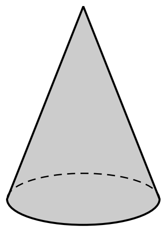
Key Concepts

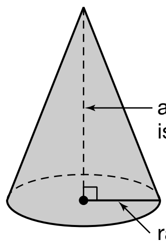
Volume of Pyramids and Cones

Objective Teach students the formulas for the volumes of pyramids and cones.

Note to the Teacher *Begin the lesson with a discussion of what cones and pyramids are. With the example of an ice cream cone in mind, guide the discussion so students understand that a cone is a 3-dimensional figure whose base is a circle, from which it rises to a single point called its* ***vertex****. A cone is shown below.*



Explain that the **altitude** of a cone is the segment from the vertex of the cone to its base that is perpendicular to the base. The length of the altitude is the **height** of the cone.



Formula for the Volume of a Cone

Note to the Teacher *It will be beneficial to your students if they do the Mini-Lab on page 490 of the Student Edition. This activity will provide them with some insight into how the formula is developed.*

The volume V of a cone is $\frac{1}{3}$ the area of the circular base B times the height h.

$$V = \frac{1}{3}Bh$$

The area of a circle is πr^2. So we can replace B in the volume formula with πr^2, giving

$$V = \frac{1}{3}\pi r^2 h.$$

Now do an example in class and give your students more examples to work on. Here is a typical example. There are more examples in the Student Edition.

Example 1 **Find the volume of a cone if the base has radius 3 inches and the height is 6 inches.**

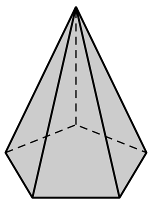
Solution We use the formula $V = \frac{1}{3}\pi r^2 h$. In this case, $r = 3$ inches and $h = 6$ inches.

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(3^2)(6) \quad \textit{Replace r with 3 and h with 6.} \\ &= \frac{1}{3}\pi(9)(6) \\ &= 18\pi \end{aligned}$$

So the volume of the cone is 18π cubic inches. If we use 3.14 to approximate the value of π, we get approximately 56.52 cubic inches for the volume of the cone.

Formula for the Volume of a Pyramid

A **pyramid** is similar to a cone except that its base is a polygon rather than a circle. The figure below is a *pentagonal pyramid*, so named because its base is a pentagon.

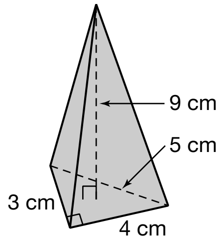


Just as for a cone, the volume V of a pyramid is $\frac{1}{3}$ of the area of the base B times the height h. Unlike cones where the base is always a circle, the computation of the value of B for a pyramid is unique, depending on the type of polygon that is the base of the pyramid. So we cannot replace B with another expression, and we simply use the formula

$$V = \frac{1}{3}Bh.$$

You should do an example or two in class, and then give more examples for the students to work on individually or in a group. Here is a typical example.

Example 2 **A triangular pyramid has a base that is a right triangle with side lengths 3, 4, and 5 centimeters, and whose height *h* is 9 centimeters as shown below. Find the volume of the pyramid.**



Solution We use the formula $V = \frac{1}{3}Bh$. The base of this pyramid is a right triangle whose area is $\frac{1}{2}(3)(4)$ or 6. So, $B = 6$.

Note to the Teacher *This is a good time to review finding the area of a right triangle.*

Use the volume formula. The height h of the pyramid is 9.

$V = \frac{1}{3}Bh$

$= \frac{1}{3}(6)(9)$ *Replace B with 6 and h with 9.*

$= 18$

So, the volume of the pyramid is 18 cubic centimeters.

Note to the Teacher *Working with the volume formula for pyramids will force your students to review the area formulas for the different bases you use. Give your students pyramids with bases that are triangles, parallelograms, and trapezoids so that computing the volumes will reinforce their knowledge of the areas of these polygons.*