Key Concepts

Slope

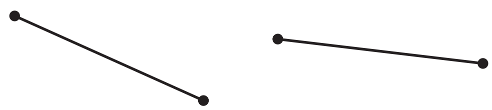
Objective Teach students to find the slope of a line.

Steepness and Slope

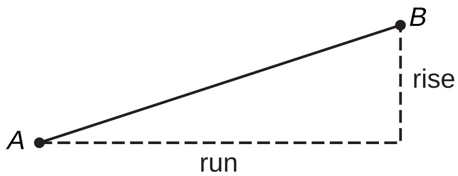
First, talk in relative terms about what is meant by slope. Give examples of the slope of a roof, an up-hill road, or a ladder. Explain that we will assign a number that allows us to measure the steepness of a straight line. Tell students that the greater the absolute value of the number is, the steeper the line will be.

Definition of Slope

Draw two straight lines on the chalkboard, each containing two points as shown. Explain that there are two numbers associated with each pair of points, namely the *rise* and the *run*.

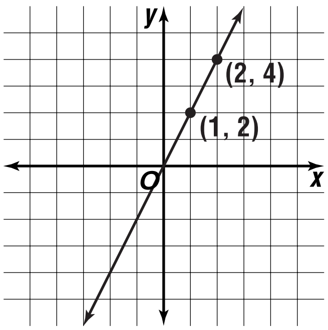


The rise is the vertical difference between point B and point A, and the run is the horizontal difference between point B and point A. You should note that the differences may be negative.



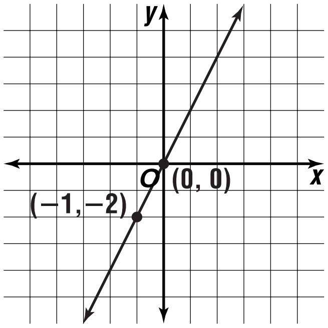
Definition of Slope	The slope is the quotient $\frac{\text{rise}}{\text{run}}$.

Let's apply this concept to a line on the coordinate plane.



Notice that the line contains the points whose coordinates are (1, 2) and (2, 4). To find the slope of the line, you must first determine the rise. To do this, calculate the difference of the *y*-coordinates. The rise is $4 - 2$ or 2. Next, determine the run. To do this, calculate the difference of the corresponding *x*-coordinates. The result is $2 - 1$ or 1. Therefore, the slope is the quotient

$$\frac{2}{1} \text{ or } 2.$$



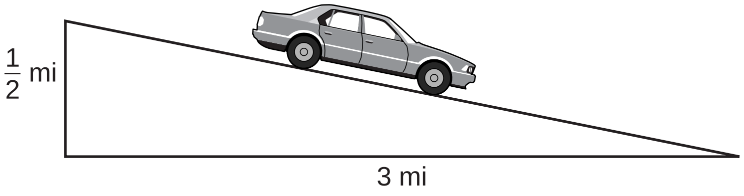
Let's choose two different points that lie on the line, say (0, 0) and (−1, −2). Find the slope.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-2 - 0}{-1 - 0} = \frac{-2}{-1} \text{ or } 2$$

Notice that the slope is the same for any pair of points on the same straight line. This means that we may speak of the slope of a line without referring to a particular pair of points.

Example **Suppose a car is traveling on a straight road that decreases in altitude $\frac{1}{2}$ mile for every 3 miles traveled horizontally. What is the slope of the road, viewed as a straight line?**

Solution Let's picture the situation.

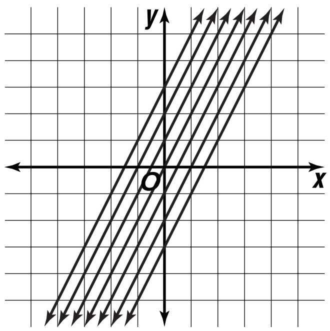


The slope is the rise, $-\frac{1}{2}$ mile, divided by the run, 3 miles. Thus, we have

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-\frac{1}{2}}{3} = \frac{-\frac{1}{2}}{\frac{3}{1}} = -\frac{1}{2} \cdot \frac{1}{3} \text{ or } -\frac{1}{6}.$$

Slope and Parallel Lines

Graph the lines given by $y = 2x$, $y = 2x + 1$, $y = 2x + 2$, $y = 2x + 3$, $y = 2x - 1$, $y = 2x - 2$, and $y = 2x - 3$.



Next, ask students to determine the slopes of these lines. They should find that all of the lines have a slope of 2.

Then ask students what they can say about the lines. They should say that the lines are parallel. Discuss the meaning of parallel lines.

Key Idea	Two lines in a plane are parallel if they never meet.

Refer to the family of graphs shown on the coordinate plane. If we select any two different lines, they will never intersect because they "stay the same distance apart" as we move up or down along the lines. The following key idea can be concluded from this example.

Key Idea	Two lines are parallel if and only if they have the same slope.

Work through the following exercises as a class or have students complete them on their own.

Exercises

Complete each of the following.

1. Tell whether the lines given by $2x + 3y = 5$ and $4x + 6y = 9$ are parallel. Explain. **Since the lines have the same slope, they are parallel.**
2. Are the lines given by $y = 3x + 4$ and $y = 3x + 2$ parallel? Why or why not? **Yes; they have the same slope.**
3. Suppose we are given two lines. The first contains the points whose coordinates are (1, 2) and (4, 7). The second contains the points whose coordinates are (0, 0) and (5, 8). Are these lines parallel? Explain. **The slope of the first line is $\frac{5}{3}$, and the slope of the second line is $\frac{8}{5}$. Since the slopes are not equal, the lines are not parallel.**