Key Concepts

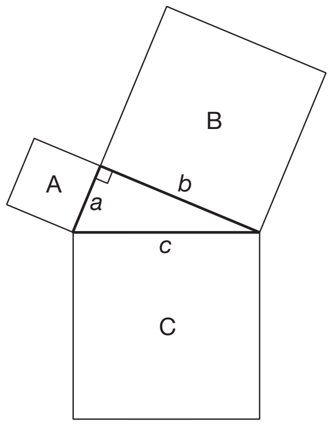
The Pythagorean Theorem

Objective Teach students the Pythagorean Theorem and how to use it.

Note to the Teacher *The Pythagorean Theorem is probably the most famous theorem in mathematics. Its history goes back at least to Pythagoras, who was a famous mathematician and philosopher in ancient Greece. He lived about 2,500 years ago. However some historians think that the theorem went back much further, as ancient writings have been found that list "Pythagorean triples," groups of three whole numbers (a, b, c) that can be the lengths of the sides of a right triangle. Mention this history to your class. As a possible extra-credit project, have students research the history of the theorem in more detail and write a report or give a presentation to the class.*

Geometric Representation

It is helpful to begin your discussion of the Pythagorean Theorem by presenting a purely geometric representation of the theorem. On the chalkboard, draw the figure shown below. The figure is a right triangle with a square emanating from each of the sides.



Point out the side lengths, a, b, and c of the right triangle, and the fact that $a < b < c$. Stress that the areas of the three squares A, B, and C are then a^2, b^2, and c^2, respectively. Provide the following series of diagrams to help students visualize the fact that the area of the largest square is equal to the sum of the areas of the two smaller squares, that is, $c^2 = a^2 + b^2$.

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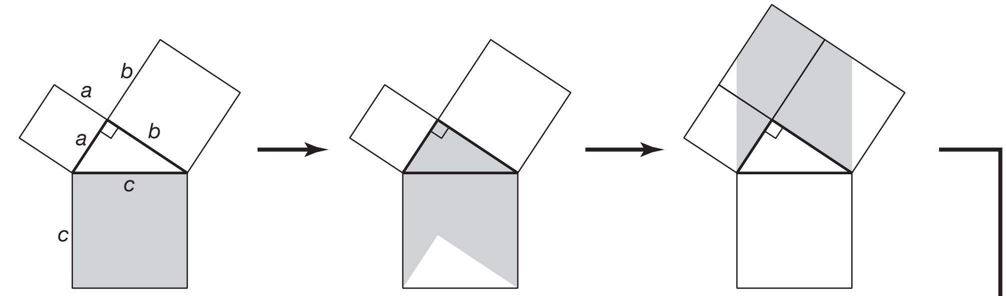
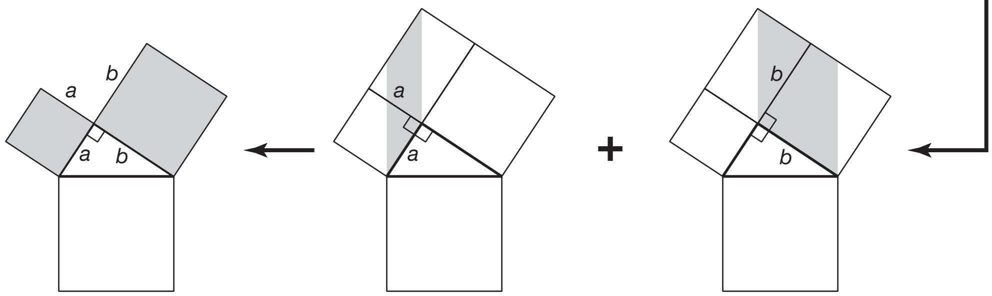


area of shaded square: c^2
area of two unshaded squares: $a^2 + b^2$

area of shaded region: c^2

area of shaded region: c^2

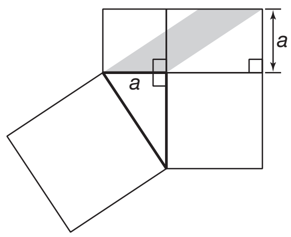


area of shaded region: $a^2 + b^2$

The shaded region is a parallelogram with base a and height a.
area of shaded region: $a \cdot a = a^2$

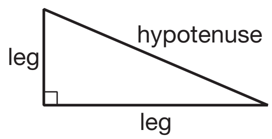
The shaded region is a parallelogram with base b and height b.
area of shaded region: $b \cdot b = b^2$

Note to the Teacher *Students may have difficulty visualizing the height of the shaded parallelograms shown in the bottom row of figures above. Remind them that the height is the perpendicular distance from the chosen base to the side opposite the base. In both figures, identify the base as the side of the parallelogram that is a leg of the right triangle. Then identify the corresponding height. For the parallelogram whose base measure is a, turning the figure as shown at the right may help students better visualize the height and its measure a.*



Before stating the theorem in its traditional form, students need to learn some terminology regarding right triangles. Write these two sentences on the chalkboard and sketch the figure below them.

