Key Concepts

Solving Multi-Step Inequalities

Objective Teach students to solve inequalities that involve more than one operation by using the properties of inequalities and to graph inequalities.

Note to the Teacher *This lesson is an extension of the methods used for solving linear equations. The methods used in this lesson are the Addition and Multiplication Properties of Inequalities, which parallel the corresponding properties for equalities. Students should remember two significant distinctions. The first is that when multiplying an inequality by a negative number, the direction of the inequality must be changed. The other is that the term "solve" is used a bit differently from the way it is used when solving equations. In equations, this term means the process of finding a particular number that is the only solution to the equation. In inequalities, the result of solving the inequality is never a single number but rather a description of a set such as* $x \geq 5$ *or* $-2 \leq x \leq 7$*. It is very important to explain this to the class.*

Solving Inequalities

Begin by reviewing with students the properties of inequalities. These were covered in Lessons 3-6 and 3-7 of the Student Edition.

Key Idea	Adding or subtracting a fixed number from each side of an inequality produces an equivalent inequality where any solution of either inequality is a solution of the other.

For example, $x - 2 \leq 1$ is equivalent to $x \leq 3$.

$$x - 2 \leq 1$$
$$x - 2 + 2 \leq 1 + 2 \quad \textit{Add 2 to each side.}$$
$$x \leq 3$$

Key Idea	Multiplying or dividing by the same positive number on both sides of an inequality produces an equivalent inequality.

For example, $3x \geq 12$ is equivalent to $x \geq 4$.

$$3x \geq 12$$
$$\frac{3x}{3} \geq \frac{12}{3} \quad \textit{Divide each side by 3.}$$
$$x \geq 4$$

Multiplying or dividing by the same negative number produces an equivalent inequality if we reverse the direction of the inequality.

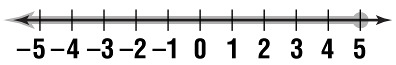
For example, $-x \geq 4$ is equivalent to $x \leq 4$.

$$-x \geq 4$$
$$(-1)(-x) \geq (-1)(4) \quad \textit{Multiply each side by } -1.$$
$$x \leq -4$$

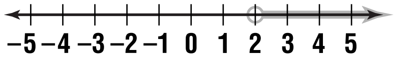
Graphing Solutions of Inequalities

Solving an inequality means describing a set of numbers that satisfy the inequality. This set is called the **solution set** for the problem. To visualize the solution set, we can draw it on a number line. Typically, we draw a number line and shade the part of the number line that corresponds to the solution set. The solution set for an inequality may or may not include an endpoint. When the endpoint is included, we draw a solid dot for the endpoint. When the endpoint is not included, we draw a circle. Some graphs of inequalities are shown below.

The solution set for $x \leq 5$ is represented by the set of points to the left of 5 on the number line. Since x is less than or equal to 5, 5 is included. So, we use a solid dot.



The solution set for $x > 2$ is represented by the set of points to the right of 2 on the number line. Since x is greater than 2, 2 is not included. So, we use a circle.

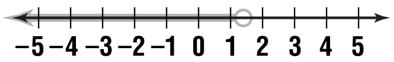


Note to the Teacher *The best way to introduce multi-step inequalities is to do a few problems on the chalkboard and then have students complete some additional ones. Here are some exercises.*

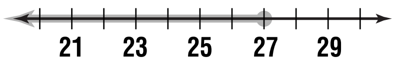
Exercises

Solve each inequality. Graph the solution on a number line.

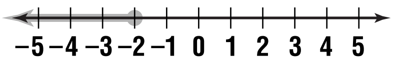
1. $3x + 4 < 8$ $\boldsymbol{x < \frac{4}{3}}$



2. $\frac{x}{3} + 2 \leq 11$ $\boldsymbol{x \leq 27}$



3. $-x + 7 \geq 9$ $\boldsymbol{x \leq -2}$



4. $1.5x + 9 > 12$ $\boldsymbol{x > 2}$