T-shirt) is always $4.00. The fixed expense varies among $25, $50, $75, or $100. The selling price can be any integer from $1 through $15. The number sold is calculated using a formula that includes a random number, but the variation is less than would actually occur.
- On page 2, the use of parentheses and red color (instead of a negative sign) to indicate loss is standard accounting practice.
- On page 4, the selling price can include fractions of dollars. New data is presented in the data table, and students can enter more data.
- Students need to understand the following terms: variable expense, fixed expense, revenue, profit, and selling price.

Extensions
- If students know people who own small businesses, they may be able to find data on selling prices, sales figures, and profit for various items. Students might interview business owners and ask how they set their prices.
- Research how prices are set in school for lunch items, concert tickets, or athletic events. Describe how price affects number sold and profit.
Exploring Polynomial Functions

The Mathematics Behind Thrill Rides

Students create several simulated tracks for rollercoaster-like rides at an amusement park. A simulated car travels their track. The car's time, distance, and speed are graphed. Students interpret the graphs in terms of the track design.

Students manipulate graphs of polynomial functions of degree 1 through 5 by changing coefficients. They observe the general shape of each degree polynomial.

Given four different track designs, students fit a polynomial function to each track by modifying coefficients and observing the resulting graph. Then they test each track and graph the car's speed. If the speed does not exceed 45 feet per second, the track is considered safe. Students evaluate the tracks by interpreting the graph of time and speed.

Interactive Activities
- Create tracks and graph time, distance, and speed.
- Model tracks with polynomial functions.
- Evaluate tracks for safe speed.

Video
- A civil engineer describes how she uses polynomial functions in designing tunnels.

Prerequisites
- Read and interpret polynomial graphs.
- Identify the degree of polynomial functions.
- Fit linear functions to data points.

Sample Screen

Objective: Explore graphs of time, distance, and speed.

1. Overview
   - The variables in this rollercoaster simulation are time, distance along the track, and speed. You will explore these variables as you design a track.
   - You will:
     - change the track shape, and
     - interpret three types of graphs:
       - Time and Speed,
       - Time and Distance, and
       - Distance and Speed.
Goals
1. Interpret graphs of time, distance, and speed.
2. Explore polynomial functions.
3. Predict the degree of a polynomial from its graph.
4. Use polynomials as models.

Content
Page 1 Introduction and Goals
Page 2 Exploration: Tracks and Graphs
Page 3 Activity: Graphing Polynomials
Page 4 Application: Fit Polynomial Functions
Page 5 Video: Civil Engineer
Page 6 Quiz

Objectives
Page 3 Exploration
Explore graphs of time, distance, and speed.

Page 4 Application
Fit polynomial functions to track curves.

Tips & Suggestions
- You may want to have students experiment with tracks and balls. Use cylindrical foam pipe covers, available at low cost in hardware stores. Cut them in half lengthwise to form a trough. Provide masking tape to connect lengths to form curved tracks. Students can tape the tracks to chairs or desks for height. Use marbles. Unsafe tracks allow the marble to “jump the track.” Students can time the run and observe changes in time when the track design is modified.
- On page 4, the first two tracks are fairly easy to fit. The last two tracks are more difficult. If students become frustrated with Tracks 3 and 4, point out the Solve It button in the lower right corner. Clicking this button will show the correct polynomial and allow students to proceed to the next screen to test the track.
- Students need to understand the following terms: polynomial and degree.

Extensions
- If there is an amusement park in your community, students may be able to interview ride operators about safe speed.
- Your school’s physics teacher may use amusement park rides as examples of physics principles. You might connect polynomials with part of the physics curriculum.
Orbits and Intersecting Paths

Interactive Activities

- Explore circles and ellipses by manipulating their graphs.
- Use an ellipse and a circle to model orbits and find intersection points.

Video

- A video describes astronomers’ predictions that a comet would collide with Jupiter.

Prerequisites

- Graph coordinate points.
- Identify points of intersection of two curves.

Students manipulate graphs of circles and ellipses, observing the changes in the values of the variables $a$, $b$, $h$, $k$, and $r$.

They learn about the elliptical orbits of comets and asteroids and the nearly circular orbit of Earth.

Students use graphical methods to construct the equation of an ellipse to model an asteroid’s orbit and the equation of a circle to model Earth’s orbit. Then they estimate the points of intersection of these two orbits.

Sample Screen
Goals
1. Explore circles and ellipses.
2. Use ellipses to represent orbits.
3. Determine the intersection of two orbits.

Content
Page 1 Introduction and Goals
Page 2 Exploration: Circles and Ellipses
Page 3 Information: Foci, Axes, and Orbits
Page 4 Application: Intersecting Orbits
Page 5 Video: Predicting Comets' Paths

Objectives
Page 2 Exploration
Explore circles and ellipses.

Page 4 Application
Find intersections of elliptical and circular orbits.

Tips & Suggestions
• Relate this activity to comets recently near Earth or other planets in our Solar System, for example Hale-Bopp in 1997.

• On page 4, the ellipse shown at first always has \( a = 151, b = 109, \) and \( b = -105, \) all in millions of miles. When the New Problem button is clicked, a new ellipse is shown. It is chosen using a formula that ensures that the maximum distance from the sun is always 255.75 million miles, the distance of the asteroid belt.

• On page 4, to be considered correct, the coordinates of the intersection point must be within 2 million miles of the actual equation value.

• Students need to understand the following terms: axis, focus, center, orbit, asteroid, and comet.

Extensions
• Research orbits of comets and asteroids that have recently approached our Solar System.
Exploring Rational Expressions

The Mathematics of Photography

Students use a rational function, called the focus equation:

\[ \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \]

They identify the distances \( p \), \( q \), and \( f \). In a simulated camera, they manipulate the lens and film positions to focus the image. Then they collect and graph the data for \( p \) and \( q \) and the focal length \( f = 10 \). They describe the graph.

Students investigate how various values of \( q \), the distance from the lens to the film, affect the focus of the picture.

They interpret the focus equation and various values of \( q \), to determine the \( q \) value that results in the highest percentage of focused pictures.

Interactive Activities
- Move a camera lens and film to focus pictures and graph data.
- Interpret a rational function graph to focus hummingbird pictures.

Video
- An electrical engineer describes how rational functions model resistors.

Prerequisites
- Solve rational equations.
- Interpret graphs of rational functions.

Sample Screen
Goals
1. Collect, graph, and analyze data.
2. Interpret graphs.
3. Solve rational equations.
4. Apply rational equations to photography.

Content
Page 1 Introduction and Goals
   Activity: Focus Equation
Page 2 Exploration: Focusing Photo Images
Page 3 Activity: Focus and q Distance
Page 4 Application: Optimizing q in a Camera
Page 5 Video: Electrical Engineer
Page 6 Quiz

Objectives
Page 2 Exploration
Use a rational equation to focus images.

Page 4 Application
Apply rational equations and graphs to photography.

Tips & Suggestions
• On page 2, be sure students understand that only a small portion of the function is graphed, 15 cm < p < 31 cm. On page 4, more of the focus equation graph is shown.
• On page 4, to enter the expression for q, first click in the box, then press the / key, and then enter the numerator.
• On page 4, after entering a new value for q, press the Enter or Return key to see the new position for the blue line.
• On page 4, the actual value of f is 5 cm. The image is focused or blurred in the following ranges: focused if 4.98 < f < 5.02; blurred if 4.9 < f < 4.98 or 5.02 < f < 5.1; badly blurred if f < 4.9 or f > 5.1.
• Students need to understand the following terms: focus, focal length, and lens.

Extensions
• Research the focal length and the q distance in cameras available in local stores. A camera shop or a manufacturer may be able to provide this information.
• Research the use of rational functions to model resistance in electrical circuits.
Interative Activities
- Graph three growth models and compare to plankton growth.
- Collect and graph plankton growth data and fit an exponential function.

Video
- A campground owner describes how her invested money grows exponentially.

Prerequisites
- Operate with exponents and $e$.
- Graph exponential functions.

Exploring Exponential and Logarithmic Functions

Exponential Growth in a Drop of Water!

Students experiment with a plankton-growth simulation, observing the growth and comparing the graphs for three different growth models: linear, quadratic, and exponential. They compare actual plankton growth and observe that the exponential function is the best model.

An animation sequence introduces the concept of plankton reproduction by cellular division. This results in a sequence, $2^n$, for positive integers $n$.

Students count simulated plankton on a microscope slide each hour for about eight to ten hours. They record and graph their data and then fit an exponential function to the data. The function is of the form $y(t) = 3k^t$, where $t$ is time in hours and $k$ is a value between about 1.22 and 1.34. Students then use their model to calculate the plankton population at 25 hours, which would be extremely tedious to count.

Sample Screen
Goals
1. Compare exponential and other growth.
2. Collect, graph, and analyze data.
3. Fit an exponential function to data.
4. Apply exponential models to biological processes.

Content
Page 1 Introduction and Goals
Page 2 Exploration: Plankton Growth
Page 3 Information: Cellular Division Sequence
Page 4 Application: Modeling Plankton Growth
Page 5 Video: Business Owner
Page 6 Quiz

Objectives
Page 2 Exploration
Explore function models for growth.

Page 4 Application
Apply exponential functions to plankton growth.

Tips & Suggestions
- On page 2, the time counter shows tenths of hours. For example, 158 means 15.8 hours. The tenths’ digit is white on a black background. To increase time by one hour, click the arrows or click the slider bar between the slider and the arrow. To increase by fractions of an hour, drag the slider.

- On page 2, the plankton simulator displays a maximum of only 2000 plankton.

- On page 2, the “actual plankton” simulated here fit the growth model $1.8^t$.

- On page 4, clicking the time arrows changes the time by one hour. To skip an hour, hold down the arrow button.

- On page 4, encourage students to collect data on ten hours of growth. Eight hours is a minimum. Ten hours of data produce a better graph that makes the curve fitting easier.

- On page 4, you can investigate a new sample of plankton with a different growth rate by returning to the first screen and clicking the New Sample button.

- On page 4, the base value that students enter lies between about 1.22 and 1.34.