1. The sides of a right triangle that are adjacent to the right angle are called the **legs** of the triangle.
2. The side opposite the right angle is called the **hypotenuse**.



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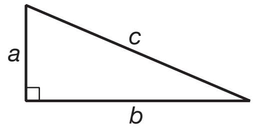
The Pythagorean Theorem	**Words:** In any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. **Symbols:** Consider a right triangle whose legs have lengths *a* and *b* and whose hypotenuse has the length *c* as shown in the figure below. Then $a^2 + b^2 = c^2$. 

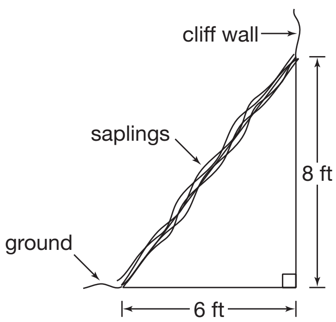
Engage your students in a classroom discussion about the following question. "Why are the geometric and algebraic descriptions of the Pythagorean Theorem equivalent?" **In a right triangle with side lengths *a*, *b*, and *c*, the areas of the squares sharing these sides are equal to a^2, b^2, and c^2, respectively.**

Here is another question you can use to challenge your class. "Can there be a right triangle whose side lengths are 4, 5, and 7 inches?" **No; by definition, the hypotenuse is the longest side of a right triangle. But $7^2 = 49$ is not equal to $4^2 + 5^2 = 16 + 25$ or 41. So the Pythagorean Theorem is not satisfied by these three numbers, and therefore a triangle with these three side lengths is not a right triangle.**

Here are some examples that require the use of the Pythagorean Theorem. Present them (or problems like them) in class, so that your students will begin to understand how the Pythagorean Theorem is used.

Example 1 **At a skills camp, a group of five scouts is to construct a lean-to shelter at the base of a cliff. The first step of the construction requires them to position a group of saplings against the cliff wall with the bottoms of the saplings 6 feet out from the cliff wall and the tops of the saplings resting against the cliff wall 8 feet above the ground. To what length should they cut each sapling? Assume that the cliff wall is at a right angle to the ground.**

Solution First, draw a figure that models the situation. Include all known measures. From the figure, we see that the saplings form the hypotenuse of a right triangle whose legs are 6 feet and 8 feet long. Let *s* denote the required length of each sapling.



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By the Pythagorean Theorem, we have

$s^2 = 6^2 + 8^2$ *Use $c^2 = a^2 + b^2$, with $a = 6$, $b = 8$, and $c = s$.*

$s^2 = 36 + 64$

$s^2 = 100$

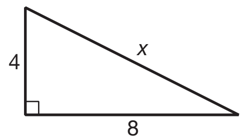
$s = \sqrt{100}$ *Take the square root of each side.*

$s = 10$ *The value of s, a length, must be positive.*

So, the saplings should each be cut approximately 10 feet long.

Often when working with the Pythagorean Theorem we have to approximate a square root. Here is an example where this is necessary. Be sure to work through at least one example like this with your students.

Example 2 **Find the length of the third side of the triangle shown below.**



Solution By the Pythagorean Theorem, we have the following.

$x^2 = 4^2 + 8^2$

$x^2 = 16 + 64$

$x^2 = 80$

$x = \sqrt{80}$

We know that $\sqrt{81} = 9$ and $\sqrt{64} = 8$. So $x = \sqrt{80}$ is much closer to 9 than it is to 8. We can approximate the value of x as $x \approx 8.9$. If we use a calculator to find the approximate value of $\sqrt{80}$, we get $x \approx 8.94427191$.

Note to the Teacher *Finish the lesson by giving your students several problems involving the Pythagorean Theorem. Its use addresses both geometric and algebraic concepts, and practices skills such as taking square roots.*

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