Key Concepts

Congruent Triangles

Objective Introduce students to the concepts of congruence and congruent triangles, and teach them how to compare angle measures and side lengths of congruent triangles.

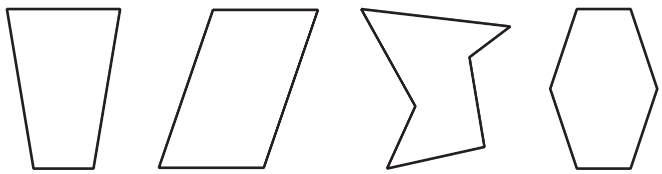
Note to the Teacher *In this lesson, students will learn the concept of congruence. Point out that two figures are congruent if they have the same shape and size. After giving this definition, ask students what they think the phrase "the same shape" means. Also ask them what measure (perimeter, area, and so on) is meant by the phrase "the same size." Use the discussion to motivate the lesson content.*

The notion of congruence is important for comparing polygons. Begin by reviewing the definition of polygon with your students.

A **polygon** is a simple closed figure in a plane formed by three or more line segments. The segments are called *sides* and their endpoints are called *vertices*. The sides and vertices of a polygon have the following properties.

1. The sides do not intersect each other except at the endpoints.
2. Each vertex is an endpoint of exactly two sides.

Here are several examples of polygons.



Point out to students that triangles are probably the most studied of all polygons in geometry courses. When studying triangles, we focus on their sides and their angles. We say that two sides of a triangle are **congruent** if their lengths are equal. We write the mathematical sentence

$$\overline{AB} \cong \overline{CD}$$

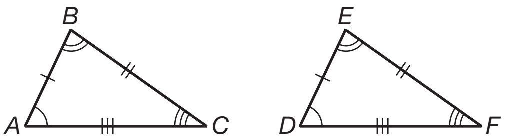
(read "segment *AB* is congruent to segment *CD*") to indicate that segments *AB* and *CD* are congruent.

We say that two angles of a triangle are **congruent** if their degree measures are equal. We write the mathematical sentence

$$\angle X \cong \angle Y$$

(read "angle X is congruent to angle Y") to indicate that angles X and Y are congruent.

We can define congruence more precisely by saying that *two polygons are congruent if all of their corresponding parts (sides and angles) are congruent.* Ask your students what they think is meant by the term *corresponding parts* used in this definition. Sketch the figure below on the chalkboard.



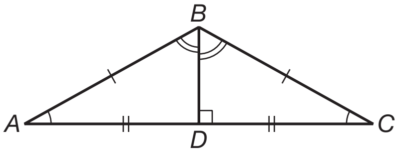
$\triangle ABC \cong \triangle DEF$

The arcs and tick marks in the figure indicate the congruence of the corresponding parts of the triangles. In the figure, $\angle A$ and $\angle D$ are corresponding angles, as are $\angle B$ and $\angle E$, and $\angle C$ and $\angle F$. Similarly, $\overline{AB} \cong \overline{DE}$ are corresponding sides, as are $\overline{BC}$ and $\overline{EF}$, and $\overline{CA}$ and $\overline{FD}$. Point out that students can also read the corresponding parts from the congruence statement shown under the triangles above.

So, in the notion of congruent triangles, the congruent angles assure that the triangles have "the same shape," and the congruent sides assure that the triangles have the "same size."

Do the following example on the chalkboard.

Example 1 **The corresponding parts of two congruent triangles are marked in the figure below. Write a congruence statement for the two triangles.**



Solution First list the pairs of congruent angles and the pairs of congruent sides.

$$\angle A \cong \angle C \qquad \overline{AB} \cong \overline{BC}$$
$$\angle ABD \cong \angle CBD \qquad \overline{AD} \cong \overline{CD}$$
$$\angle ADB \cong \angle CDB \qquad \overline{BD} \cong \overline{BD}$$

Note to the Teacher *Point out that any segment or angle is congruent to itself, as shown by the statement $\overline{BD} \cong \overline{BD}$ above. Emphasize that $\overline{BD}$ is a side of both smaller triangles shown in the figure.*

The congruence statement can then be written by matching the vertices of the congruent angles. Vertex *A* goes with vertex *C*, vertex *B* goes with itself, and vertex *D* goes with itself. So, the congruence is written as

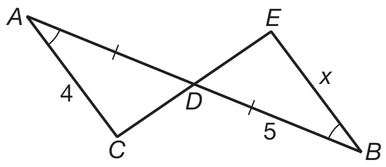
$$\triangle ADB \cong \triangle CDB.$$

Point out that if two triangles are known to be congruent, then there are six pairs of congruent corresponding parts, three pairs of angles and three pairs of sides, as shown in Example 1. But what must we be able to show about two triangles in order to declare that they are congruent? In certain circumstances, showing just three of the six congruencies is enough. Three such circumstances are given below.

Congruent Triangles	Two triangles must be congruent if the following corresponding parts of the triangles are shown to be congruent: **1.** all three pairs of corresponding sides. (This situation is known by the letters *SSS*, which stand for Side-Side-Side.) **2.** two pairs of corresponding angles and the corresponding sides included between them. (This situation is known by the letters *ASA*, which stand for Angle-Side-Angle.) **3.** two pairs of corresponding sides and the corresponding angles included between them. (This situation is known by the letters *SAS*, which stand for Side-Angle-Side.)

Now present several examples like Example 2 on the next page. There are other good examples like this one in the Student Edition.

Example 2 **In the figure below, $\angle A \cong \angle B$ and $\overline{AD} \cong \overline{BD}$. Find the value of x.**



Solution Notice the pair of vertical angles that share point D as their vertex. From the discussion in Lesson 5-1 of the Student Edition, we know these angles are congruent. We also are given that $\angle A$ is congruent to $\angle B$, and that $\overline{AD}$ is congruent to $\overline{BD}$. Looking at the figure, we can see that the corresponding parts have the pattern ASA discussed on the previous page. So the two triangles are congruent; that is, $\triangle ADC \cong \triangle BDE$. This congruence statement shows that $\overline{BE}$ is congruent to $\overline{AC}$. Since the length of $\overline{AC}$ is given as 4 in the figure, the length of $\overline{BE}$ must also be 4. Therefore, $x = 4$.

Here is a question for class discussion that will help solidify your students' understanding of the concept of congruence. Suppose two triangles are congruent and we know that the perimeter of one of the triangles is 10 feet. What is the perimeter of the other triangle? Explain your answer. **Two congruent triangles have equal perimeters, so the second triangle also has a perimeter of 10 feet. This is true because congruent triangles have congruent corresponding sides. In other words, each of the three sides of one triangle has the same length as its corresponding side on the other triangle. Since the perimeter is the sum of the lengths of the sides, the two congruent triangles must have equal perimeters.**