Extensions
- Research growth rates of other biological organisms.
- Research and compare interest rates at local banks.
Investigating Sequences and Series

Anatomy of Geometric Sequences

Students explore fractal trees by setting the initial length (the trunk) and the length ratio. They choose the number of stages. The fractal tree is drawn and data on the number and length of branches is recorded in a table. Students use these data to write equations that describe the fractal image.

Students are shown a simplified diagram of the bronchial system (the lungs), which consists of a fractal tree. They measure the branches at several stages, calculate the ratios of branch length, and calculate the average ratio. They use this average ratio to draw a fractal tree that models the bronchial system.

Sample Screen
Goals
1. Collect and analyze data.
2. Explore ratios in geometric sequences.
3. Investigate fractal trees.
4. Create a fractal model of lungs.

Content
Page 1 Introduction and Goals
Page 2 Exploration: Fractal Trees
Page 3 Information: Fractals in Medicine
Page 4 Application: Fractal Model of Lungs
Page 5 Video: Biochemist
Page 6 Quiz

Objectives
Page 2 Exploration
Explore fractals and geometric sequences.

Page 4 Application
Create a fractal model of the bronchial system.

Tips & Suggestions
• On page 2, the fractal tree is created by drawing two branches, reduced by the ratio $r$, at the end of the first branch. One branch is 60 degrees to the left; the other is 60 degrees to the right.

• On page 4, only three digits plus a decimal point fit in the boxes. For example, 0.72 or .723 will fit, but 0.723 will not fit. You can decide whether you prefer using the leading zero or having an additional decimal place of accuracy.

• On page 4, student measurements may not be exact. In real situations, measurements do include some error. The lengths do not exactly fit one ratio. Students average the ratios and use the average value to build the fractal tree model.

• Students need to understand the following terms: fractal, fractal tree, and branch.

Extensions
• Research other uses of fractal models.

• Find other examples of geometric sequences in nature.
Calculating Probabilities

Students collect data on success and failure in a simulated television game show. They investigate two different strategies for playing and compare the experimental probabilities of winning.

They review the formula for calculating the theoretical probability of success, based on the number of successes in the total number of outcomes.

Students calculate the theoretical probabilities of outcomes in a Hopi game of chance called Totolospi. They play the game to collect data. They calculate the experimental probabilities for the outcomes and compare these to the theoretical probabilities.

Sample Screen
Goals
1. Collect and analyze data.
2. Calculate probabilities.
3. Compare experimental and theoretical probabilities.

Content
Page 1 Introduction and Goals
Page 2 Exploration: Game Show Probability
Page 3 Information: Outcomes and Successes
Page 4 Application: Probabilities of a Game
Page 5 Video: Probability in Epidemiology
Page 6 Quiz

Objectives
Page 2 Exploration
Explore probabilities in games.

Page 4 Application
Apply probability to a game of chance.

Tips & Suggestions
• The game show activity on page 2 uses only experimental probabilities. On the student worksheet, the After the Computer Session portion directs students to calculate the theoretical probabilities in this game.
• On page 2, a small number of games may result in an experimental probability of winning that is quite different from the theoretical probability.
• On page 4, students first calculate theoretical probabilities and then play the game to collect experimental data.
• On page 4 when playing the game, be sure students take time between tosses to see what is occurring. The column of the player who will toss is colored yellow. The result of the most recent toss is colored red (player 1) or blue (player 2).
• On page 4, the experimental probabilities are calculated using the sum of both players’ tosses.
• Students need to understand the following terms: event, outcome, success, failure, independent, mutually exclusive, theoretical, experimental, and sample.

Extensions
• Research the uses of probability and sampling in epidemiology, political polls, environmental studies, or other fields.
Exploring Trigonometric Functions

Mathematics and Forest Fires: A Hot Combination

Interactive Activities
• Explore angle measurement in radians and degrees.
• Apply trigonometry to locating forest fires.

Video
• A civil engineer explains how she uses trigonometry to calculate angles for drainage pipes.

Prerequisites
• Recognize angles in multiples of 45 degrees.
• Interpret sine and cosine ratios in right triangles.

Students begin by exploring radian measures of angles and the unit circle. They compare degree and radian measures and write formulas to convert from one measurement unit to the other.

They learn how to use the sine and cosine functions along with angle measures and distance measures to calculate the coordinates of a vertex of a right triangle.

Students construct a protractor to measure angles in radians. Then they use the protractor to measure the angle of a simulated forest fire from the fire lookout town. They calculate the $x$ and $y$ legs of the right triangle using trigonometric functions. These coordinates enable the firefighters to drop water on the fire to extinguish it.

Sample Screen
Goals
1. Measure angles in degrees and radians.
2. Construct a protractor.
3. Apply trigonometric functions to locating fires.
4. Solve problems involving right triangles.

Content
Page 1  Introduction and Goals
Page 2  Exploration: Radian Measure
Page 3  Information: Right Triangles
Page 4  Application: Locating Forest Fires
Page 5  Video: Civil Engineer
Page 6  Quiz

Objectives
Page 2 Exploration
Use a square root expression to find distance.

Page 4 Application
Fit square root and quadratic functions to distance data.

Tips & Suggestions
• On page 2, dragging the arrow from the zero position counterclockwise creates positive angles. To create negative angles, return to the zero position and then drag the arrow clockwise.
• On page 4, the protractor the students create measures angles in radians, not degrees.
• On page 4, clicking the New Problem button moves the fire to a new location.
• Students need to understand the following terms: degree, radian, protractor, sine, and cosine.

Extensions
• Research the mathematics of water drainage systems.
• Research the process of fighting forest fires.
Using Trigonometric Graphs and Identities

Trigonometric Functions: A Sound Tool

Students experiment with a simulated signal generator and amplifier. They change the number of cycles and the volume of the sound signals and observe the graphs of the sounds they create. They relate the shape of the graph to the values for cycles (pitch) and amplitude (volume).

The general equation is \( y = a \sin b \theta \).

They see how oscilloscopes are used to test audio equipment.

Students take the role of a quality control engineer and test the quality of speakers. For each test signal, students determine the number of cycles, \( b \), and the amplitude, \( a \), for the sine function. The function is graphed and students observe whether or not they have matched the graph of the test signal. Then they compare the graphs of the simulated speaker output and the test signal to decide whether the speaker passes the test.

Sample Screen
Goals
1. Explore the sine function and its graph.
2. Identify amplitude and cycles.
3. Find sine functions to model sounds.

Content
Page 1 Introduction and Goals
Page 2 Exploration: Cycles and Amplitude
Page 3 Information: Oscilloscopes
Page 4 Application: Testing Speaker Quality
Page 5 Video: Harpist
Page 6 Quiz

Objectives
Page 2 Exploration
Explore amplitude and cycles in sound signals.

Page 4 Application
Use trigonometric functions to test speaker quality.

Tips & Suggestions
• On page 2, be sure students understand the similarities and differences between cycles and frequency.
• On page 4, the amplitudes are always multiples of 0.5. However, values within 0.1 of the actual value are considered correct.
• On page 4, the speaker passes Test 1 and Test 2 100% of the time. However, it passes Test 3 and Test 4 just 70% of the time. The test signals change each time the activity is done.
• Students need to understand the following terms: cycle, amplitude, frequency, pitch, volume, absolute value, and vibration.

Extensions
• Research the frequencies of radio broadcasts or cell phones.
• Experiment with computer software that graphs sounds.
• Research electronically generated music.
The Mathematics of Sonar: A Deep Subject

At the Computer

Depth Measurement
1. Explain why depth measurements are important to ocean vessels.

Graph Distance and Depth
2. Sketch the graph of distance and depth.

<table>
<thead>
<tr>
<th>Depth (ft)</th>
<th>Distance from buoy (feet)</th>
</tr>
</thead>
</table>

a. How deep is the deepest part of the ocean along the ship’s path? _________
b. What is the shallowest part of the ocean floor along the ship’s path? _________

Mark these points on your graph.

3. Sketch a cross section of the ocean floor along the ship's path. Use the same x-axis scale. Show sea level and the ocean floor below.

Graph Distance and Time
4. Sketch the graph of depth and time.

<table>
<thead>
<tr>
<th>Depth (ft)</th>
<th>Ping Time (seconds)</th>
</tr>
</thead>
</table>

a. What type of graph is this? _________
b. What variable is graphed on the horizontal axis? ____________________________
c. Explain the term ping time.

______________________________
______________________________
Sonar and Equations
5. Record the two equations you created.
   Distance from Buoy = 
   Depth of Ocean = 

6. Summarize the main points in the video about sonar.

The Speed of Sound In Water
7. Record the equation for depth, including the value for the speed of sound.
   \[ d = \]

8. Record the speed of sound in water. Include the units of measurement.

Graphs and Map
9. On the contour map, what is the difference between the darker cells and the light cells?

10. Explain how the graph of distance and depth helped you decide where to place the contour map cells.

Sonar and Torpedoes
11. Based on the video, describe how Navy torpedoes use sonar.
Math Journal

12. Suppose you are building a new sonar receiver to calculate ocean depth. List in order the steps that will be needed to make the calculations. The first two steps are given. Use the depth equation and the speed of sound in water.

1. Record the start time of the sonar ping.
2. Send the sonar ping.
Driving Across America

At the Computer

Fuel Efficiency

1. Record the ratio for fuel efficiency.

2. Record the values for intercept and slope of the line for each segment. Record the terrain type (uphill, downhill, or flat).

<table>
<thead>
<tr>
<th>Segment</th>
<th>x-Intercept</th>
<th>Slope</th>
<th>Terrain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Which terrain results in the best fuel efficiency (highest MPG)? ____________
   Which terrain results in the worst fuel efficiency (lowest MPG)? ____________

4. Describe the slope of the line representing best fuel efficiency. ____________

5. Record the slope of the fitted line that best fits all the points.
   Slope ____________

6. Calculate the fuel efficiency (MPG) for the whole trip by using the coordinates of the last point of the graph.
   Coordinates ( __________ , __________ ) Record the MPG. __________
   Compare the slope of the fitted line and the MPG value for the whole trip.

