Key Concepts

Multiplying and Dividing Monomials

Objective Introduce the algebra of polynomials by using the laws of exponents to multiply and divide monomials.

Note to the Teacher *The main ideas in this lesson are the laws for multiplying and dividing powers. In this lesson, we will deal with monomials that are powers of a single variable.*

Powers

Begin by reviewing powers and exponents. Explain that if a is a variable, then a^2 represents $a \cdot a$, a^3 represents $a \cdot a \cdot a$, and more generally, a^b represents

$$\underbrace{a \cdot a \cdot \ldots \cdot a}_{b \text{ factors}}.$$

Remind students that a is called the base, b is called the exponent, and a^b is called the power. Review that the power a^4 is read "a raised to the fourth power." In general, when we write a^b, we say "a raised to the b power."

Laws of Exponents and Multiplying Monomials

When multiplying monomials, we must analyze the product of the two powers of the same base. Consider $x^2 \cdot x^3$. Let's analyze this by multiplying various powers of 2 together.

$$2^2 \cdot 2^3 = 4 \cdot 8 = 32 = 2^5$$
$$2^4 \cdot 2^5 = 16 \cdot 32 = 512 = 2^9$$

In both cases, the exponent of the resulting power is the sum of the exponents in the two factors. For $2^2 \cdot 2^3$, $5 = 2 + 3$, and for $2^4 \cdot 2^5$, $9 = 4 + 5$.

The table shows what happens when a power of 2 is multiplied by "2 to the first power," which is 2. Recall that any number raised to the first power is the number itself.

n	2^n	$2^n \cdot 2^1 = 2^n \cdot 2$
1	$2^1 = 2$	$2 \cdot 2 = 4$ or 2^2
2	$2^2 = 4$	$4 \cdot 2 = 8$ or 2^3
3	$2^3 = 8$	$8 \cdot 2 = 16$ or 2^4
4	$2^4 = 16$	$16 \cdot 2 = 32$ or 2^5
5	$2^5 = 32$	$32 \cdot 2 = 64$ or 2^6
6	$2^6 = 64$	$64 \cdot 2 = 128$ or 2^7

Notice each power that results. Do you see a pattern? Each resulting power can be found by adding 1 to the exponent of the original power. For example, $2^3 \cdot 2^1 = 2^{3+1}$ or 2^4. Using symbols, we write $2^n \cdot 2^1 = 2^{n+1}$.

The following table shows what happens when a power of 2 is multiplied by "2 to the second power" or 4.

n	2^n	$2^n \cdot 2^2 = 2^n \cdot 4$
1	$2^1 = 2$	$2 \cdot 4 = 8$ or 2^3
2	$2^2 = 4$	$4 \cdot 4 = 16$ or 2^4
3	$2^3 = 8$	$8 \cdot 4 = 32$ or 2^5
4	$2^4 = 16$	$16 \cdot 4 = 64$ or 2^6
5	$2^5 = 32$	$32 \cdot 4 = 128$ or 2^7
6	$2^6 = 64$	$64 \cdot 4 = 256$ or 2^8

Again, notice each power that results. In this case, each power can be found by adding 2 to the exponent of the original power. For example, $2^3 \cdot 2^2 = 2^{3+2}$ or 2^5. Using symbols, we write $2^n \cdot 2^2 = 2^{n+2}$.

This confirms our earlier observation that when we multiply two powers that have the same base, the exponent of the resulting power is the sum of the exponents in the two factors.

Now is a good time to have the class explore this idea for themselves. Instruct students to choose their own bases and exponents. Then have them evaluate both $a^b \cdot a^c$ and a^{b+c} to verify that they are equal. Why is this true? Remember that exponents are a shorthand that represents a repeated product of the same number or variable. So,

$$2^2 \cdot 2^3 = \underbrace{2 \cdot 2}_{\text{2 factors}} \cdot \underbrace{2 \cdot 2 \cdot 2}_{\text{3 factors}} = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{\text{5 factors}} = 2^5.$$

In general, when we multiply a^b and a^c, we'll get

$$a^b \cdot a^c = \underbrace{a \cdot a \cdot \ldots \cdot a}_{b \text{ factors}} \cdot \underbrace{a \cdot a \cdot \ldots \cdot a}_{c \text{ factors}} = \underbrace{a \cdot a \cdot \ldots \cdot a}_{b+c \text{ factors}} = a^{b+c}.$$

Key Idea	When we multiply a power of *a* times another power of *a*, the result is a power of *a*, where the exponent is the sum of the exponents of the two factors. In symbols, $a^b \cdot a^c = a^{b+c}$. This holds true for any number *a* and positive integers *b* and *c*.

