Key Concepts

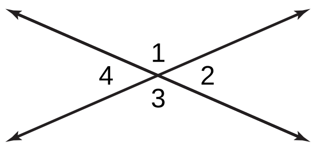
Angle Relationships and Parallel Lines

Objective Introduce the relationship between vertical, adjacent, complementary, and supplementary angles, as well as angles formed by two parallel lines and a transversal.

Note to the Teacher *In this lesson, students will learn some of the basic concepts of* ***plane geometry****. Plane geometry is the branch of geometry that deals with plane figures. It is one of the oldest areas of mathematics with a very rich history dating back to between 2200 and 2600 years ago. Plane geometry has roots in the work of Thales, Pythagorus, Euclid, and Archimedes.*

Pairs of Angles

In this lesson, students will learn the special relationships that pairs of angles can have. Be sure to use the correct terminology when discussing these angles. The first pair of angles we are going to talk about is **vertical angles**. Vertical angles occur when two lines intersect. The opposite angles are vertical. Draw the following vertical angles on the chalkboard.



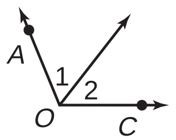
In the figure, angles 1 and 3, and angles 2 and 4 are vertical angles. The Vertical Angle Theorem follows. This theorem is said to have been discovered around 550 B.C. in ancient Greece by the great mathematician Thales. Write the theorem on the board and discuss it with students.

Vertical Angle Theorem	Vertical angles have equal measures.

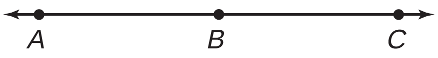
Recall that angles with the same measure are **congruent**. Therefore, the Vertical Angle Theorem can be restated as *vertical angles are congruent.*

Have students use a straightedge to draw two pairs of vertical angles. Remind students that this can be done by drawing two lines that intersect. Next, have students measure each pair of angles to verify that they are congruent.

When two angles in a plane have the same vertex, share a common side, and do not overlap, they are called **adjacent angles**. In the figure at the right, $\angle 1$ and $\angle 2$ are adjacent angles, and $m\angle AOC = m\angle 1 + m\angle 2$.

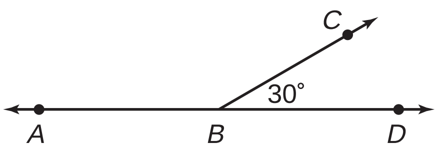


The measure of the angle that forms a straight line is 180°. This is sometimes called a **straight angle**. Refer to the figure below. If points *A*, *B*, and *C* all lie on a line,



then $m\angle ABC = 180°$. If the sum of the measures of two angles is 180°, we say that they are **supplementary**. Do the following example on the board.

Example 1 **If $m\angle CBD = 30°$, find $m\angle ABC$.**



Solution We know that $m\angle ABC + m\angle CBD = m\angle ABD$. Points *A*, *B*, and *D* lie on a straight line so we also know that $m\angle ABD = 180°$. Therefore, $\angle ABC$ and $\angle CBD$ are supplementary. The following equation results.

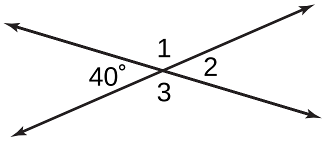
$$m\angle ABC + m\angle CBD = 180°$$

Since $m\angle CBD = 30°$, the following equation results.

$$m\angle ABC + 30° = 180°$$
$$m\angle ABC = 150°$$

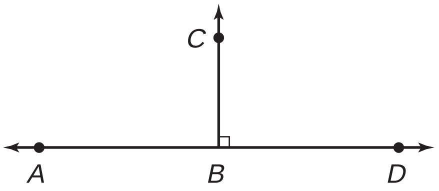
We can use the concepts of supplementary and vertical angles together to solve problems. Consider the following example.

Example 2 **Refer to the figure below. What are the measures of ∠1, ∠2, and ∠3?**



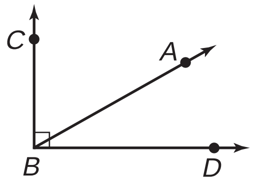
Solution The angle that measures 40° and ∠2 are vertical angles. So, by the Vertical Angle Theorem, $m\angle 2 = 40°$. Angles 2 and 3 are supplementary, so their sum is 180°. That is, $40° + m\angle 3 = 180°$. Therefore, $m\angle 3 = 140°$. Since ∠1 and ∠3 are vertical angles, $m\angle 1 = 140°$.

A 90° angle is called a **right angle**. Since 90° + 90° = 180°, an angle that is supplementary to a 90° angle also has a measure of 90°. If two line segments intersect at a 90° angle as shown,



then both ∠*ABC* and ∠*DBC* measure 90°. When two lines intersect to form a right angle, they are said to be **perpendicular**.

If the sum of the measures of two angles is 90°, then the angles are complementary.