7. What does the slope of this fitted line represent?

   ____________
Slope
8. Explain how to calculate the slope of the line through two given points.

Linear Functions as Models
9. Record the data table.

<table>
<thead>
<tr>
<th>(d) miles</th>
<th>(g) gallons</th>
<th>change in (d)</th>
<th>change in (g)</th>
<th>MPG</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

10. Record your equation for each segment.

<table>
<thead>
<tr>
<th>equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

11. Suppose you were driving this van on a route through flat terrain.
   Which equation would model this trip best? (refer to the map) _______________
   How many miles per gallon would you expect? _______________
   If you had 12 gallons of gas in the tank, how far could you go? _______________

12. Record your equation for the whole trip. _______________

13. Explain what the slope of this line represents.

14. Use the equation of the line to predict the amount of gas you will need to travel 1800 miles.
   Suppose gasoline costs $1.25 per gallon. How much will the total gasoline cost?

Using Slope in Engineering
15. Based on the video, summarize the reasons for designing a bridge with a slope of 4%.
After the Computer Session

16. Commercial passenger jets often reach a cruising altitude of about 40,000 feet. They know that the glide slope for landing should be 3% to insure a smooth ride. How many miles from the airport should the pilot begin the descent for landing?

   Draw a diagram and show your calculations below. (Hints: 40,000 feet is close to 8 miles. Change 3% into decimal form.)

Math Journal

17. Explain in your own words why the slope of a line can represent fuel efficiency or miles per gallon on the graph with gasoline used on the horizontal axis and distance traveled on the vertical axis.
Mathematics: A Moving Experience

At the Computer

Three-Dimensional Mathematics

1. List the “building blocks” that the CAD technician in the video uses in her work.

Points in Space

2. Graph the point (7, 10, 6) on the computer. Sketch the point on this grid.

3. Move the blue dot to each octant. Record the point coordinates here.
   - Octant 1: __________  Octant 2: __________
   - Octant 3: __________  Octant 4: __________
   - Octant 5: __________  Octant 6: __________
   - Octant 7: __________  Octant 8: __________

Planes

4. On the graph above, sketch one of the planes you explored. Record its equation.
An Equation for Rental Costs

5. Record the equation you created for the total cost of the truck rental.

\[ \square = \square \times \square + \square \times \square + \square \]

Equations and Graphs

6. Record the cost equations that you wrote for the two rental companies.

Happy Rental: \( z = \square x + \square y + \square \)

Ready Rental: \( z = \square x + \square y + \square \)

7. Describe what the green line represents. What is true about the points that lie on that line?

8. Find a rule that describes the conditions when one truck rental company costs less than the other. Show your work. Explain your rule.

9. Write your rule in words, using \textit{days} and \textit{miles}.

\quad \text{(rental company)}

Applications

10. List three variables that might be used in an equation in biology research.
After the Computer Session

The x-y Plane in Three Dimensions
11. List the three variables and their letter names used in the cost equations in this activity.

________________________________________________________________________

________________________________________________________________________

12. Explain why the x-y plane is especially important in analyzing the rental costs.

________________________________________________________________________

________________________________________________________________________

13. If the blue dot is higher than the red dot, then the blue cost is higher than the red cost. Explain why this is so.

________________________________________________________________________

________________________________________________________________________

Math Journal
14. Compare the x-y plane in three dimensions with the two-dimensional x-y grid. How are they alike and different?

________________________________________________________________________

________________________________________________________________________

15. What parts of this computer activity did you find difficult? What parts were easy for you? What concepts or ideas were new to you?

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
Mathematics: The Sky Is The Limit

At the Computer

Aircraft Attitude
1. Explain what the attitude of a jet means. Describe the attitude of 0 degrees.

Matrices, Points, and Rotations
2. Suppose point A has coordinates (4, −9).
   a. Write this point as a column matrix.
   b. Find the coordinates of its image point A' after a 90-degree counterclockwise rotation. Write A' in matrix form.

Rotation Matrices
3. Record the eight matrices in the Matrix Library along with their angles of rotation.

\[
\begin{bmatrix}
\end{bmatrix} \quad \begin{bmatrix}
\end{bmatrix} \quad \begin{bmatrix}
\end{bmatrix} \quad \begin{bmatrix}
\end{bmatrix} \\
\begin{bmatrix}
\end{bmatrix} \quad \begin{bmatrix}
\end{bmatrix} \quad \begin{bmatrix}
\end{bmatrix} \quad \begin{bmatrix}
\end{bmatrix} \\
\begin{bmatrix}
\end{bmatrix} \quad \begin{bmatrix}
\end{bmatrix} \quad \begin{bmatrix}
\end{bmatrix} \quad \begin{bmatrix}
\end{bmatrix}
\]
Rotations
4. Explain how you can find a clockwise rotation that has the same resultant position as a given counterclockwise rotation. Give an example to illustrate your process.

Test Run
5. Describe what happened in the first test run of your program.

6. If you found mistakes in your program, describe them and explain how you fixed them.

Matrix Applications
7. Based on the video, explain how a seismologist uses matrices.
After the Computer Session

Rotation Matrices

8. Several of the Rotation Matrices include only 0, +1, and −1 as elements. Identify the angles that they represent. Explain.

9. Several of the Rotation Matrices include 0.71 and −0.71 as elements. Explain where these numbers come from. (Hint: Can they be rewritten as fractions or as roots?) You may want to draw a diagram.

Math Journal

10. Describe two new things that you learned about matrices.
Complex Numbers and Fractals

At the Computer

Fractal Images
1. In your own words, describe the fractal images you see in the video.

Graphing Complex Numbers
2. Graph the six complex numbers shown in the Explore section. Label the points and the axes.

Iteration
3. Explain the difference between an escaping point and a prisoner point.

4. In your own words, explain iteration with respect to functions.

5. Describe the process you used here to create a fractal image.
Multiplying Complex Numbers

6. Explain why the FOIL method can be used to multiply complex numbers.

Julia Sets

7. Record the values for \( c \) and \( f(z) \).

\[
\begin{array}{c|c}
\text{Iteration} & \text{\( f(z) \)} \\
\hline
0 & \\
1 & \\
2 & \\
3 & \\
10 & \\
100 & \\
\end{array}
\]

8. Draw a rough sketch of the Julia set fractal you created.

\( c = \)

Applications of Complex Numbers

9. Based on the video, explain how an electrical engineer uses complex numbers.
After the Computer Session

Complex Numbers

10. Multiply these two complex numbers and simplify your answer.
   
   \((2 - 3i)(-3 + 5i)\)

11. You are exploring the iterated function \(f(z) = z^2 + c\). Suppose you choose a random point in the complex plane within some reasonable boundaries. Is it more likely that the point is a prisoner point or an escaping point? That is, is it more likely to belong to the fractal or not? Explain your answer.

   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________

12. Explain why fractals have been mathematically investigated only in relatively recent years.

   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________

Math Journal

13. Was the concept of iteration new to you? If so, describe what you find most interesting or unusual about it. If not, describe what you already knew about iteration and where you learned it.

   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________
Maximize Your Profits—
It’s the Quadratic Way!

**At the Computer**

**Selling Price**
1. Explain why selling price is important to business owners.

---

**Expenses and Revenue**
2. Write definitions in your own words.
   - Variable Expense: ________________________________
   - Fixed Expense: ________________________________
   - Revenue: ________________________________

**Profits**
3. Record the data for the greatest profit and for the largest number of shirts sold.

<table>
<thead>
<tr>
<th>Selling Price</th>
<th>Number Sold</th>
<th>Profit or (Loss)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greatest Profit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Most Shirts Sold</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. “The way to make the most profit is to sell the most shirts.” Do you agree or disagree with this statement? Explain why and include examples from your data.

---

4. “The way to make the most profit is to sell shirts at very high prices.” Do you agree or disagree with this statement? Explain why and include examples from your data.

---

**Changing Prices**
5. List three changes that might require retailers to change their selling price.

---
Profit Function
Record the functions. Recall that $x$ is the number sold.

6. Number Sold $N(x) =$

7. Revenue $R(x) =$

8. Variable Expenses $V(x) =$

9. Profit $P(x) =$
   Simplify the expression.

10. Sketch the graph of your Profit function. Mark the point that represents maximum profit. Mark the price that produces that profit. Label the axes.

Maximum profit __________________
Price that produces this profit __________________

Applications to Traffic Flow
11. Describe how two equations combine to create a quadratic function for Traffic Flow.

___________________________________________________________________________
___________________________________________________________________________
After the Computer Session

Profit and Number Sold

12. Use your equation for Number Sold in the T-shirt scenario, along with the maximum profit data. Calculate the number of shirts sold at the price that creates the maximum profit.

13. Suppose that data on traffic density for a certain stretch of highway shows that \( N(v) = 120 - v \), where \( N \) is traffic density and \( v \) is traffic speed. Write a function for Traffic Flow. Graph the function on a graphing calculator or by hand. Find the speed that gives the maximum density.

Math Journal

14. In your own words, explain how linear functions that describe rates can lead to quadratic functions. For example, selling price per item or vehicles per mile.
The Mathematics Behind Thrill Rides

At the Computer

Safe Thrills
1. Based on your own experience with amusement park rides, list three factors that affect the safety of the rides.

Track Design
2. After you create several tracks, sketch your favorite track design.

After you sketch the graphs below, mark on the track those places where the car’s speed is increasing.

3. Sketch the three graphs that were created by the track you sketched above.

<table>
<thead>
<tr>
<th>Speed</th>
<th>Distance</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time</td>
<td>Time</td>
</tr>
<tr>
<td></td>
<td>Distance</td>
<td></td>
</tr>
</tbody>
</table>

4. Summarize how the graphs are related to the shape of the track.
Polynomials

5. Create polynomials in which all coefficients are 0, except the coefficient of the first term. Describe how the graph changes when you change the first coefficient.

