

# Key Concepts



## The Percent Proportion

**Objective** Teach the concept of percent, how percents are used, and how they are related to fractions.

**Note to the Teacher** *In this lesson, your students will learn about the concept of percent and how to calculate a percent.*

Begin the lesson by doing an example like this one.

**Example 1** In a class with 20 students, 12 are girls. What is the percentage of girls in the class?

**Solution** Set up a proportion problem using the ratio of girls to all students in the class. Letting  $x$  denote the percentage of girls in the class, the proportion is

$$\frac{12}{20} = \frac{x}{100}$$

To solve this proportion, cross multiply and solve for  $x$ .

$$100 \cdot 12 = 20 \cdot x \quad \text{Cross multiply.}$$

$$1,200 = 20x$$

$$\frac{1,200}{20} = \frac{20x}{20} \quad \text{Divide each side by 20.}$$

$$60 = x$$

So, 60% of the students in the class are girls.

Now do another example to make sure students understand how fractions and ratios can be expressed as percents.

**Example 2** Express the fraction  $\frac{3}{7}$  as a percent.

**Solution** Set up the proportion problem

$$\frac{3}{7} = \frac{x}{100}$$

where  $x$  denotes the percent. Now solve for  $x$ .

$$100 \cdot 3 = 7 \cdot x \quad \text{Cross multiply.}$$

$$\frac{300}{7} = \frac{7x}{7} \quad \text{Divide each side by 7.}$$

$$42.86 \approx x \quad \text{Use a calculator; round to the nearest hundredth.}$$

So,  $\frac{3}{7}$  is approximately 42.86%.

In everyday life, we encounter percents in many situations. The following example presents a common one.

**Example 3** Akili and his mother went out to lunch and their bill was \$18. His mother said that she wanted to leave a 15% tip for the waitress. How much she should leave as a tip?

**Solution** Since Akili's mother wants to leave a 15% tip, she needs to compute 15% of \$18. Substituting a multiplication symbol for the word "of" in the phrase "15% of \$18" gives the multiplication expression

$$15\% \times \$18.$$

To find the value of this expression, recall that 15% can be expressed as the fraction  $\frac{15}{100}$ . Replacing 15% by  $\frac{15}{100}$  gives

$$\begin{aligned} 15\% \times \$18 &= \frac{15}{100} \times \$18 \\ &= \frac{15 \times \$18}{100} \\ &= \frac{\$270}{100} \\ &= \$2.70 \end{aligned}$$

So, a 15% tip for an \$18 bill is \$2.70.

In the following example, the percent is one of the known values in the percent proportion.

**Example 4** 180 is 30% of what number?

**Solution** Let  $x$  denote the number we are trying to find. Remember, when we see the word "of" we should substitute a multiplication sign. So we can translate the problem statement into the equation

$$180 = 30\% \cdot x.$$

Now, by changing percent to a fraction, we have

$$180 = \frac{30}{100} \cdot x \quad \text{Replace } 30\% \text{ with the fraction } \frac{30}{100}.$$

$$180 = \frac{30x}{100} \quad \text{Express the right side as a single fraction.}$$

$$100 \cdot 180 = 100 \cdot \frac{30x}{100} \quad \text{Multiply each side by } 100.$$

$$18,000 = 30x$$

$$\frac{18,000}{30} = \frac{30x}{30} \quad \text{Divide each side by } 30.$$

$$600 = x$$

So, 180 is 30% of 600.

Ask students how we could verify that the answer in Example 4 is correct. Lead students to see that the statement “180 is 30% of 600” means that the equation  $180 = \frac{30}{100} \cdot 600$  must be true. To check the answer we can do the multiplication to see if we get 180.

Here is one more example involving a restaurant tip.

**Example 5** Aunt Marlene took her sister’s family out to a restaurant for dinner. When dinner was over and the bill arrived, she decided that she was very happy with the service, so she would leave an 18% tip. She computed the tip and left \$9.50 for the waiter. How much was the bill for the dinner, to the nearest dollar?

**Solution** Let  $x$  represent the amount of the bill. Since Marlene left an 18% tip which amounted to \$9.50, we know that \$9.50 is 18% of  $x$ . Substituting a multiplication sign for the word “of,” we have the equation  $\$9.50 = 18\% \cdot x$ .

To solve for  $x$ , we again replace the percent with an equivalent fraction.

$$\$9.50 = \frac{18}{100} \cdot x \quad \text{Replace } 18\% \text{ with the fraction } \frac{18}{100}.$$

$$\$9.50 = \frac{18x}{100} \quad \text{Express the right side as a single fraction.}$$

$$100 \cdot \$9.50 = 100 \cdot \frac{18x}{100} \quad \text{Multiply each side by 100.}$$

$$\$950 = 18x$$

$$\frac{\$950}{18} = \frac{18x}{18} \quad \text{Divide each side by 18.}$$

$$\$52.78 \approx x \quad \text{Use a calculator.}$$

So the bill for dinner was approximately \$53.

**Note to the Teacher** *Computing with percents is an extremely important skill. Make sure your students are given lots of problems to work. This will help them solidify their understanding of the concept of percent, as well as make them more proficient with their computations.*

