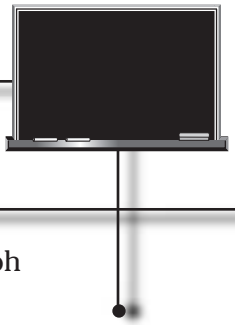
Key Concepts



Graphing Quadratic Functions

Objective Introduce students to quadratic functions and how to graph them.

Note to the Teacher *Much of the mathematics that your students have learned so far has involved linear functions. Linear functions are functions that involve an x-term and a constant term, written in the form*

$$y = ax + b.$$

As your students should know by now, the graph of a linear function is a line.

In this lesson your students will be introduced to quadratic functions, which are one type of **nonlinear functions** (functions whose graphs are not lines).

Quadratic Functions

A quadratic function is a function that not only involves an x-term and a constant term like a linear function, but it also has an x^2-term. Write the following definition on the chalkboard.

Quadratic Function	A **quadratic function** is a function that can be described by an equation of the form $y = ax^2 + bx + c$, where $a \neq 0$.

Write the following examples on the chalkboard. Ask your students which are quadratic and which are not. (Those in the first row are quadratic and those in the second row are not quadratic.)

$y = x^2$ $\quad y = 3x^2 + 4$ $\quad y = 5x^2 - 2x + 10$

$y = 3x - 1$ $\quad y = 2x^3 - x$ $\quad y = x^4 - 3x^3 + 4x - 1$

Graphing Quadratic Functions: Parabolas

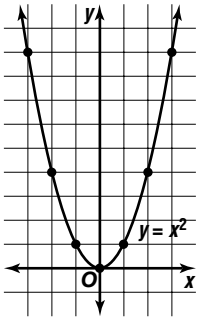
The graph of a quadratic function is called a **parabola**.

Note to the Teacher *Graph a couple of simple parabolas on the chalkboard by plotting points found using a function table. Have your students also graph a few parabolas "by hand." Then graph two or three more complicated quadratic functions using a graphing calculator. Have your students graph at least one of these quadratic functions on a graphing calculator.*

Tell your students that the most basic parabola is the graph of the function $y = x^2$. On the chalkboard, build a function table for $y = x^2$ like the one shown below.

x	y	(x, y)
-3	$(-3)^2 = 9$	$(-3, 9)$
-2	$(-2)^2 = 4$	$(-2, 4)$
-1	$(-1)^2 = 1$	$(-1, 1)$
0	$0^2 = 0$	(0, 0)
1	$1^2 = 1$	(1, 1)
2	$2^2 = 4$	(2, 4)
3	$3^2 = 9$	(3, 9)

On the chalkboard, graph the ordered pairs on a coordinate plane. Then draw a smooth curve that goes through each of the points. This curve is the graph of $y = x^2$.



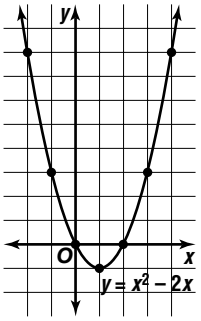
Now present the following examples. Point out that the functions are more complicated than the function $y = x^2$, but their graphs will look similar to the parabola above.

Example 1 **Graph $y = x^2 - 2x$.**

Solution Construct a function table.

x	y	(x, y)
-2	$(-2)^2 - 2(-2) = 8$	$(-2, 8)$
-1	$(-1)^2 - 2(-1) = 3$	$(-1, 3)$
0	$0^2 - 2(0) = 0$	$(0, 0)$
1	$1^2 - 2(1) = -1$	$(1, -1)$
2	$2^2 - 2(2) = 0$	$(2, 0)$
3	$3^2 - 2(3) = 3$	$(3, 3)$
4	$4^2 - 2(4) = 8$	$(4, 8)$

Graph the ordered pairs and connect the points with a smooth curve.



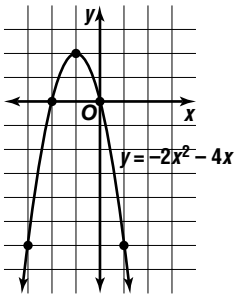
In the following example, the parabola opens downward rather than upward as in the previous two graphs.

Example 2 **Graph $y = -2x^2 - 4x$.**

Solution Construct a function table.

x	y	(x, y)
-3	$-2(-3)^2 - 4(-3) = -6$	$(-3, -6)$
-2	$-2(-2)^2 - 4(-2) = 0$	$(-2, 0)$
-1	$-2(-1)^2 - 4(-1) = 2$	$(-1, 2)$
0	$-2(0)^2 - 4(0) = 0$	$(0, 0)$
1	$-2(1)^2 - 4(1) = -6$	$(1, -6)$

Graph the ordered pairs and connect the points with a smooth curve.



Conclude the lesson by giving your students several examples of quadratic functions to graph using grid paper. Have them construct a function table, compute ordered pairs, graph them on a coordinate plane, and draw the parabola. This can be done individually or in groups. Here are some good examples, and there are many other good examples in the Student Edition.

$$y = 2x^2 - 1 \qquad y = -3x^2 - 4 \qquad y = 4x^2 + x$$

Next, give your students other quadratic functions that are more complicated. For example, you might provide them with quadratic functions whose coefficients are fractions, like $y = \frac{1}{2}x^2 - \frac{3}{4}x + 1$, that they graph using a graphing calculator. Have a discussion about the shapes of all these graphs. Ask your students if they see any common features in the shapes of these graphs. Students may note, for example, that the graphs are all open at one end, have a highest or lowest point (the *vertex*), and that they are *symmetric*. Point out that students can observe this symmetry by folding the paper down the middle of the parabola through its vertex and noting that the two "sides" of the parabola match up. Inform your students that these properties of parabolas will be studied in greater detail when they take an Algebra 1 course.