6. Describe how the graph changes when you change the constant term of polynomials.

7. Record your observations about the shapes of polynomial graphs.

<table>
<thead>
<tr>
<th>Positive Leading Coefficient</th>
<th>Negative Leading Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Degree</td>
<td></td>
</tr>
<tr>
<td>Second Degree</td>
<td></td>
</tr>
<tr>
<td>Third Degree</td>
<td></td>
</tr>
<tr>
<td>Fourth Degree</td>
<td></td>
</tr>
<tr>
<td>Fifth Degree</td>
<td></td>
</tr>
</tbody>
</table>

Polynomial Models

8. Record the polynomials that fit each track.

<table>
<thead>
<tr>
<th>Track Number</th>
<th>Degree of Polynomial</th>
<th>Polynomial Function</th>
<th>Safe?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9a. Sketch the distance and speed graph for an unsafe track.

Mark the distance where the speed is too high.
Explain the significance of the red horizontal line.
Polynomials and Highways

9b. Based on the video, briefly explain how polynomials are used in highway tunnel construction.

After the Computer Session

Working Backwards

10. Here is the graph of time and speed created by a car on a roller coaster track. Based on the graph, sketch the track.

Math Journal

11. When you fitted polynomials to the four tracks in the computer activity, were some tracks more difficult to fit? If so, which ones were more difficult? Explain why.

12. Describe the process you used to fit a polynomial to the track curve.
Orbits and Intersecting Paths

At the Computer

Earth’s Orbit
1. Describe Earth’s orbit.

Circles
2. Describe how $h$, $k$, and $r$ change when you drag the green sphere on the circumference. Describe how they change when you drag the green sphere at the center.

3. Construct the circle $x^2 + y^2 = 25$ on the computer. Use the graph to decide whether or not each of the following points lie on the circle.
   a. $(-3, 4)$  
   b. $(3.2, -2.4)$  
   c. $(2, 4.6)$

Ellipses
4. Describe how $h$, $k$, $a$, and $b$ change when you drag the green spheres at the right, at the top, and at the center.
   Horizontal Ellipse
   
   Vertical Ellipse

5. Construct the ellipse $\frac{(x + 2.4)^2}{5.9^2} + \frac{(y - 1)^2}{4.3^2} = 1$.
   Which of the following points lie on the ellipse?
   a. $(1, 4.5)$  
   b. $(-1, 5)$  
   c. $(-6, -3)$
Analyzing Conic Sections (continued)

**Axes**
6. Describe the positions of the foci of an ellipse.

**Intersections**
7. Record the equations of the orbits of Earth and the asteroid.

8. Explain how you determined the values for $a$ and $b$ in the equation of the ellipse.

9. Record the two points of intersection.

**Collisions**
10. Summarize the video information on the Shoemaker-Levy comet.
After the Computer Session

Algebraic Methods

11. Here are two orbits that may intersect. Use algebraic methods to show whether they do or do not intersect at the three points given below. Show your work. Explain your method.

Equations of orbits: \( x^2 + y^2 = 25 \)

\[
\frac{x^2}{16} + \frac{y^2}{25} = 1
\]

a. \((0, -5)\)

b. \((4, 3)\)

c. \((3, -4)\)

Math Journal

12. Summarize what you already knew about the orbits of comets and asteroids before you did this computer activity. List the new ideas and facts that you learned.
Exploring Rational Expressions

The Mathematics of Photography

At the Computer

Variables
1. Record the definitions of the three variables in the focus equation.
   \[ p \quad q \quad f \]
2. Record the equation used in photography that relates distances and focal length.

Graphing Data
4. Sketch the graph of the data points \((p, q)\). Be sure to label the axes.

Focus
5. For one of your photos that is in focus, record the equation with the \(p\) and \(q\) values.
6. For one of your photos that is out of focus, record the expression with \(p\) and \(q\) values.
Equation or Not?

7. Describe the photo taken when the \( q \) distance makes the focus equation true for the given \( p \) and \( f \) values. Describe the photo when the equation is not true.

---

Changing \( q \)

8. Solve the focus equation for \( q \). Record it here and enter it on the screen.

---

9. Record the data for several values of \( q \).

<table>
<thead>
<tr>
<th>Number of Photos</th>
<th>( q = )</th>
<th>( q = )</th>
<th>( q = )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Taken</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In Focus</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slightly Blurry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Very Blurry</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

10. Which value of \( q \) gives the best chance that the photos will be in focus? How does the blue line for that value of \( q \) relate to the function graph?

---

Applications in Engineering

11. Based on the video, explain how rational functions are used in electrical engineering.
**Exploring Rational Expressions (continued)**

**After the Computer Session**

**Evaluating Equations**

12. Using your calculator, verify that the equation you recorded in exercise 5 is true. You may need to round off your answer. Show your work.

13. Using your calculator, verify that the equation you recorded in Exercise 6 is not true. You may need to round off your answer. Show your work.

**Math Journal**

14. Inexpensive cameras usually have a fixed focus that you cannot change. Explain how the camera manufacturer might use the mathematics in this chapter to decide what focus setting to use. The goal is to make the largest possible number of photos be in focus or nearly in focus.
Exponential Growth in a Drop of Water!

At the Computer

Sea Creatures
1. In your own words, write a brief definition of plankton.

Population Growth
2. Record your data in the table.

<table>
<thead>
<tr>
<th>Number of Hours</th>
<th>Linear Model</th>
<th>Quadratic Model</th>
<th>Exponential Model</th>
<th>Plankton</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Sketch the graphs of the three growth models. Label each curve.

4. Describe how the three models are different.

5. Which growth model best fits the actual plankton population growth?
Exploring Exponential and Logarithmic Functions
(continued)

**Multiplying Plankton**

6. Record the number of plankton at each time period. Write the numbers as a sequence. Describe the pattern. What will the population be in the next time period?

**Exponential Functions**

7. Record the exponential function that fits the data.

8. Sketch the data points and the curve that fits them. Label the axes.

9. How many plankton will there be in 25 hours? How many new plankton would you have had to count in the 25th hour?

**Applications in Finance**

10. Record the general exponential function that represents the value of invested money. List the meaning of each variable. Record the specific equation used in the video.
After the Computer Session

Using the Financial Function
11. Using your calculator and the function shown in the video, verify the value of the investment after 1 year, 10 years, 20 years, and 30 years.

12. Using your calculator and the general exponential function for investment, calculate the value of $1000 invested at 5% interest for 1 year, 10 years, and 30 years.

13. Sketch a graph of both investments, from Exercises 11 and 12. Describe the differences.

Math Journal
14. Suppose the population growth of group Q is best modeled by a quadratic function and the growth of group E is best modeled by an exponential function. Which group’s population is growing at the faster rate? Explain. Describe and compare the graphs for each group’s population growth.
Anatomy of Geometric Sequences

At the Computer

Mathematics in Medicine
1. List one use of mathematics in medical research.

Fractal Trees
2. After you create several fractals, choose your favorite.
   Record the variable settings and the data table.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Number of Branches</th>
<th>Length of Branch</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Initial Length: ________
Length Ratio: ________
Stages: ________

3. How does changing the Initial Length affect the fractal image?
   How does changing the Length Ratio affect the fractal image? How is the Length Ratio related to the geometric sequence of branch lengths?

4. Write functions for the number of branches, N, after n stages.

<table>
<thead>
<tr>
<th>Exponential Function</th>
<th>Recursive Function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Write a recursive function that gives the length of a branch, \( L \). \( L_0 \) is the initial length. Use \( r \) for the length ratio.

---

**Path Length**

6. Set the Initial Length at 80.0 and the Length Ratio at 0.70. Set the Stage at 4. What is the length of the path from the beginning of the initial segment to the end of the branch drawn in stage 4? Use a calculator.

---

7. Write an equation for the length of the path in Exercise 6 for any length ration \( r \). Use the function from Exercise 5. Evaluate the equation for \( r = 0.70 \). Do your answers match?

---

**Medical Research**

8. List one way in which fractals are used in medical research.

---

**Fractal Models**

9. Record the branch lengths and ratios.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Branch Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

---

10. Record the average ratio and the initial branch length.

---

**Applications in Biochemistry**

11. Based on the video, summarize how fractals are used by biochemists.
After the Computer Session

Fractal Models

12. Use the data below. Calculate the average ratio of consecutive branch lengths.

<table>
<thead>
<tr>
<th>Stage</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch Length</td>
<td>100</td>
<td>59</td>
<td>36.58</td>
<td>23.05</td>
<td>13.37</td>
</tr>
</tbody>
</table>

13. Write an equation to find the length, \( L \), of the branch in stage \( n \).

Math Journal

14. In your own words, explain how fractal trees and geometric sequences are related.
Calculating Probabilities

At the Computer

Probabilities
1. In your own words, explain what it means if the probability of an event is one-half (0.5).

Door Game
2. Record your results for both strategies.

Always Switch Doors

<table>
<thead>
<tr>
<th>Number of Games</th>
<th>Number of Wins</th>
<th>Number of Losses</th>
<th>Probability of Winning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Never Switch Doors

<table>
<thead>
<tr>
<th>Number of Games</th>
<th>Number of Wins</th>
<th>Number of Losses</th>
<th>Probability of Winning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Describe how the two strategies are different.
Investigating Discrete Mathematics and Probability
(continued)

Tossing Coins
4. Record the equation for the probability of success, \( s \).

5. Record the total number of outcomes for tossing three coins.

Totolspi
6. Record the theoretical probabilities you entered in the table. After you play the game, enter the experimental probabilities also.

<table>
<thead>
<tr>
<th></th>
<th>Theoretical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(FFF) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P(RRR) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P(Lose) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Theoretical and Experimental
7. After you calculated both theoretical and experimental probabilities, you probably found that they are different. Explain why they are different.

Probability and Sampling
8. Suppose you take a sample of ten cigarette smokers from the population of all cigarette smokers. You find that seven of them have lung cancer. Can you conclude that 0.70 or 70% of all cigarette smokers have lung cancer? Explain why or why not. Use the example of the yellow and red candies to support your position.
After the Computer Session

The Door Game Strategies
Calculate the theoretical probabilities of the two strategies in the Door Game. Recall that you choose one of three doors. One of the other two doors opens and you see that there is no car there. You can keep your chosen door or switch to the other closed door. You can refer to these as the chosen door, the open door, and the switch door.