Example 1 **Find $x^5 \cdot x^7$.**

Solution $x^5 \cdot x^7 = x^{5+7}$ or x^{12}

Example 2 **Find $7^9 \cdot 7^6$.**

Solution $7^9 \cdot 7^6 = 7^{9+6}$ or 7^{15}

Laws of Exponents and Dividing Monomials

What happens when we divide powers? Let's analyze this by dividing various powers of 2.

$$\frac{2^5}{2^2} = \frac{32}{4} = 8 = 2^3$$

$$\frac{2^7}{2^3} = \frac{128}{8} = 16 = 2^4$$

Note to the Teacher *Ask students to make a conjecture about dividing powers that have the same base.*

In both cases, the result is the original base raised to the power given by the difference of the two exponents. For $\frac{2^5}{2^2}$, $3 = 5 - 2$, and for $\frac{2^7}{2^3}$, $4 = 7 - 3$.

The table on the left shows what happens when a power of 2 is divided by 2^1. The table on the right shows what happens when a power of 2 is divided by 2^2

n	2^n	$\frac{2^n}{2^1} = \frac{2^n}{2}$
2	$2^2 = 4$	$\frac{4}{2} = 2$ or 2^1
3	$2^3 = 8$	$\frac{8}{2} = 4$ or 2^2
4	$2^4 = 16$	$\frac{16}{2} = 8$ or 2^3
5	$2^5 = 32$	$\frac{32}{2} = 16$ or 2^4
6	$2^6 = 64$	$\frac{64}{2} = 32$ or 2^5
7	$2^7 = 128$	$\frac{128}{2} = 64$ or 2^6

n	2^n	$\frac{2^n}{2^2} = \frac{2^n}{4}$
3	$2^3 = 8$	$\frac{8}{4} = 2$ or 2^1
4	$2^4 = 16$	$\frac{16}{4} = 4$ or 2^2
5	$2^5 = 32$	$\frac{32}{4} = 8$ or 2^3
6	$2^6 = 64$	$\frac{64}{4} = 16$ or 2^4
7	$2^7 = 128$	$\frac{128}{4} = 32$ or 2^5

In the table on the left, notice the powers that result. Each resulting power can be found by subtracting 1 from the exponent of the original power. In the table on the right, each resulting power can be found by subtracting 2 from the exponent of the original power. This agrees with our original observation that when we divide two powers with the same base, the exponent of the resulting power is the difference of the exponents of the two dividends.

Consider another case. Let's divide a^4 by a^2. To do this, expand a^4 into $a \cdot a \cdot a \cdot a$ and a^2 into $a \cdot a$. Next place them into the fraction

$$\frac{a \cdot a \cdot a \cdot a}{a \cdot a}.$$

We can now cancel two a's from both numerator and denominator.

$$\frac{a \cdot \cancel{a} \cdot \cancel{a} \cdot a}{\cancel{a} \cdot \cancel{a}} = \frac{a \cdot a}{1} = a^2$$

Note to the Teacher *Be sure to tell students that cancellation is a shorthand process involving the properties of fractions. Also, point out that any number raised to the first power is that number itself.*

$$\frac{a \cdot a \cdot a \cdot a}{a \cdot a} = \frac{a \cdot a}{1} \cdot \frac{a \cdot a}{a \cdot a} = a \cdot a \cdot 1 = a^2$$

After canceling, we find that $\frac{a^4}{a^2} = a^{4-2}$ or a^2.

Note to the Teacher *At this point, have students explore this idea with various examples of their own choosing. Provide particular examples for students who are having difficulties.*

Why is this true? In general, when we write the quotient and expand it into products of a's, the result is

$$\frac{a^b}{a^c} = \frac{\overbrace{a \cdot a \cdot \ldots \cdot a}^{b \text{ factors}}}{\underbrace{a \cdot a \cdot \ldots \cdot a}_{c \text{ factors}}} = \frac{\overbrace{a \cdot a \cdot \ldots \cdot a}^{b-c \text{ factors}}}{1} \cdot \frac{\overbrace{a \cdot a \cdot \ldots \cdot a}^{c \text{ factors}}}{\underbrace{a \cdot a \cdot \ldots \cdot a}_{c \text{ factors}}} = \overbrace{a \cdot a \cdot \ldots \cdot a}^{b-c \text{ factors}} \cdot 1 = a^{b-c},$$

which shows why this fact is valid.

Key Idea

When we divide a power of *a* by another smaller power of *a*, the result is a power of *a,* in which the exponent is the difference of the exponents of the two dividends. In symbols,

$$\frac{a^b}{a^c} = a^{b-c} \text{ when } b > c.$$

This holds true for any nonzero number *a* and whole numbers *b* and *c*.

Example 3 **Find $\frac{m^7}{m^2}$.**

Solution $\frac{m^7}{m^2} = m^{7-2}$ or m^5

Example 4 **Find $\frac{6^5}{6^2}$.**

Solution $\frac{6^5}{6^2} = 6^{5-2}$ or 6^3

End of Lesson