Key Concepts

Powers of Monomials

Objective Teach students to find powers of monomials.

Note to the Teacher *In this lesson, we will discuss how to find powers of monomials. However, before doing so, we will review how to multiply monomials. It is important for students to understand how to multiply monomials so that they can fully comprehend how to find powers of monomials. The key ideas in this lesson are the laws of exponents. These laws allow us to simplify monomial expressions.*

Monomials

Begin by reviewing powers and exponents. Remind students that if x is a variable, then x^2 represents $x \cdot x$, x^3 represents $x \cdot x \cdot x$, and more generally x^n represents

$$\underbrace{x \cdot x \cdot \ldots \cdot x}_{n \text{ factors}}.$$

Next, review with students that a **monomial** is an expression that is either a real number, a variable, or a product of numbers and variables. Tell students that numbers are referred to as constants. Remind them that when a variable occurs more than once, we will typically write the expression using exponents. Some examples of monomials are shown below.

$$5x^2 \qquad 7ab \qquad -3c^3 \qquad 2.4g^5h \qquad z^{100} \qquad -\frac{1}{2}m^3n^4p$$

Here are a few examples of expressions that are *not* monomials.

$$3x + 1 \qquad \frac{1}{x^2 + 2} \qquad \sqrt{x + 3} \qquad m - n \qquad \frac{ab}{4} + 3$$

Monomials are frequently used in geometry problems. Consider the following example.

Example 1 **Suppose the length of a rectangle is three times its width. Write an expression that represents the area of the rectangle in terms of x.**

Solution Let x represent the width. Then the expression $3x$ represents the length. Now, we can use the following expression to find the area.

$$A = \ell w \quad \textit{Area of a rectangle}$$
$$= (3x)(x) \quad \textit{Replace } \ell \textit{ with } 3x \textit{ and } w \textit{ with } x.$$
$$= 3x^2$$

Here's another example.

Example 2 **Park Ridge Middle school is having a bake sale. The cost of a cookie is $0.50. If each customer buys the same number of cookies, what is the total revenue?**

Solution Using the variable c to represent the number of customers, and the variable x to represent the number of cookies each customer buys, the total amount of cookies sold is given by the expression cx. Since each cookie sells for $0.50, the total revenue is $0.50cx$. This expression is a monomial. It is the product of the constant 0.50 and two variables c and x.

Laws of Exponents and Multiplying Monomials

Next, let's review how to multiply monomials. Recall that when we multiply a power of x times another power of x, the result is a power of x, where the exponent is the sum of the exponents of the two factors. In symbols,

$$x^m \cdot x^n = x^{m+n}.$$

This holds true for any number x and positive integers m and n. Here are some examples.

Example 3 **Find $m^7 \cdot m^6$.**

Solution $m^7 \cdot m^6 = m^{7+6}$ or m^{13}

Example 4 **Find $6^3 \cdot 6^8$.**

Solution $6^3 \cdot 6^8 = 6^{3+8}$ or 6^{11}

In each example shown below, notice that we group constants and variables together.

Example 5 **Find $3x^5 \cdot 4x^2$.**

Solution $3x^5 \cdot 4x^2 = (3 \cdot 4)(x^5 \cdot x^2)$
$= 12x^7$

Example 6 **Find $9y^8 \cdot (-y^7)$.**

Solution $9y^8 \cdot (-y^7) = (9 \cdot -1)(y^8 \cdot y^7)$
$= -9y^{15}$

Note to the Teacher *Make sure students practice this kind of multiplication.*

Powers of a Monomial

Now, recall what it means to raise a number to a power.

2^n	$(2^n)^2$
$2^1 = 2$	$4 = 2^2$
$2^2 = 4$	$16 = 2^4$
$2^3 = 8$	$64 = 2^6$
$2^4 = 16$	$256 = 2^8$
$2^5 = 32$	$1024 = 2^{10}$
$2^6 = 64$	$4096 = 2^{12}$

Notice that when we square a power of 2, the answer is found by *doubling* the power. Let's apply this concept to monomials. Consider the following.