Never Switch Strategy You keep your chosen door and ignore the opening of one door.
9. a. How many doors are there? total possible outcomes ________
   b. How many of them hide the car? number of successes ________
   c. What is the probability that your door hides the car? 
      \[ P(\text{chosen door}) = \] 

Always Switch Strategy You switch doors, after the opening of one door.
10. a. What is the probability that the door you choose does not hide the car? 
    (Hint: Refer to your result in Exercise 9.)
    \[ P(\text{not chosen door}) = \]
    b. What is the probability that either the open door or the switch door hides the car? (Hint: This is the same as the probability that the chosen door does not hide the car.)
    \[ P(\text{open door or switch door}) = \]
    c. Since these are mutually exclusive events, write the probability as a sum.
    \[ P(\text{open door or switch door}) = P( ) + P( ) = \]
    d. What is \( P(\text{open door}) \)? (Hint: This is easy.) \( P(\text{open door}) = \)
    Use this value in the equation in step c.
    e. What is the probability that the switch door hides the car?
    \[ P(\text{switch door}) = \]

11. Compare these theoretical probabilities to your experimental results that you recorded on this worksheet.

Math Journal
12. Describe another real-world situation that uses probability, other than those in this chapter.
Mathematics and Forest Fires: A Hot Combination

At the Computer

Forest Fires
1. List two variables that are used to locate forest fires.

Unit Circle and Radians
2. About how many radians equal the circumference of a circle? __________
3. Record the angle measurement data on the table.

<table>
<thead>
<tr>
<th>Angle (Degrees)</th>
<th>Angle (Radian)</th>
<th>Radians</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
<td>45</td>
<td>$\frac{\pi}{4}$</td>
<td>0.7854</td>
</tr>
<tr>
<td>90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>135</td>
<td></td>
<td></td>
</tr>
<tr>
<td>180</td>
<td></td>
<td></td>
</tr>
<tr>
<td>225</td>
<td></td>
<td></td>
</tr>
<tr>
<td>270</td>
<td></td>
<td></td>
</tr>
<tr>
<td>315</td>
<td></td>
<td></td>
</tr>
<tr>
<td>360</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

_Radians and Degrees_
4. a. Record the formula for converting angle measure from degrees to radians.

b. Use the formula to convert 450 degrees to radian measure.

5. a. Record the formula for converting angle measure from radians to degrees.

b. Use the formula to convert $\frac{3}{4}\pi$ radians to degrees.
Sine and Cosine
6. Record the sine and cosine functions and label the triangle.
   \[ \sin \theta = \quad \cos \theta = \]

Fire Fighting
7. Record the values for \( r \), \( q \), \( \sin q \), and \( \cos q \). Calculate \( x \) and \( y \) and record those values. Sketch the triangle and label it.
   
   \[
   \begin{align*}
   \text{Range, } r & = \\
   \text{Angle } \theta & = \\
   \cos \theta & = \quad \sin \theta = \\
   x & = \\
   y & = 
   \end{align*}
   \]

Trigonometry and Highways
8. Based on the video, describe a civil engineering problem that is solved using trigonometry.

   __________________________________________
   __________________________________________
   __________________________________________
   __________________________________________
After the Computer Session

Math Journal

9. In your own words, explain how radian measure and degree measure differ.

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

10. Explain why right triangles are an especially useful type of triangle for specifying locations and for engineering.

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________
Trigonometric Functions: A Sound Tool

At the Computer

Music Industry
1. Describe what an amplifier does.

Amplitude and Cycles
2. For the function \( y = a \sin b \theta \), record the following:
   - number of cycles
   - frequency
   - amplitude

3. Record two sound waves that have very different graphs. Record the number of cycles, the amplitude, and the equation for each.
   - a. Cycles
   - Amplitude
   - \( y = \) __________

   - b. Cycles
   - Amplitude
   - \( y = \) __________

Evaluating Sound
4. Describe what an oscilloscope does. Describe how it is used in speaker manufacturing.
Using Trigonometric Graphs and Identities
(continued)

Testing Speakers
5. Record the data from the table as you perform each test.

<table>
<thead>
<tr>
<th>Test</th>
<th>a</th>
<th>b</th>
<th>Pass/Fail</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. For the test(s) that fail, describe how the speaker graph is different from
the signal generator graph.

Trigonometry and Music
7. Based on the video, summarize how amplitude and frequency relate to
playing the harp or other string instrument.
After the Computer Session

8. The number of cycles per second for a sound is 600. Calculate the frequency.

9. Graph the equation $y = 3 \sin 5\theta$. Mark the amplitude and the cycles in the interval of $2\pi$.

Compare the graph of $y = -3 \sin 5\theta$ with the graph above.

Math Journal

10. Explain how pitch, frequency, and the graph of $y = a \sin b\theta$ are related.

11. Explain how volume, amplitude, and the graph of $y = a \sin b\theta$ are related.
Chapter 1: Analyzing Equations and Inequalities

1. Ships must avoid running aground.

2. Depth (ft)
   - Distance from buoy (feet)
   - Depth
   - Below Sea Level
   - Sea Level
   - a. 1600 ft
   - b. about 500 ft

3. Depth (ft)
   - Distance from buoy (feet)
   - Sea Level
   - a. linear
   - b. \( \frac{\text{ping time}}{2} \)
   - c. It is the time the sonar ping takes to travel from ship to ocean floor and back to ship.

5. Distance from Buoy = Speed of Boat \( \times \) Time from the Buoy
   Depth of Ocean = Speed of Sound in Water \( \times \) \( \frac{\text{ping time}}{2} \)

6. All ships use sonar. The fathometer constantly sends signals. A receiver records the time of the pulse or signal and calculates the ocean depth.

7. \( d = \frac{4800t}{2} \)

8. 4800 feet per second
9. Darker cells mean deeper depths; lighter cells mean shallower depths.

10. The graph shows the depth along the ship’s path at various distances from the buoy. Choose a depth tile to match the depth shown on the graph.

11. The torpedo sends a sonar pulse to a target and it bounces back. A computer calculates the distance to the target. This keeps the torpedo on target and avoids hitting the ocean floor.

12. Sample Response
   3. Receive the sonar return ping.
   4. Record the end time of the ping.
   5. Subtract the start time from the end time to get the ping time (seconds).
   6. Divide the ping time by 2. (time to travel one way)
   7. Multiply by 4800 feet per second, the speed of sound in water.
   8. This is the depth in feet, the distance below the surface.

**Chapter 2: Graphing Linear Relations and Functions**

1. \[ \frac{\text{distance traveled}}{\text{gasoline used}} \quad \text{or} \quad \frac{\text{miles}}{\text{gallon}} \]

2. Sample Data

<table>
<thead>
<tr>
<th>Segment</th>
<th>x-Intercept</th>
<th>Slope</th>
<th>Terrain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>9.1</td>
<td>uphill</td>
</tr>
<tr>
<td>2</td>
<td>1.8</td>
<td>20.5</td>
<td>downhill</td>
</tr>
<tr>
<td>3</td>
<td>-0.6</td>
<td>11.1</td>
<td>uphill</td>
</tr>
<tr>
<td>4</td>
<td>4.7</td>
<td>29.1</td>
<td>downhill</td>
</tr>
</tbody>
</table>

3. downhill; uphill
4. greater, steeper
5. 14.7
6. (9.4, 140); 14.8 MPG; They are about equal.
7. It is the average miles per gallon for the whole trip.
Answer Key

8. Find the change in the y (vertical axis) values. Find the change in the x (horizontal axis) values. Divide the vertical change by the horizontal change.

9.  

<table>
<thead>
<tr>
<th></th>
<th>d miles</th>
<th>g gallons</th>
<th>change in d</th>
<th>change in g</th>
<th>MPG</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>744</td>
<td>39</td>
<td>744</td>
<td>39</td>
<td>19.1</td>
</tr>
<tr>
<td>2</td>
<td>1276</td>
<td>72</td>
<td>532</td>
<td>33</td>
<td>16.1</td>
</tr>
<tr>
<td>3</td>
<td>2134</td>
<td>107</td>
<td>858</td>
<td>35</td>
<td>24.5</td>
</tr>
<tr>
<td>4</td>
<td>3102</td>
<td>106</td>
<td>968</td>
<td>53</td>
<td>18.3</td>
</tr>
</tbody>
</table>

10. (Use any point on the line segment.)

<table>
<thead>
<tr>
<th></th>
<th>equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$d - 0 = 19.1(g - 0)$ or $d - 744 = 19.1(g - 39)$</td>
</tr>
<tr>
<td>2</td>
<td>$d - 744 = 16.1(g - 39)$ or $d - 1276 = 16.1(g - 72)$</td>
</tr>
<tr>
<td>3</td>
<td>$d - 2134 = 24.5(g - 107)$ etc.</td>
</tr>
<tr>
<td>4</td>
<td>$d - 3102 = 18.3(g - 106)$ etc.</td>
</tr>
</tbody>
</table>

11. Equation #3; 24.5 MPG; 294 miles

12. $d = 20g$

13. The slope, 20, is the average miles per gallon for the trip.

14. $\frac{1800}{20} = 90$ gallons, $90 \times 1.25 = 112.50$

15. The bridge slope must be steep enough to fit the navigation channel, but flat enough so that cars won’t slip in winter road conditions.

16. 3% = .03 slope

\[ \frac{8}{x} = 0.03 \quad x = \frac{8}{0.03} = \text{about 267 miles} \]

17. See students’ work.
Chapter 3: Solving Systems of Linear Equations and Inequalities

1. planes, cylinders, spheres

2. 

3. Octant 1: (7, 10, 6) Octant 2: (7, 10, 6)
   Octant 3: (7, -3, -4) Octant 4: (7, 2, -4)
   Octant 5: (-2, 10, 6) Octant 6: (-2, -3, 6)
   Octant 7: (-2, -3, -4) Octant 8: (-2, 10, -4)

4. Answers will vary; \( z = 2.0x + 1.2y + 7.2 \).

5. 

