# Key Concepts

## Ordered Pairs and Relations

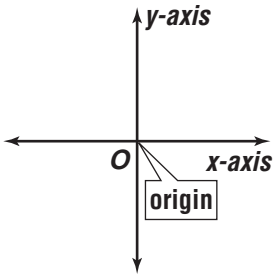
**Objective** Teach students to plot points named by ordered pairs of coordinates, and understand and graph relations.

**Note to the Teacher** *Graphing is the most commonly used method of representing data pictorially. The ideas of relations and functions are crucial not only in algebra but in calculus and other areas of mathematics. These concepts are somewhat abstract, so it is important to illustrate both concepts with numerous pictorial examples.*

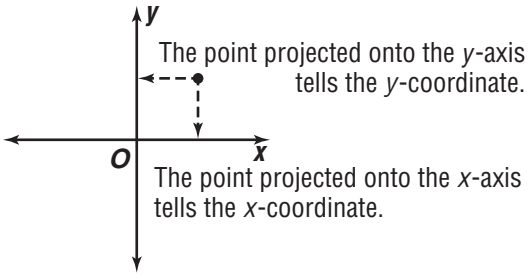
### Plotting Points

Remind students that numbers can be represented as points on a number line. Review this concept by plotting a few points on a number line. Then ask students "If points on a number line correspond to numbers, what do points in a plane represent?"

Suppose we draw two lines in a plane at right angles to each other. These are called *axes* and where they meet is called the *origin.* The horizontal line is usually the *x*-axis and the vertical line is the *y*-axis.




For any point in this plane, we can find two numbers to represent the location that point by *projecting* the point onto each of the axes. These numbers are the *coordinates* of the point.

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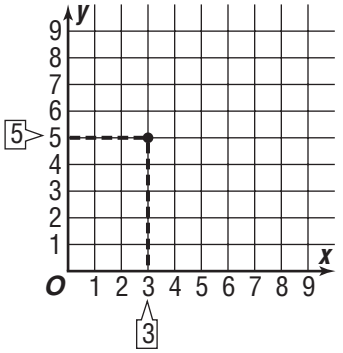
Using a similar logic, we can also take any two numbers, such as 3 and 5, and find a point whose $x$-coordinate is 3 and whose $y$-coordinate is 5.

Remember that each of the axes is actually a number line.




To locate this point, first locate 3 on the horizontal axis and draw a vertical line through it.

Then find 5 on the vertical axis and draw a horizontal line through it.

The intersection of the lines you drew is the point whose coordinates are 3 and 5.

Have the class plot a number of points.

**Note to the Teacher** *You may want to point out that by using graph paper, students can make more accurate graphs. The grid lines on the graph paper represent the horizontal and vertical lines they would draw to locate integers on the axes.*

Have the students plot several points whose coordinates you name. Introduce ordered pair notation, (x, y). Stress that the x-coordinate is always named first. Have students look at the points they have just graphed and write the ordered pair that names each point. Also point out that the coordinates of the origin are (0, 0).

**Note to the Teacher** *Point out that the ordering of the coordinates is important. (3, 5) does not name the same point as (5, 3).*

## Relations

Explain that a *relation* is just a collection of ordered pairs. Relations may be finite or infinite. Finite means that all ordered pairs of the relation can be listed. Infinite means that it is not possible to list all the ordered pairs of the relation.

**Example 1** **Determine whether each set is a relation and, if so, whether it is *finite* or *infinite*.**

**a. {(1, 2), (3, 6), (−1, −2)}**

**b. {all points whose $x$-coordinate is equal to its $y$-coordinate}**

**c. {all points that are located one unit from the origin}**

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**Solution** All three sets are relations.

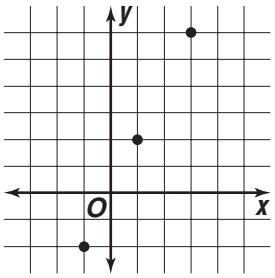
{(1, 2), (3, 6), (−1, −2) is a finite relation because you can name all the points in the relation.

The last two sets are infinite relations because you cannot name all the points in the relation.

Next explain that since relations are sets of ordered pairs, they can be graphed. To graph a finite relation, just plot each of the ordered pairs in the relation.

**Example 2** **Graph the relation {(1, 2), (3, 6), (−1, −2)} from Example 1.**

**Solution** When we plot these three points, we get this graph.



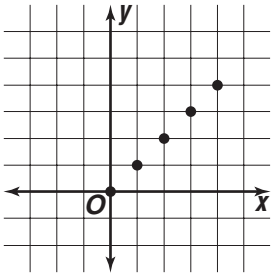

Remind students that a line or a curve is made up of an infinite number of points. To graph relations that are infinite, we plot several points in the relation and then sketch the line or curve suggested by those points.