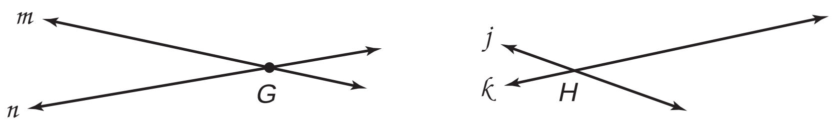


∠CBA and ∠ABD are complementary.

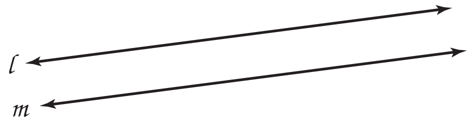
Parallel Lines

When we refer to lines, we will always mean infinite lines. That is, lines that go on without end in both directions.

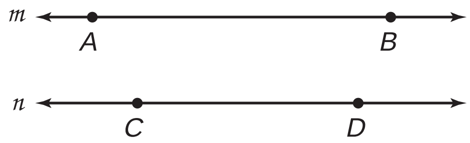
Many pairs of lines in a plane will eventually intersect each other.



However, if two lines in a plane never intersect, they are **parallel**. The figure below shows an example of parallel lines.

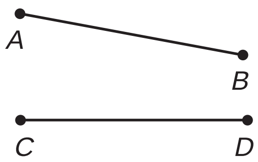


If two lines are parallel to each other, then the segments contained in each of the lines are parallel to each other.

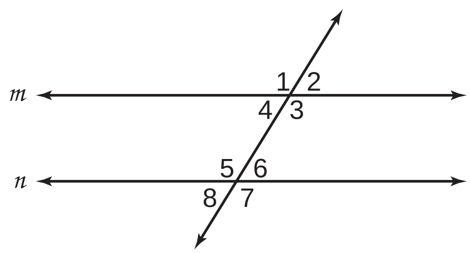


Line m contains $\overline{AB}$ and line n contains $\overline{CD}$. Since $m \parallel n$, $\overline{AB} \parallel \overline{CD}$.

In the figure below, $\overline{AB}$ and $\overline{CD}$ are not parallel even though they do not intersect. This is because if we extend the lines that contain $\overline{AB}$ and $\overline{CD}$, they will eventually intersect.



When two parallel lines are intersected by a third line, called a **transversal**, eight angles are formed. Four are *interior* angles and four are *exterior* angles.



Angles 3, 4, 5, and 6 are interior angles. Angles 1, 2, 7, and 8 are exterior angles. These angles can be paired up as follows.

- **alternate interior angles:** $\angle 4$ and $\angle 6$, and $\angle 3$ and $\angle 5$
  Alternate interior angles are nonadjacent interior angles found on opposite sides of the transversal.
- **alternate exterior angles:** $\angle 1$ and $\angle 7$, and $\angle 2$ and $\angle 8$
  Alternate exterior angles are nonadjacent exterior angles found on opposite sides of the transversal.

- **corresponding angles:** $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, and $\angle 4$ and $\angle 8$
  Corresponding angles are angles that have the same position on two different parallel lines cut by a transversal.

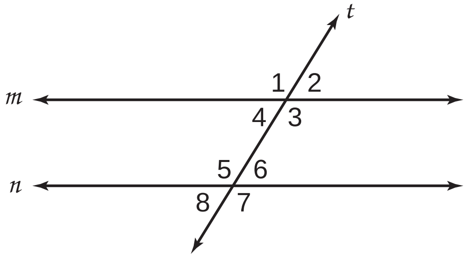
The main properties of these angle relationships are listed below.

- Corresponding angles are congruent.
- Alternate interior angles are congruent.
- Alternate exterior angles are congruent.

Have your students draw lines that are parallel on a sheet of paper. You may wish to have them use two lines on a sheet of ruled notebook paper. Then have them draw a transversal. Next, instruct students to use a protractor to measure the corresponding, alternate interior, and alternate exterior angles to verify the properties.

The previous concepts are incorporated in the following example.

Example 3 **In the figure below, $m \parallel n$, t is a transversal, and $m\angle 2 = 30°$. Find each of the other angle measures.**



Solution Notice that $\angle 1$ and $\angle 2$ are supplementary. The following equation results.

$m\angle 1 + m\angle 2 = 180°$

$m\angle 1 = 180° - m\angle 2$	*Subtract $m\angle 2$ from each side.*
$m\angle 1 = 180° - 30°$	*Replace $m\angle 2$ with 30°.*
$m\angle 1 = 150°$	*Subtract.*

$\angle 1$ and $\angle 3$, and $\angle 2$ and $\angle 4$ are pairs of vertical angles. So, by the Vertical Angle Theorem, we have

$$m\angle 3 = m\angle 1 = 150°$$

and

$$m\angle 4 = m\angle 2 = 30°.$$

$\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, and $\angle 4$ and $\angle 8$ are pairs of corresponding angles. By the properties, we know that they are congruent to each other.

$m\angle 1 = m\angle 5 = 150°$

$m\angle 2 = m\angle 6 = 30°$

$m\angle 3 = m\angle 7 = 150°$

$m\angle 4 = m\angle 8 = 30°$

To solidify the concepts introduced in this lesson, you may wish to assign a few more exercises similar to the ones in the Student Edition.

End of Lesson