6. Sample Data

   Happy Rental: \( z = 14x + 0.23y + 54 \)
   Ready Rental: \( z = 6x + 0.69y + 12 \)

7. It is the intersection of the two planes. For points on this line, the number of days and miles are values where the two rental costs are equal.
8. Happy \[ z = 14x + .23y + 54 \]
    Ready \[ z = 6x + .69y + 12 \]
    (Roundoff of graph coordinates causes discrepancies.)

<table>
<thead>
<tr>
<th>Graphic Method</th>
<th>Symbolic Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Locate two points on the green line.</td>
<td>Set expressions for ( z ) equal to each other.</td>
</tr>
<tr>
<td>(1,102) and (10, 254)</td>
<td>14x + .23y + 54 = 6x + .69y + 12</td>
</tr>
<tr>
<td>Write equation of line.</td>
<td>Solve for ( y ).</td>
</tr>
<tr>
<td>( y = 102 = (254-102)/(10-1)(x-1) )</td>
<td>( y = 17.4x + 91.3 )</td>
</tr>
<tr>
<td>( y = 16.9x + 85.1 )</td>
<td></td>
</tr>
</tbody>
</table>

Choose a point where Happy is lower. \( (1202, 114) \)
Use that point to write the inequality. \[ 202 > 16.9 (1) + 85.1 \]
\[ y > 16.9x + 85.1 \]
Happy (red) costs less when \( y > 16.9x + 85.1 \)

9. Happy costs less when the number of miles is greater than 16.9 times the number of days plus 85.1.

10. time, temperature, number of bacteria
11. \( x \), number of days; \( y \), number of miles; \( z \), total cost
12. The \( x-y \) plane represents the independent variables, days and miles.
13. The \( z \)-axis value determines the height of a point in the plane. The \( z \)-axis represents the total cost.
14. See students' work.
15. See students' work.

**Chapter 4: Using Matrices**

1. Altitude is the position of a plane relative to the horizon. An altitude of 0 degrees means that the wings are parallel to the horizon.

2. If \[ A = \begin{bmatrix} 4 \\ -9 \end{bmatrix} \] then \[ A' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ -9 \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \end{bmatrix} \].

3. \[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
.71 & -1 \\
.71 & 1
\end{bmatrix}
\begin{bmatrix}
0 & -1 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
-1 & 0 \\
0 & -1
\end{bmatrix}
\begin{bmatrix}
.71 & .71 \\
-.71 & -.71
\end{bmatrix}
\]

\[
0^\circ \\
45^\circ \\
90^\circ \\
135^\circ \\
180^\circ \\
225^\circ \\
270^\circ \\
315^\circ
\]

4. Subtract the counterclockwise rotation value from 360.
   Counterclockwise rotation of 225\(^\circ\). 360 - 225 = 135 clockwise degrees

5. Answers will vary.
6. Answers will vary.
7. See students’ work.
8. See students’ work.
9. See students’ work.
10. See students’ work.

Chapter 5: Exploring Polynomials and Radical Expressions

1. Responses will vary.
2. \[ -2 + 5i \bullet 5 \quad \text{imaginary axis} \]
   \[ 0 + 3i \bullet 3 \quad \bullet 2 + 3i \]
   \[ -6 + 0i \bullet \quad \quad \text{real axis} \]
   \[ -6-5-4-3-2-1 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \]
   \[ -3 - 4i \bullet -4 \quad \bullet 5 - 3i \]

3. When an escaping point is iterated in a function, the function values (points) become farther and farther away from the starting point. When a prisoner point is iterated, the points get closer together.
4. Iteration means to repeat. You use the function value as the next \( x \)-value and repeat the calculations. Then you repeat this process.
5. Choose a starting point. Test to see if it is a prisoner point. If so, graph that point (in black) on the grid. Continue choosing, testing, and graphing prisoner points until the figure appears.
6. Each complex number is a binomial with a real and a complex part. The FOIL method multiplies binomials.
7. \( c = 0.1 - 0.6i \)

<table>
<thead>
<tr>
<th>Iteration</th>
<th>( f(z) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( 0 + 1i ) or ( i )</td>
</tr>
<tr>
<td>1</td>
<td>( -0.9 - 0.6i )</td>
</tr>
<tr>
<td>2</td>
<td>( 0.55 + 0.48i )</td>
</tr>
<tr>
<td>3</td>
<td>( 0.17 - 0.07i )</td>
</tr>
<tr>
<td>10</td>
<td>( 0.132 - 0.051i )</td>
</tr>
<tr>
<td>100</td>
<td>( -0.1758 - 0.1572i )</td>
</tr>
</tbody>
</table>

8. Drawings will vary. The difficulty of drawing fractal images by hand becomes clear.
Answer Key

9. An electrical engineer uses complex numbers to represent impedance when modeling capacitors and resistors.

10. \((2 - 3i)(-3 + 5i) = (2)(-3) + (2)(5i) + (-3i)(5i) + (-3i)(5i)\)
   \[= -6 + 10i - 15i^2 - 15i\]
   \[= -6 - 5i + 15\]
   \[= 9 - 5i\]

11. It is more likely to be an escaping point. The Julia set, the prisoner points, creates a relatively small figure on the plane.

12. The iteration of functions involves so many calculations that the speed of computers is needed to perform the calculations. The fractal image requires the graphics capabilities of computer monitors; they are too complicated to draw by hand.

13. See students’ work.

Chapter 6: Exploring Quadratic Functions and Inequalities

1. Selling price helps determine how much profit businesses make.

2. Variable Expense: The wholesale cost of each shirt.
   Fixed Expense: The constant cost of doing business.
   Revenue: The money you get from selling shirts.

3. Sample Data

<table>
<thead>
<tr>
<th>Selling Price</th>
<th>Number Sold</th>
<th>Profit or (Loss)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greatest Profit</td>
<td>$8</td>
<td>29</td>
</tr>
<tr>
<td>Most Shirts Sold</td>
<td>$1</td>
<td>56</td>
</tr>
</tbody>
</table>

3. Disagree. The most shirts sold led to a loss, not a profit. The selling price, $1, was below the variable expenses of $4 per shirt.

4. Disagree. When the price was $15, only 1 shirt was sold. There was a loss. The revenue was far below the fixed expenses.

5. competitors’ stores; change in customer preference; increased demand; neighborhood change

Sample Responses

6. Number Sold \(N(x) = -4.4x + 63\)

7. Revenue \(R(x) = x(-4.4x + 63)\)

8. Variable Expenses \(V(x) = 4(-4.4x + 63)\)

9. \(P(x) = x(-4.4x + 63) - 4(-4.4x + 63) - 100\)
   \[= -4.4x^2 + 63x + 17.6x - 252 - 100\]
   \[= -4.4x^2 + 80.6x - 352\]
10. Maximum profit $17
Price that produces this profit about $9

11. Two linear functions are multiplied together to create a quadratic function.

12. \( N(x) = -4.4x + 63 \)
    \[ N(9) = -4.4(9) + 63 = -39.6 + 63 = 23.4 \]
Number of Shirts sold at the $9 price is 23.

13. \( F(v) = v(120 - v) = 120v - v^2 \)
The maximum flow is 3600.
The speed that gives this density is 60 mph.

14. See students’ work.

Chapter 7: Exploring Polynomial Functions

1. Responses will vary. age of equipment, maintenance, operator, design, speed.

2. Sample Response

3. 

   ![Graphs showing speed and distance over time]
4. Graphs of speed increase when the track goes downhill and decrease for uphill sections. The distance and time graph is steadily increasing.

5. The graphs stretch out or narrow depending on the coefficient size. The U-shaped graphs open upward (+) or downward (−) depending on the sign. The other graphs begin positive or negative depending on the sign.

6. The graphs move up or down along the z-axis.

7. Answers will vary.

<table>
<thead>
<tr>
<th>Positive Leading Coefficient</th>
<th>Negative Leading Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>line slanting upward</td>
<td>line slanting downward</td>
</tr>
<tr>
<td>U-shaped opening up</td>
<td>U-shaped opening down</td>
</tr>
<tr>
<td>leftmost point negative</td>
<td>leftmost point positive</td>
</tr>
<tr>
<td>U-shaped with “bump”, up</td>
<td>U-shaped with “bump”, down</td>
</tr>
<tr>
<td>leftmost point negative</td>
<td>leftmost point positive</td>
</tr>
</tbody>
</table>

8. | Track Number | Degree of Polynomial | Polynomial Function | Safe? |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>$2x^2 - 5x - 13$</td>
<td>U</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>$-1x^3 + 6x^2 + 2x - 4$</td>
<td>S</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>$-1x^4 - 4x^3 + 1x^2 + 4x + 4$</td>
<td>S</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>$1x^4 - 1x^3 - 7x^2 + 9x + 3$</td>
<td>U</td>
</tr>
</tbody>
</table>

9a. Speed
(foot/second)

Answers will vary.
The red line is 45 ft. per sec.
Speeds above that line are unsafe.

9b. The curve of a tunnel, the slope down and then up, is a polynomial.

10. See students’ work.

11. See students’ work.
Chapter 8: Analyzing Conic Sections

1. Earth's orbit is a nearly circular path around the Sun.
2. When you change the circumference, only $r$ changes.
   When you drag the center, $h$ and $k$ change, but $r$ does not.
3. a. yes  b. no  c. yes
4. Horizontal Ellipse: The right sphere changes $a$; the top sphere changes $b$; the center changes $h$ and $k$.
   Vertical Ellipse: The right sphere changes $b$; the top sphere changes $a$; the center changes $h$ and $k$.
5. a. yes  b. yes  c. no
6. The foci lie on the major axis. The sum of the distances from the foci to any point on the ellipse is constant.
7. \[ \frac{(x + 100)^2}{152^2} + \frac{y^2}{110^2} = 1 \quad x^2 + y^2 = 93^2 \]
8. Find the length of both axes. Use one half of the longer length for $a$. Use one half of the shorter axis for $b$.
9. $(-20, 90)$ and $(-20, -90)$
10. In 1992 the Shoemaker-Levy comet had a close encounter with Jupiter. Jupiter's gravity broke the comet into pieces.
    Astronomers predicted that it would collide with Jupiter. In 1994 the pieces of the comet smashed into Jupiter's atmosphere.
11. a. yes  b. no  c. no
    Responses will vary. One method is to use the values of $x$ and $y$ in both equations, evaluate the expressions, and see if the equations are true statements. All three points lie on the circle, but only the first point lies on the ellipse.
12. See students' work.