To determine $(x^4)^3$, write the powers as products.

$$(x^4)^3 = x^4 \cdot x^4 \cdot x^4 = \underbrace{x \cdot x \cdot x \cdot x}_{\text{4 factors}} \cdot \underbrace{x \cdot x \cdot x \cdot x}_{\text{4 factors}} \cdot \underbrace{x \cdot x \cdot x \cdot x}_{\text{4 factors}} = \underbrace{x \cdot x \cdot \ldots \cdot x \cdot x}_{\text{12 factors}} = x^{12}$$

In this case, we are raising the monomial to the third power. Therefore, we *triple* the exponent 4 to get an exponent of 12. Here is a general rule.

Definition of Power to a Power	When we raise a power of a number or variable to another power, the result is that same number raised to the power which is the product of the two powers. This is the power of a power rule. In symbols, we write $(x^m)^n = x^{mn}$.

Example 7 **Find $(x^8)^7$.**

Solution $(x^8)^7 = x^{8 \cdot 7}$ or x^{56}

Example 8 **Find $(15^3)^6$.**

Solution $(15^3)^6 = 15^{3 \cdot 6}$ or 15^{18}

Note to the Teacher *It is a good idea to work through several examples by expanding the powers into products, so that reasoning behind the key idea is reinforced.*

Why is this true? The following diagram explains this idea in general terms.

$$(x^m)^n = \underbrace{x^m \cdot x^m \cdot \ldots \cdot x^m \cdot x^m}_{n \text{ factors}} = \underbrace{\underbrace{x \cdot x \cdot \ldots \cdot x \cdot x}_{m \text{ factors}} \cdot \underbrace{x \cdot x \cdot \ldots \cdot x \cdot x}_{m \text{ factors}} \cdot \ldots \cdot \underbrace{x \cdot x \cdot \ldots \cdot x \cdot x}_{m \text{ factors}}}_{n \text{ factors}}$$

Point out that we have n groups of factors, each of which is itself a group of m factors. So, we have mn factors all together. Thus, the result is x^{mn}.

We now know how to find powers of powers. Since monomials are generally products of powers, we will also need to discuss **powers of products**. This will enable us to find powers of monomials. Write down the expression $(a \cdot b)^2$ on the board. Then ask students to try to simplify the expression. An explanation for the answer is as follows.

$$\begin{aligned}(a \cdot b)^2 &= (a \cdot b)(a \cdot b)\\ &= a \cdot b \cdot a \cdot b\\ &= a \cdot a \cdot b \cdot b\\ &= a^2 \cdot b^2\end{aligned}$$

So, $(a \cdot b)^2 = a^2 \cdot b^2$.

Consider the following. We know that $6^3 = 216$. On the other hand, $6 = 2 \cdot 3$, and $(2 \cdot 3)^3 = 2^3 \cdot 3^3 = 8 \cdot 27$ or 216.

Definition of Power of a Product	A product raised to a power is the product of the two factors raised to the given power. This is the power of a product rule. In symbols, we write $(a \cdot b)^m = a^m \cdot b^m$.

Why is this true? The following diagram illustrates the general rule.

$$(a \cdot b)^m = \underbrace{(a \cdot b) \cdot (a \cdot b) \cdot \ldots \cdot (a \cdot b)}_{m \text{ factors}} = \underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ factors}} \cdot \underbrace{b \cdot b \cdot \ldots \cdot b}_{m \text{ factors}} = a^m b^m$$

Point out that in the middle expression, it is clear that we have exactly m factors each of a and b.

We can now combine these two ideas into one.

Key Idea	The expression $(a^m \cdot b^n)^p = a^{mp} \cdot b^{np}$ holds true for any whole numbers *m, n,* and *p*. This is the power of a monomial rule.

We can now find powers and products of any monomials. This is a good time to present several exercises and have students practice the technique.

Exercises

Simplify.

1. $(2xb^2)^3$
$8x^3b^6$

2. $(3xy)^5(x^2y)^4$
$243x^{13}y^9$

3. $(a^3b^2)^2 \cdot b^7$
a^6b^{11}

4. $100y^2z^2 \cdot (3z)^5$
$24{,}300y^2z^7$

End of Lesson