**Example 3** **Graph the relation {all points whose *x*-coordinate is equal to its *y*-coordinate}.**

**Solution** First find several ordered pairs that belong to this relation.

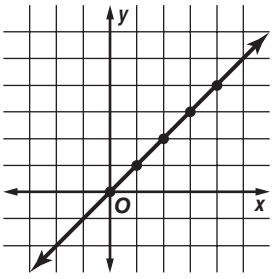
Some of these would include (0, 0), (1, 1), (2, 2), (3, 3), (4, 4)

Now graph these points.




Notice that the points suggest a straight line.

Sketch a line through these points to represent the relation.

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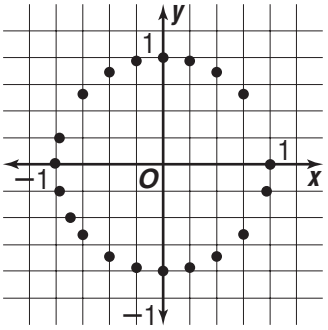
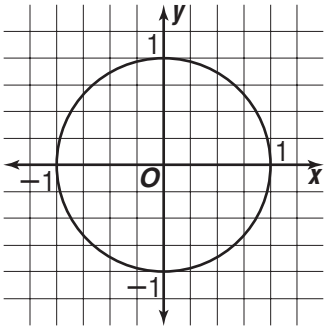
**Example 4** **Graph the relation {all points that are one unit from the origin}.**

**Solution** Students may remember that all points a given distance from a given point is the definition of a circle. So the graph of this relation is a circle whose center is the origin and whose radius is 1 unit. However, students may not have been exposed to this concept and you need to use a unit of measure to locate several points that are one unit from the origin. Graph enough points to suggest the circle. Then complete the curve




## How can relations help you visualize data?

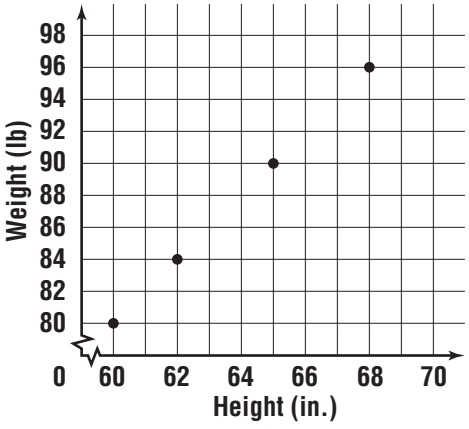
Graphing relations is an excellent way to visualize data and sometimes draw conclusions about the data.

**Example 5** **Several students had their heights and weights recorded. This data can be expressed as ordered pairs with the first coordinate being the height and the second being weight. Some of the ordered pairs are shown below.**

**(60, 80), (62, 84), (65, 90), and (68, 96)**

**Graph the data and describe any trends you observe.**

**Solution** When the points are graphed, they seem to lie on a line. The upward slant of the line as we move from left to right tells us that the taller the person is the heavier that person is likely to be.

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Sometimes data is given in a table and ordered pairs can be formed from the data as in the next example.

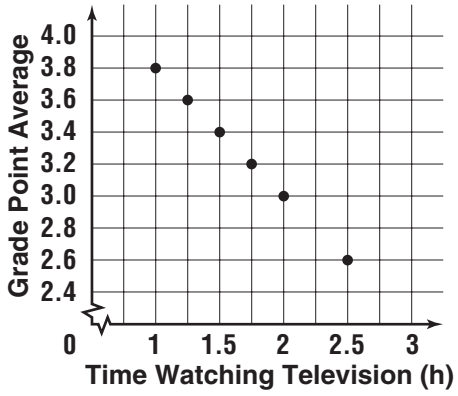
**Example 6** **Six students were asked about the average amount of time they spend watching television each evening. Then they were asked about their grade point average. The table shows the results of this survey. Graph the data and determine if the data suggests a trend.**

| Student | Time Spent Watching Television (hours) | Grade Point Average |
|---|---|---|
| Marcia | 1 | 3.8 |
| Ramon | 1.25 | 3.6 |
| Susan | 1.5 | 3.4 |
| Robert | 1.75 | 3.2 |
| Elaine | 2 | 3.0 |
| Derek | 2.5 | 2.6 |

**Solution** For each student, we can form an ordered pair with the first coordinate corresponding to the TV-watching time and the second coordinate corresponding to the grade point average.

{(1, 3.8), (1.25, 3.6), (1.5, 3.4), (1.75, 3.2), (2, 3.0), (2.5, 2.6)}

Now plot these ordered pairs.




Note that the points form a linear pattern that slopes downward as we move from left to right. This pattern suggests that the more time a student spends watching television, the lower his or her grade point average is likely to be.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