Chapter 9: Exploring Rational Expressions

1. $p$: the distance from the object to the camera lens
   $q$: the distance from the lens to the film
   $f$: the focal length; the distance from the lens to the focal point
2. \[ \frac{1}{p} + \frac{1}{p} = \frac{1}{f} \]
3. 

![Diagram of lens and focal point](image)
4. The graph shows a downward curve.

5. \( \frac{1}{15.0} + \frac{1}{29.8} \neq \frac{1}{10.0} \)

6. \( \frac{1}{15.0} + \frac{1}{20.0} \neq \frac{1}{10.0} \)

7. When the equation is true, the photo is in focus. When the equation is not true, the photo is blurry.

8. \( q = \frac{5p}{p - 5} \)

9. Sample Data

<table>
<thead>
<tr>
<th>Number of Photos</th>
<th>( q = 16.0 )</th>
<th>( q = 7 )</th>
<th>( q = )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Taken</td>
<td>12</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>In Focus</td>
<td>1</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Slightly Blurry</td>
<td>7</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Very Blurry</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

10. The value 7 gives a better chance than 16. The blue line at 7 is almost on top of the horizontal portion of the function graph. So more of the blue line points are close to function points.

11. Electrical engineers use rational functions to model total resistance when resistors are connected in parallel.

12. Response will vary.

13. Response will vary.

14. See students' work.
Chapter 10: Exploring Exponential and Logarithmic Functions

1. Plankton are microscopic organisms living in ocean water.

2. | Number of Hours | Linear Model | Quadratic Model | Exponential Model | Plankton |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>9</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>25</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>100</td>
<td>1024</td>
<td>1024</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>400</td>
<td>1,048,576</td>
<td>1,048,576</td>
</tr>
</tbody>
</table>

3. Number of Plankton

4. Sample Response. The linear graph is almost flat. It increases the slowest. It show the slowest growth rate. The exponential graph curves up steeply. It shows the fastest growth rate. The quadratic graph curves up gently. It shows medium fast growth.

5. Exponential

6. 1, 2, 4, 8, 16, 32 The pattern is to multiply the previous number by 2. Each value is an integer power of 2, $2^n$. The next value is 64.

7. Sample; $y(t) = 3(1.25)^t$

8. Number of Plankton
9. \(y(25) = 794\) plankton; \(y(24) = 634\) plankton; new plankton = \(y(25) - y(24) = 160\)

10. \(A = Pert;\) \(A\) is money after \(t\) years; \(P\) is the amount invested; \(r\) is the interest rate; \(t\) is the time in years; \(a = 1000e^{0.08t}.\)

11. \(A = 1000e^{0.08t};\) \(A(1) = 1083;\) \(A(10) = 2225;\) \(A(20) = 4953;\) \(A(30) = 11,023\)

12. \(A = 1000e^{0.05t};\) \(A(1) = 1051;\) \(A(10) = 1648;\) \(A(30) = 4482\)

13. \[
\begin{array}{c|c|c}
\text{Time (years)} & \text{Main Function} & \text{Interest Function} \\
\hline
0 & 0 & 0 \\
10 & 4000 & 2225 \\
20 & 6000 & 4953 \\
30 & 8000 & 11,023 \\
\end{array}
\]

The function that represents 8% interest rises much more steeply than the one that represents 5% interest.

14. See students’ work.

**Chapter 11: Investigating Sequences and Series**

1. One use is mathematical modeling of biological processes.

2. Sample Data

<table>
<thead>
<tr>
<th>Stage</th>
<th>Number of Branches</th>
<th>Length of Branch</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>39.0</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>25.350</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>16.478</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>10.710</td>
</tr>
<tr>
<td>5</td>
<td>32</td>
<td>6.962</td>
</tr>
<tr>
<td>6</td>
<td>64</td>
<td>4.525</td>
</tr>
<tr>
<td>7</td>
<td>128</td>
<td>2.941</td>
</tr>
<tr>
<td>8</td>
<td>256</td>
<td>1.912</td>
</tr>
<tr>
<td>9</td>
<td>512</td>
<td>1.243</td>
</tr>
</tbody>
</table>

Initial Length: 60
Length Ratio: 0.65
Stages: 9
3. When the initial length is larger, the fractal image is larger, 
but the shape is the same.; When the ratio changes, the 
shape of the fractal image changes. The ratio determines the 
branch lengths. The product of a branch length and the ratio 
equals the next branch length.

4.  

<table>
<thead>
<tr>
<th>Exponential Function</th>
<th>Recursive Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N(n) = 2^n )</td>
<td>( N_n = 2N_{n-1} )</td>
</tr>
</tbody>
</table>

5. \( L_n = r \, L_{(n-1)} \), where \( L_0 \) is initial length.
6. \( 56 + 39.2 + 27.44 + 19.208 = 141.848 \)
7. \( L_1 + L_2 + L_3 + L_4 = rL_0 + r(rL_0) + r(r(rL_0)) + r(r(r(rL_0))) \)  
\[ = L_0(r + r^2 + r^3 + r^4) \]  
\[ = 80(1.7731) = 141.848 \]  
Yes, they match.
8. Fractal models of lungs are compared with MRI pictures of 
real lungs.
9. Sample Data

<table>
<thead>
<tr>
<th>Stage</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch Length</td>
<td>74</td>
<td>51.6</td>
<td>39</td>
<td>30.2</td>
<td>20</td>
</tr>
</tbody>
</table>

| Ratios | .697 | .756 | .774 | .662 |

10. Sample Response. 0.722
11. Biochemists use fractals to model the surfaces and structure of proteins in order to 
better understand proteins.
12. Ratios 0.59 0.62 0.63 0.58

\[
\text{Average } \frac{2.42}{4} = 0.605 = 0.61
\]
13. \( L_n = 0.61L_{(n-1)} \), where \( L_0 \) is the initial length.
14. See students’ work.
Chapter 12: Investigating Discrete Mathematics and Probability

1. There is an equal chance that the event will or will not occur.

2. Sample Data

   **Always Switch Doors**
   
<table>
<thead>
<tr>
<th>Number of Games</th>
<th>Number of Wins</th>
<th>Number of Losses</th>
<th>Probability of Winning</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>13</td>
<td>7</td>
<td>0.65</td>
</tr>
<tr>
<td>101</td>
<td>71</td>
<td>30</td>
<td>0.70</td>
</tr>
<tr>
<td>200</td>
<td>120</td>
<td>80</td>
<td>0.60</td>
</tr>
</tbody>
</table>

   **Never Switch Doors**
   
<table>
<thead>
<tr>
<th>Number of Games</th>
<th>Number of Wins</th>
<th>Number of Losses</th>
<th>Probability of Winning</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>9</td>
<td>11</td>
<td>0.45</td>
</tr>
<tr>
<td>101</td>
<td>36</td>
<td>65</td>
<td>0.36</td>
</tr>
<tr>
<td>200</td>
<td>63</td>
<td>137</td>
<td>0.32</td>
</tr>
</tbody>
</table>

3. The Always-Switch strategy wins more often than it loses. It wins about two thirds of the time. The Never-Switch strategy wins only about one third of the time.

4. \[ P(s) = \frac{s}{s + f} \]

5. 8

6. Sample Data

<table>
<thead>
<tr>
<th></th>
<th>Theoretical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(FFF) )</td>
<td>0.125</td>
<td>0.049</td>
</tr>
<tr>
<td>( P(RRR) )</td>
<td>0.125</td>
<td>0.143</td>
</tr>
<tr>
<td>( P(\text{Lose}) )</td>
<td>0.750</td>
<td>0.805</td>
</tr>
</tbody>
</table>

7. Each roll of the dice is an independent event. The theoretical probability tells the chance that an event occurs on any roll. If you roll the dice a large number of times, then the experimental probability will be close to the theoretical.

8. No, you cannot conclude that the percentage of the whole population is the same as that of the sample. This sample is like pulling 7 yellows and 3 reds from the mixed candies. The next sample could have just 3 yellows, or 3 smokers with lung cancer.

9. a. 3  
   b. 1  
   c. \( P(\text{chosen door}) = \frac{1}{3} = 0.33 \)
Answer Key

10. a. \( P(\text{not chosen door}) = \frac{2}{3} = 0.67 \)
   
   b. \( P(\text{open door or switch door}) = 0.67 \)
   
   c. \( P(\text{open door or switch door}) = P(\text{open door}) + P(\text{switch door}) = 0.67 \)
   
   d. \( P(\text{open door}) = 0 \)
   
   e. \( P(\text{switch door}) = 0.67 \)

11. Responses will vary. When a large number of games are played, the experimental results come close to the theoretical results.

