Key Concepts

Solving Equations

Objective Teach students to solve equations using the multiplication and division properties of equality.

Note to the Teacher *Make sure your class understands very clearly how the multiplication and division properties of equality are used to solve one- and two-step equations. Another skill developed in this lesson is the ability to write an equation for a situation described in a word problem. It is important that students have ample opportunities to practice this equation-writing skill.*

Begin the lesson with the following word problem.

Example 1 **Aki is three times as old as Hector. If Aki is 24 years old, how old is Hector? Write an equation to model this situation.**

Solution The problem states that two quantities are equal to each other. In words,

three times Hector's age is equal to Aki's age.

We know that Aki is 24 years old, but we do not know Hector's age. When we do not know a quantity in a problem, we choose a variable to represent that unknown value and then work with the known values in the problem to determine the unknown quantity.

In this case, let's use h to represent Hector's age. The quantity "three times Hector's age" can then be represented by $3h$. Since three times Hector's age is equal to Aki's age, we can write the equation

$$3h = 24$$

to model this situation.

Solving Equations

A number is a **solution** to an equation if a true numerical statement results when the number is substituted for the variable in the equation.

How can we determine the value of h that is the solution of the equation $3h = 24$? To find this value of h, we will need to use an important property of equations that your students have already studied.

Key Idea	If an equation is multiplied or divided by a nonzero number, the result is an **equivalent equation**. Any solution of the new equation will be a solution of the original equation.

The Key Idea above is a statement of the **multiplication and division properties of equality**. In the case of the equation $3h = 24$, we would like to isolate the variable h on the left side so that the equation reads

$$h = ?.$$

In this equation, the question mark is a numerical quantity we have to determine that does not involve the variable h. Once we have the equation in this form, we consider the equation solved, since we will know that h is equal to the number on the right side. For the equation $3h = 24$, we need to divide each side by 3 to isolate the variable.

$$\frac{3h}{3} = \frac{24}{3}$$

The division property of equality tells us that any solution of this new equation is a solution of the original equation. Since $\frac{3h}{3} = h$ and $\frac{24}{3} = 8$, this new equation simplifies to

$$h = 8.$$

Since h was chosen to represent Hector's age, we now know that Hector is 8 years old.

Now have students solve another equation using the division property of equality.

Example 2 **Solve $7x = 112$.**

Solution Divide each side by 7.

$$\frac{7x}{7} = \frac{112}{7}$$

Since $\frac{7x}{7} = x$ and $\frac{112}{7} = 16$, the equation becomes $x = 16$.

So the solution of the equation is 16.

The following example involves solving an equation using the multiplication property of equality.

Example 3 **Solve $\frac{y}{12} = 4$.**

Solution Since the variable y is divided by 12 in the equation, multiply each side of the equation by 12.

$$12 \cdot \frac{y}{12} = 12 \cdot 4$$

Since $12 \cdot \frac{y}{12} = y$ and $12 \cdot 4 = 48$, the equation becomes $y = 48$.

The solution of the equation is 48.

Why Does This Work?

For an equation of the form

$$\text{coefficient} \cdot x = \text{number},$$

the division property of equality states that we can divide each side of the equation by the coefficient to obtain the equivalent equation

$$x = \frac{\text{number}}{\text{coefficient}},$$

which is the solution of the equation.

For an equation of the form

$$\frac{x}{\text{divisor}} = \text{number},$$

the multiplication property of equality states that we can multiply each side of the equation by the divisor to obtain the equivalent equation

$$x = \text{divisor} \cdot \text{number},$$

which is the solution of the equation.

Note to the Teacher *This is a good point to stop and let the students work through some examples for themselves.*

Example 4 **Solve $16y = 48$.**

Solution $16y = 48$

$\frac{16y}{16} = \frac{48}{16}$ *Divide each side by 16.*

$y = 3$

Example 5 **Solve $1.5t = 6$.**

Solution $1.5t = 6$

$\frac{1.5t}{1.5} = \frac{6}{1.5}$ *Divide each side by 1.5.*

$t = 4$

Example 6 **Solve $\frac{s}{3} = 17$.**

Solution $\frac{s}{3} = 17$

$3 \cdot \frac{s}{3} = 3 \cdot 17$ *Multiply each side by 3.*

$s = 51$

Writing Equations for Word Problems

Learning how to translate from a word problem to an equation is a very important skill. The following examples will provide your students with practice in this skill.

Example 7 **Ms. Jackson's class makes up $\frac{1}{25}$ of the student body at Hillview Middle School. If there are 27 students in her class, how many students go to Hillview Middle School?**

Solution Let s represent the number of students at Hillview Middle School. Then $\frac{1}{25}$ of this number can be represented as $\frac{1}{25} \cdot s$ or $\frac{s}{25}$. Since this expression represents the number of students in Ms. Jackson's class, we can write the equation $\frac{s}{25} = 27$ to model this problem.

We can solve this equation by multiplying each side by 25.

$25 \cdot \frac{s}{25} = 25 \cdot 27$

$s = 675$

There are 675 students going to Hillview Middle School.

Example 8 **At Midtown Grocery, an average of 56 pounds of apples are sold each day. This is four times the average number of pounds of apples sold at Uptown Grocery each day. What is the average number of pounds of apples sold at Uptown Grocery each day?**

Solution If U represents the number of pounds of apples sold each day at Uptown Grocery, then the equation $4U = 56$ models this situation.

This equation can be solved by dividing each side by 4.

$$\frac{4U}{4} = \frac{56}{4}$$

$$U = 14$$

Uptown Grocery sells an average of 14 pounds of apples each day.

Example 9 **At City College, there are two thirds as many students majoring in mathematics as there are majoring in history. If there are 100 students majoring in mathematics, how many students are majoring in history?**

Solution Let h represent the number of history majors. Then the equation $\frac{2}{3}h = 100$ models this situation.

We multiply each side by $\frac{3}{2}$ to solve the equation.

$$\frac{3}{2} \cdot \frac{2}{3}h = \frac{3}{2} \cdot 100 \quad \frac{3}{2} \cdot \frac{2}{3} = 1$$

$$h = 150$$

There are 150 students majoring in history at City College.