Key Concepts

Solving Equations by Multiplying or Dividing

Objective Teach students to solve multiplication and division equations and to translate verbal problems into equations.

Note to the Teacher *This lesson uses one of the most important ideas in algebra, the Multiplication Property of Equality. Make sure students understand how it is being used. Another key skill that this lesson uses and develops is the translation of a verbal problem into an equation. It is important to practice this skill.*

Solving Equations by Multiplying or Dividing

Begin with a word problem.

Luisa is three times as old as Justin. If Luisa is 24 years old, how old is Justin?

In this problem, two quantities are equal to each other. In words, three times Justin's age is equal to Luisa's age. Luisa's age is 24, but we don't know Justin's age. When we do not know a quantity, we choose a letter as a placeholder for that value so we can work with the other numbers in the problem and perhaps determine the unknown quantity. The letter is called a **variable**. In this case, let's write j for Justin's age. The quantity "three times Justin's age" is now represented by $3j$. Since three times Justin's age is equal to Luisa's age, we now write

$$3j = 24.$$

This expression is called an **equation**. If an equality results when a number is substituted for j, the number is a **solution** to the equation.

In order to determine j, we'll need to use an important property of equations.

Key Idea	Multiplying or dividing any equation by a nonzero number results in a true equation. Any solution of the original equation will be a solution of the new equation, and any solution of the new equation will be a solution of the original equation.

These ideas are called the Multiplication and Division Properties of Equality. In $3j = 24$, we want to isolate the variable j on the left side, so that the equation reads

$$j = ?$$

where the ? is a quantity we have to determine that doesn't involve the variable j. Once the equation is in this form, it is solved since we know that j is equal to the number on the right side.

Let's solve the equation to determine Justin's age.

$$\begin{aligned} 3j &= 24 \\ \frac{3j}{3} &= \frac{24}{3} && \textit{Divide each side by 3.} \\ j &= 8 && \textit{Simplify.} \end{aligned}$$

So Justin is 8 years old.

Let's solve another equation using the Division Property of Equality.

Example 1 **Solve $7x = 112$.**

Solution
$$7x = 112$$
$$\frac{7x}{7} = \frac{112}{7} \quad \textit{Divide each side by 7.}$$
$$x = 16 \quad \textit{Simplify.}$$

The solution is 16.

The next example shows how to solve an equation using the Multiplication Property of Equality.

Example 2 **Solve $\frac{y}{12} = 4$.**

Solution
$$\frac{y}{12} = 4$$
$$12 \cdot \frac{y}{12} = 12 \cdot 4 \quad \textit{Multiply each side by 12.}$$
$$y = 48 \quad \textit{Simplify.}$$

The solution is 48.

How does this work? To answer this question, think of the general pattern. For an equation of the form (coefficient) $\cdot\, x$ = number, we divide the whole equation by the coefficient to get $x = \frac{\text{number}}{\text{coefficient}}$.

Have students complete the following exercises.

Exercises

Solve each equation.

1. $16y = 48$ **3** **2.** $1.5t = 6$ **4** **3.** $\frac{s}{3} = 17$ **51**

Translating Verbal Problems into Equations

Learning how to translate from verbal problems into equations is a very important skill. Here are some exercises to give to your students that will reinforce these skills.

Example 3 **Ms. Jones' class makes up one twenty-fifth of the student body of Grant Middle School. Suppose Ms. Jones' class has 27 students. Write an equation that can be used to determine the student body.**

Solution We know the number of students in Ms. Jones' class is equal to one twenty-fifth of the student body. Since we do not know the number of students in the student body, let s represent this amount. We can now write the following equation.

$$\underbrace{\textit{The number of students in Ms. Jones' class}}_{27} \quad \underbrace{\textit{is equal to}}_{=} \quad \underbrace{\textit{one twenty-fifth of the student body.}}_{\frac{1}{25}s}$$

Thus, $27 = \frac{1}{25}s$ can be used to determine the student body.

Example 4 **At the fruit market, Bob sells 56 pounds of apples per day. This amount is 4 times as many as the number of pounds of apples Joanne sells per day. Write an equation that can be used to find the amount of apples Joanne sells.**

Solution You know that the amount Bob sells is 4 times as many as the amount Joanne sells.

$$\underbrace{\textit{The amount Bob sells}}_{56} \quad \underbrace{\textit{is}}_{=} \quad \underbrace{\textit{4 times the amount Joanne sells.}}_{4j}$$

So, $56 = 4j$ or $4j = 56$ can be used to find the amount Joanne sells.

Exercises

4. At City College, there are two thirds as many students studying mathematics as history. Suppose there are 100 math students. Write an equation that represents the situation. Then find how many history students there are. $\frac{2}{3}h = 100$; $h = 150$

5. There are 15 times as many students as teachers at Lincoln Park Elementary School. Let s and t represent the number of students and teachers at the school. Write an equation relating these two variables. $15t = s$

Note to the Teacher *It's very important for students to understand Exercises 4 and 5. Students will frequently make the mistake of writing 15s = t as a solution to Example 5 because in the statement of the problem, the number 15 comes up nearer to the word "students" than the word "teachers." This is worth explaining after the class has tried the problem.*