Chapter 13: Exploring Trigonometric Functions

1. angle and distance
2. about 6 radians
3. | Degree | \( \pi \) Radians | Radians |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
<td>45</td>
<td>( \frac{1}{4} \pi )</td>
<td>0.7854</td>
</tr>
<tr>
<td>90</td>
<td>( \frac{2}{4} \pi )</td>
<td>1.57</td>
</tr>
<tr>
<td>135</td>
<td>( \frac{3}{4} \pi )</td>
<td>2.3562</td>
</tr>
<tr>
<td>180</td>
<td>( \frac{4}{4} \pi )</td>
<td>3.1416</td>
</tr>
<tr>
<td>225</td>
<td>( \frac{5}{4} \pi )</td>
<td>3.9270</td>
</tr>
<tr>
<td>270</td>
<td>( \frac{6}{4} \pi )</td>
<td>4.71</td>
</tr>
<tr>
<td>315</td>
<td>( \frac{7}{4} \pi )</td>
<td>5.4978</td>
</tr>
<tr>
<td>360</td>
<td>( \frac{8}{4} \pi )</td>
<td>6.2832</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Degree</th>
<th>( \pi ) Radians</th>
<th>Radians</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
<td>-45</td>
<td>( -\frac{1}{4} \pi )</td>
<td>-0.7854</td>
</tr>
<tr>
<td>-90</td>
<td>( -\frac{2}{4} \pi )</td>
<td>-1.57</td>
</tr>
<tr>
<td>-135</td>
<td>( -\frac{3}{4} \pi )</td>
<td>-2.3562</td>
</tr>
<tr>
<td>-180</td>
<td>( -\frac{4}{4} \pi )</td>
<td>-3.1416</td>
</tr>
<tr>
<td>-225</td>
<td>( -\frac{5}{4} \pi )</td>
<td>-3.9270</td>
</tr>
<tr>
<td>-270</td>
<td>( -\frac{6}{4} \pi )</td>
<td>-4.71</td>
</tr>
<tr>
<td>-315</td>
<td>( -\frac{7}{4} \pi )</td>
<td>-5.4978</td>
</tr>
<tr>
<td>-360</td>
<td>( -\frac{8}{4} \pi )</td>
<td>-6.2832</td>
</tr>
</tbody>
</table>

4. a. radians = \( \frac{\pi \text{ degrees}}{180} \)
   
   b. 7.85 radians

5. a. degrees = \( \frac{180 \text{ radians}}{\pi} \)
   
   b. \( \frac{3}{4} \pi \); 135 degrees

6. \( \sin \theta = \frac{y}{r} \) \quad \cos \theta = \frac{x}{r} \)

7. Sample Data
   
   Range, \( r = 58.69 \)
   
   Angle \( \theta = 0.779 \)
   
   \( \cos \theta = 0.712 \) \quad \sin \theta = 0.703 \)
   
   \( x = 41.78 \) \quad \( y = 41.25 \)
8. Trigonometry is used to calculate the angle of descent for drainage pipes. The pipe length (hypotenuse) and the difference in elevation (y) are known. Trigonometric functions are used to find the angle.

9. See students’ work.

10. See students’ work.

Chapter 14: Using Trigonometric Graphs and Identities

1. An amplifier changes a sound signal to change the volume.

2. number of cycles \( \frac{b}{2\pi} \)
   frequency \( \frac{b}{2\pi} \)
   amplitude \( |a| \)

3. a. Cycles 1 Amplitude 1
   b. Cycles 3.0 Amplitude 4

   \[ y = 1 \sin 10 \]

4. An oscilloscope graphs sound as a sine function. It is used to test the quality of speakers.
5. | Test | a   | b   | Pass/Fail |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
<td>1</td>
<td>Pass</td>
</tr>
<tr>
<td>2</td>
<td>2.5</td>
<td>4</td>
<td>Pass</td>
</tr>
<tr>
<td>3</td>
<td>2.0</td>
<td>1</td>
<td>Fail</td>
</tr>
<tr>
<td>4</td>
<td>3.0</td>
<td>0.5</td>
<td>Fail</td>
</tr>
</tbody>
</table>

6. The failed speaker graph has uneven curves. Both the height and the width of the curves vary.

7. A long string, when plucked, vibrates slowly. This low frequency produces a low pitch. A short string vibrates more rapidly. This makes a higher pitch. Plucking hard gives the string a large range of motion. This higher amplitude means a louder volume. Dampening a string lowers its range of motion, producing a smaller amplitude and softer volume.

8. \[
\frac{600}{2\pi} = \frac{300}{\pi} = 95.5
\]

9. 

![Diagram of a sine wave with labeled cycles and amplitude](#)
9. This graph would have the same number of cycles and the same amplitude. It would begin at (0,0) and curve down to −3, then up to +3. Each value would be the negative of the graph shown above.

10. See students’ work.

11. See students’ work.
Troubleshooting Guide/Windows

When I double-click the Algebra start-up icon, nothing happens.

1. Make sure the desired CD-ROM is inserted into the CD-ROM drive.
2. Quit any other programs that are running, including screen savers.
3. Try launching an entirely different CD program (not a Glencoe CD) to determine if there is a hardware problem.
4. Go to your CD-ROM drive (probably D: or E:). Then double-click the desired file in your file listing.

The program takes a long time to launch.

Approximately one minute of wait time is normal. The performance of the program has been increased by loading as much information as possible at the beginning. (See suggestions above under “The program will not launch . . . “)

The video and/or audio sputters during playback.

1. Quit all other programs that may be running, including screen savers.
2. Check that QuickTime for Windows 2.0 or higher appears in Program Manager as a program group or within the Accessories program group.
   Note: Although the video on this CD-ROM is high-quality, video played from any CD-ROM application will not play as smoothly as it would on videotape.

The playback sound is too loud or too soft.

1. If you have speakers, be sure they are plugged in and turned up.
2. If you are using headphones, check that they are plugged in.
3. Check that your machine’s software volume control is at an appropriate setting.
4. Go to the Program Group for your Audio Card within Program Manager.
5. Adjust the slider to an appropriate volume level.

The printer is not working.

1. Be sure your printer is plugged in and turned on.
2. Try printing from another application to learn if the problem might be hardware or cabling.
3. Contact your system administrator or hardware technician to be sure the correct print drivers for your system have been installed.

Certain colors look incorrect in the graphic images in Windows 95.

If you have customized your desktop colors in Windows 95, set them back to the default settings.
Troubleshooting Guide/Macintosh

The following extensions are required for Glencoe Language Products:

- Apple CD-ROM (or the CD-ROM extension for your non-Apple CD drive)
- Apple Multimedia Tuner
- QuickTime 2.0 or greater
- QuickTime PowerPlug (for Power Macs only)
- Sound Manager
- General Controls
- Any printer extension your particular printer needs
- Network extension (if the system is on a network)

The CD-ROM icon does not appear on the desktop.

1. Check your Extensions folder in the System folder to be sure that the required extensions are not missing or turned off.
2. If they are not missing or turned off, check your SCSI connections if you have an external CD-ROM drive.
3. Go to your hard disk and find a utility program that checks SCSI devices (such as SCSI Probe or FWB Toolkit).
4. Scan your SCSI chain with the program to see what devices (CD, external hard drives, etc.) are connected.
5. If your CD-ROM drive appears from within the utility program, select the Mount command.

If the steps listed above do not remedy the issue, reinstall the CD-ROM software drivers from your System installation disk.

The program will not launch on my computer when I double-click the start-up icon.

1. Make sure the CD-ROM is inserted into your CD-ROM drive.
2. Quit all other programs that may be running, including screen savers.
3. Check all cabling to your hardware.
4. Confirm that you have enough free RAM:
   Go to the Finder. (Apple icon at the upper left corner of the Screen.)
   Pull down the Apple menu and select About this Macintosh. Check the largest unused block. If the computer does not have at least 5,085K, you need more RAM.
5. If you do not have enough free RAM, you may be able to add to it by disabling Extensions that automatically load when you start your Mac. See the special section at the end of the Troubleshooting Guide.
6. Try launching an entirely different CD program (not a Glencoe CD).
The program takes a long time to launch.

1. Approximately one minute of wait time is normal. The performance of the program has been increased by loading as much information as possible at the beginning.
2. Make sure you have enough free RAM.

The video and/or audio sputters during playback.

1. Check that the Extensions folder within your System folder includes QuickTime 2.1 or higher, Indeo Video, and Intel Raw Video.
2. Quit all other programs that may be running, including screen savers.
3. Make sure that you have enough free RAM. (See suggestions in the special section at the end of this section.)

Note: Although the video on this CD-ROM is high-quality, video played from any CD-ROM application will not play as smoothly as it would on videotape.

The playback sound is too soft or too loud.

1. If you are using speakers or headphones, be sure they are plugged in and set at an appropriate volume.
2. Check that your machine’s software volume control is at an appropriate setting:
   Pull down the Apple menu. Select Control Panels. Double-click Sound. A Sound window will display. Choose Volumes from its internal pulldown menu. Adjust the slider to an appropriate volume.

The printer is not working.

1. Be sure your printer is plugged in and turned on.
2. Try printing from some other application to learn if the problem might be hardware or cabling.
3. Contact your system administrator or hardware technician to be sure the correct print drivers for your system have been installed.
For Macintosh users only:
If you do not have enough free RAM, it is possible to make slightly more available by disabling unused extensions. This is tricky business, because it may affect other programs you run. It is recommended that you have your system administrator or hardware technician help you.

For users of System 7.0 or 7.1
1. Open the System folder on your hard drive.
2. Find and duplicate your Extensions folder. Rename it “My Original Extensions.”
3. Create a new folder and name it “Extensions Disabled.” Open the Extensions folder and move all extensions except those required for the Glencoe Algebra Products to the “Extensions Disabled” folder. You can find a list of required extensions on page 106.
4. Restart your Macintosh.

Once again, the process outlined above will affect other programs you run on your machine. Before launching another program, move the items from your “Extensions Disabled” folder back into your Extensions folder, throw away the “My Original Extensions” folder, and restart your Macintosh.

If you have any trouble, throw away the “Extensions” and “Extensions Disabled” folders, but do not empty the trash. Rename “My Original Extensions” to “Extensions.” Restart your Macintosh and then empty the trash.

For users of System 7.5 or those with the Extensions Manager Program
1. Select the Apple pulldown menu.
2. Select Control Panels and Extensions Manager.
3. Save the current set of extensions by choosing Save Set from the pulldown menu under “Sets.” For example, you might call the set “My Original Extensions.”
4. Select All Off from the pulldown menu under “Sets.”
5. Click to select only those extensions listed on page 106.
6. Save the new set and name it “Glencoe Algebra Products.”
7. Restart your Macintosh.
8. Remember to activate your original set of extensions from the “Sets” pulldown menu and restart your Macintosh before launching any other programs.
Customer Service Guide

Need Help?
As with all Glencoe technology products, assistance—should you need it—is only a phone call away. Contact customer service at 1-800-334-7344 or the technology support hotline at 1-800-437-3715.