# Key Concepts

## Writing Inequalities

**Objective** Teach students to translate verbal problems involving inequalities into mathematical sentences.

**Note to the Teacher** *This lesson involves more verbal skills than mathematical. The emphasis is on translating problems rather than solving them. The best way to teach these ideas is with plenty of examples done on the board and by the students.*

### Writing Inequalities

Here are a few examples that can be discussed as a class or completed by the students.

**Example 1** **A parking garage charges an initial parking fee of \$2 and an additional fee of \$1.50 per hour. Suppose you have \$8. What inequality describes the length of time you can afford to park at this garage?**

**Solution** If $t$ is the amount of time in hours you park in the garage, then the amount of money you will owe is the sum of the initial fee of \$2 and the additional fee of \$1.50 per hour. Algebraically, this can be written as

$$2 + 1.5t \text{ dollars.}$$

Since you have only \$8, the amount of money you spend must be less than or equal to \$8. Thus, we write

$$2 + 1.5t \leq 8.$$

Now, solve the inequality.

$$2 + 1.5t \leq 8$$

$$2 + 1.5t - 2 \leq 8 - 2 \quad \textit{Subtract 2 from each side.}$$

$$1.5t \leq 6$$

$$\frac{1.5t}{1.5} \leq \frac{6}{1.5} \quad \textit{Divide each side by 1.5.}$$

$$t \leq 4$$

So, we can park at most 4 hours at the garage.

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**Example 2** **At Pine Hills Middle School, there are ten more than fourteen times as many students as teachers. There are fewer than 458 students. Write and solve an inequality that describes the greatest possible number of teachers at the school.**

**Solution** Let $t$ represent the number of teachers. The number of students is given by

$$\underbrace{\textit{ten}}_{10}\ \underbrace{\textit{more than}}_{+}\ \underbrace{\textit{fourteen times the number of teachers}}_{14t}$$

Since the number of students is fewer than 458, the following equation results.

$$\underbrace{\textit{Ten}}_{10}\ \underbrace{\textit{more than}}_{+}\ \underbrace{\textit{fourteen times the number of teachers}}_{14t}\ \underbrace{\textit{is fewer than}}_{<}\ \underbrace{\textit{458}}_{458}.$$

Now, solve the inequality.

$$\begin{aligned} 10 + 14t &< 458 \\ 10 + 14t - 10 &< 458 - 10 && \textit{Subtract 10 from each side.} \\ 14t &< 448 \\ \frac{14t}{14} &< \frac{448}{14} && \textit{Divide each side by 14.} \\ t &< 32 \end{aligned}$$

So, $t < 32$ describes the greatest possible number of teachers.

**Note to the Teacher** *In the statement "fourteen times as many students," the words "fourteen times" appears adjacent to the word "students". So, students may write that fourteen times the number of students plus ten is equal to the number of teachers. This kind of statement is often confusing to students. It is a good idea to emphasize how to translate this kind of phrase. Also, point out that the phrase "fewer than" translates into the mathematical symbol <.*

**Example 3** **In a forest, there are at least 3 times as many finches as nightingales. There are 10,000 finches in the forest. Write and solve an inequality that describes the possible number of nightingales.**

**Solution** Let $n$ represent the number of nightingales. Since there are at least 3 times as many finches as nightingales, we know that the number of finches is greater than or equal to three times the number of nightingales. In algebraic terms, we have

$$10{,}000 \geq 3n.$$

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Solve the inequality.

$$10{,}000 \geq 3n$$

$$\frac{10{,}000}{3} \geq \frac{3n}{3} \quad \textit{Divide each side by 3.}$$

$$3333\frac{1}{3} \geq n$$

Since you can have only a whole number of nightingales, the amount is less than or equal to 3333.

**Example 4** **Jarvis is traveling to St. Louis, Missouri, at a rate of 40 miles per hour. He began his trip in a town that is located 30 miles from Chicago, Illinois, and is between Chicago and St. Louis. Currently, Jarvis is between 170 and 210 miles from Chicago. Write an inequality that describes the possible values of time in hours Jarvis has been traveling.**

**Solution** Let $h$ represent the number of hours Jarvis has been traveling. Since he started out 30 miles outside Chicago and has been driving at a rate of 40 miles each hour, the number of miles he has traveled is

$$30 + 40h.$$

We know that this value must lie between 170 and 210. So, we can write

$$170 \leq 30 + 40h \leq 210.$$

Solve the inequality.

$$\begin{aligned} 170 &\leq 30 + 40h &&\leq 210 \\ 170 - 30 &\leq 30 + 40h - 30 &&\leq 210 - 30 \quad \textit{Subtract 30 from each part.} \\ 140 &\leq 40h &&\leq 180 \\ \frac{140}{40} &\leq \frac{40h}{40} &&\leq \frac{180}{40} \quad \textit{Divide each part by 40.} \\ 3.5 &\leq h &&\leq 4.5 \end{aligned}$$

So, Jarvis has been traveling at least 3.5 hours and at most 4.5 hours.

**Note to the Teacher** *Discuss the interpretation of the word "between" as a double inequality. Also, discuss phrases like "at most," "no fewer than," and "no more than," and their interpretations in mathematical symbols.*

End of Lesson

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