Key Concepts

Multiplying Fractions

Objective Teach students to multiply fractions.

Note to the Teacher *Multiplying fractions is a little more difficult conceptually than adding fractions, but the arithmetic procedure is much easier. The discussion here will begin with the multiplication of a fraction by a whole number, proceed to the multiplication of two fractions each with numerator of 1, and then finish with the multiplication of any pair of fractions.*

Multiplying a Fraction by a Whole Number

Suppose we want to multiply the fraction $\frac{1}{8}$ by the number 3. We can think of multiplying the fraction by 3 as taking "3 copies of $\frac{1}{8}$." We can model this multiplication by drawing a rectangle, dividing it into 8 equal pieces, and then shading 3 of them.

The shaded pieces represent $\frac{3}{8}$ of the entire rectangle. Notice that this answer can be found by multiplying the numerator of the fraction $\frac{1}{8}$ by 3 and retaining the denominator.

Key Idea	To multiply a fraction by a whole number, multiply the numerator in the fraction by the whole number and place this product over the denominator.

Example 1 $2 \times \frac{1}{5} = \frac{2 \times 1}{5}$ or $\frac{2}{5}$

Example 2 $\frac{2}{7} \times 3 = \frac{2 \times 3}{7}$ or $\frac{6}{7}$

Example 3 $4 \times \frac{3}{11} = \frac{4 \times 3}{11}$

$= \frac{12}{11}$ or $1\frac{1}{11}$

Why Does This Procedure Work?

Point out that the multiplication of two whole numbers can be thought of as repeated addition. For example,

$$2 \times 3 = 3 + 3 \quad 3 \times 7 = 7 + 7 + 7 \quad 5 \times 9 = 9 + 9 + 9 + 9 + 9$$

The multiplication of a fraction by a whole number can be thought of in the same way.

$$3 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \qquad \frac{2}{7} \times 6 = \frac{2}{7} + \frac{2}{7} + \frac{2}{7} + \frac{2}{7} + \frac{2}{7} + \frac{2}{7}$$

Remind students that when we add fractions with like denominators, we add them by adding the numerators and retaining the denominator. Point out that the repeated additions involve adding fractions with like denominators since the fractions are the same.

$$3 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \quad \text{and} \quad \frac{2}{7} \times 6 = \frac{2}{7} + \frac{2}{7} + \frac{2}{7} + \frac{2}{7} + \frac{2}{7} + \frac{2}{7}$$

$$= \frac{1 + 1 + 1}{2} \qquad\qquad = \frac{2 + 2 + 2 + 2 + 2 + 2}{7}$$

$$= \frac{3 \times 1}{2} \text{ or } \frac{3}{2} \qquad\qquad = \frac{2 \times 6}{7} \text{ or } \frac{12}{7}$$

The last step of the process in each example above shows that multiplying a fraction by a whole number is accomplished by multiplying the whole number times the numerator and keeping the same denominator.

Multiplying Two Fractions Whose Numerators Are Both 1

Key Idea	To multiply two fractions whose numerators are both 1, multiply the denominators and keep the numerator equal to 1.

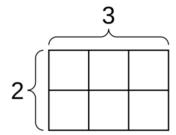
Example 4 $\frac{1}{3} \times \frac{1}{5} = \frac{1}{3 \times 5}$ or $\frac{1}{15}$

Example 5 $\frac{1}{4} \times \frac{1}{7} = \frac{1}{4 \times 7}$ or $\frac{1}{28}$

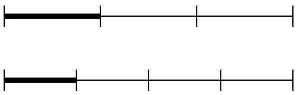
Example 6 $\frac{1}{9} \times \frac{1}{3} = \frac{1}{9 \times 3}$ or $\frac{1}{27}$

Why Does This Procedure Work?

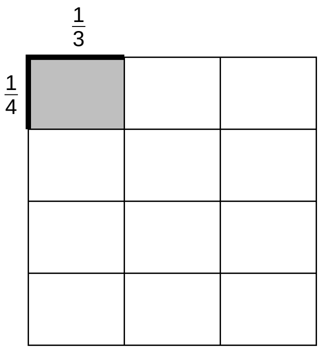
Recall that one way to model the multiplication of two whole numbers is to draw a rectangle whose length and width are the two numbers. After dividing the rectangle into unit squares, counting the unit squares gives the product of the two whole numbers. For example, consider the product 2×3 modeled below.



The model shows that $2 \times 3 = 6$. Modeling the product of two fractions is slightly different. Begin with two segments of the same length. Divide one segment into the number of equal parts indicated by the denominator of the first fraction. Divide the other segment into the number of equal parts indicated by the denominator of the second fraction. Suppose that we want to multiply $\frac{1}{3}$ and $\frac{1}{4}$. Divide one segment into 3 equal parts and the other into 4 equal parts. Since the numerator of each fraction is 1, darken one of the parts of each segment. This is shown in the figure below.



Next, we use the segments as two of the adjacent sides of a square. The marks on the segments are used to divide the square into smaller regions that are all the same size. Finally, we shade the smaller region formed by the two darkened parts of the segments.



The shaded region represents the product of the fractions. Point out that it is one of the 12 regions of equal size into which the square is divided. Since each side of the square represents a length of 1, the square has an area of 1 square unit. Therefore, the shaded region represents the fraction $\frac{1}{12}$. The model shows that $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$.

Multiplying Fractions in General

But what if the fractions do not have 1 as their numerators? The following procedure can be used to multiply any two fractions.

Key Idea	To multiply two fractions, multiply the numerators together to find the numerator of the product, and multiply the denominators together to find the denominator of the product. Simplify the resulting fraction, if possible.

Example 7 $\frac{2}{3} \times \frac{4}{7} = \frac{2 \times 4}{3 \times 7}$

$= \frac{8}{21}$

Example 8 $\frac{4}{5} \times \frac{3}{8} = \frac{4 \times 3}{5 \times 8}$

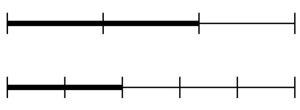
$= \frac{12}{40}$ or $\frac{3}{10}$

Example 9 $\frac{7}{10} \times \frac{5}{8} = \frac{7 \times 5}{10 \times 8}$

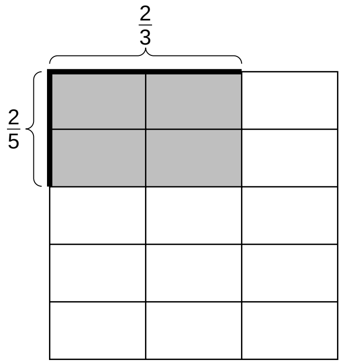
$= \frac{35}{80}$ or $\frac{7}{16}$

Why Does This Procedure Work?

Here again we can draw a model. For example, to model the multiplication of $\frac{2}{3}$ and $\frac{2}{5}$, we start with two line segments of the same length. Divide one segment into 3 equal parts and darken 2 of the parts. Divide the other segment into 5 equal parts and darken 2 of these parts.



The two segments are then used as adjacent sides of a square and the marks are used to divide the square into 15 regions of equal size. The region formed by the two darkened parts of the segments is then shaded.



The shaded portion of the entire square includes 4 of the 15 small regions of equal size. Since each small region represents the fraction $\frac{1}{15}$, the shaded portion represents the fraction $\frac{4}{15}$, which is the product of $\frac{2}{3}$ and $\frac{2}{5}$.

Have students write their own problems involving the multiplication of two fractions. Then have them solve their problems, explaining each step in the process. Ask students to present their completed problems to the rest of the class.