Key Concepts

Mean, Median, and Mode

Objective Introduce students to the concepts of mean, median, and mode of a collection of data.

Note to the Teacher *The concepts introduced in this lesson are very important measures of data. They are used to interpret and draw conclusions about the data collected in a variety of situations.*

Summarizing Data

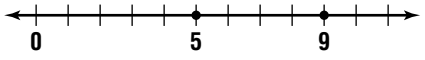
Begin the lesson with a general discussion of data, and point out that it is usually helpful to identify a number that *summarizes* a large collection of data. Point out that we do not want to have to examine a list of the weights of everyone in the United States in order to draw conclusions about diet and exercise in this country. It would be easier to have a single number that is representative of the collection of weights of all persons in the United States.

The Mean

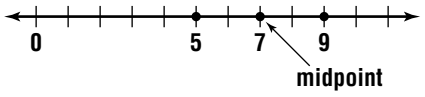
Introduce the **mean** (or *average*) of two numbers by representing it *graphically* on the number line. Stress that the mean of two numbers is represented by the point that lies exactly midway between the points representing the two numbers on the number line.

Example 1 **Find the mean of 5 and 9 using a number line.**

Solution First draw a number line and place dots at 5 and 9.



Then, identify the point on the number line that is halfway between the two dots and draw a third dot at this location.



The third dot is located at 7, which is two units to the right of 5 and two units to the left of 9 on the number line. Therefore, the mean of 5 and 9 is 7.

Now state the *arithmetic* definition of the mean. Tell the class that to find the mean of any two numbers, they should add the numbers together and divide by 2. Stress that we divide by 2 because there are two numbers being added.

Example 2 **Find the mean of 5 and 9 arithmetically.**

Solution First add 5 and 9 together.

$5 + 9 = 14$

Then divide this sum by 2 (there are 2 numbers) to get the mean.

$14 \div 2 = 7$

The mean of 5 and 9 is 7.

Note to the Teacher *This is an excellent time for the class to find the mean by these methods for themselves. Give them a few pairs of numbers, and ask them to determine the mean both graphically and arithmetically, and to compare the results.*

Key Idea	The mean of a collection of numbers is found by adding the numbers together, and then dividing by however many numbers there are in the collection.

Example 3 **Find the mean of the collection {1, 3, 5, 7}.**

Solution First add the numbers together.

$1 + 3 + 5 + 7 = 16$

Since there are four numbers in the collection, divide this sum by 4.

$16 \div 4 = 4$

The mean of the collection is 4.

Example 4 **What is the mean of the collection of numbers {2, 5, 7, 11, 15}?**

Solution $2 + 5 + 7 + 11 + 15 = 40$ *Add the numbers.*

$40 \div 5 = 8$ *There are 5 numbers in the collection.*

The mean is 8.

The Median

The **median** is another measure that can be used to summarize the numbers in a collection. Intuitively, it is the middle number of the collection of numbers when they are written in order, either from least to greatest or from greatest to least.

Example 5 **What is the median of the collection of numbers {5, 11, 7, 15, 2}?**

Solution First arrange the numbers in the collection in order. Below, the numbers are shown in order from least to greatest using inequality symbols.

$$2 < 5 < 7 < 11 < 15$$

From this list, the middle number can be identified. The number 7 is the third number from each end of the list. So, 7 is the median of this collection of numbers.

In the previous example, the middle number was easily identified because there was an odd number of members in the collection. When there is an even number of members in a collection, the collection will have two middle numbers. In this case, the median is defined to be the mean of the two middle numbers.

Example 6 **Find the median of the collection {1, 3, 4, 6, 8, 9}.**

Solution First, notice that the numbers are already in order from least to greatest. Also notice that there are six numbers in this collection. The two middle numbers are 4 and 6.

The median of the collection is the mean of these two numbers.

$$\frac{4 + 6}{2} = \frac{10}{2} \text{ or } 5$$

The median of the collection is 5.

Note to the Teacher *It is now a good idea to convince students that the median and the mean of a collection of numbers are often different. The following two examples can be used to reinforce this fact.*

Example 7 **Find the mean and the median of each collection.**

a. {1, 2, 2, 3, 3, 3, 4, 4, 5} **b. {1, 1, 1, 1, 1, 1, 1, 1, 100}**

Solutions **a.** The numbers in the collection are in order and there are nine numbers. So the median is the middle number, which is 3.

Now compute the mean.

$$\frac{1 + 2 + 2 + 3 + 3 + 3 + 4 + 4 + 5}{9} = \frac{27}{9} \text{ or } 3$$

The mean is also 3. For this collection of numbers, the median and the mean are the same.

b. The numbers in this collection are also in order and again there are nine numbers. So the median is the middle number, which is 1.

The mean is

$$\frac{1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 100}{9} = \frac{108}{9} \text{ or } 12.$$

The mean is 12. So for this collection, the median and the mean are two very different numbers.

Note to the Teacher *Here you can point out that one difference between the two collections of numbers in Example 7 is that the second collection has the* ***outlier*** *100, which is a number much greater than the other numbers in the collection. Stress that an outlier such as this has a great effect on the mean but little effect on the median, since the median is obtained by ordering the members of the collection.*

The Mode

The **mode** of a collection of numbers is the most frequently occurring number (or numbers) in the collection.

Example 8 **What is the mode of the collection {1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 4, 4, 4, 5}?**

Solution The mode is 2 since it occurs five times while 1 and 4 each occur three times, 3 occurs twice, and 5 occurs once.

Example 9 **Identify the mode of the collection {1, 1, 2, 3, 4, 5, 6, 6}.**

Solution The collection has two modes, 1 and 6, since both occur twice while the other numbers only occur once.

Point out that many collections of numbers do not have a mode. There is no mode for a collection in which all of the members occur the same number of times.

Example 10 **Explain why the collection {1, 2, 3, 4, 5, 6, 7, 8, 9} has no mode.**

Solution Since each number in the collection occurs the same number of times (once), there is no mode.

Example 11 **Does the collection {2, 2, 4, 4, 6, 6, 8, 8} have a mode? Explain.**

Solution No. The numbers 2, 4, 6, and 8 each occur the same number of times (twice), so the collection has no mode.

The Range

There is another value, called the *range*, that is useful in describing a collection of numerical data. The **range** of a collection of data is the difference between the greatest and least values in the collection. The range describes how "spread out" the data are.

Example 12 **What is the range of the collection {5, 4, 7, 1, 2, 9, 15}?**

Solution The greatest value in the collection is 15 and the least value is 1. Therefore, the range is $15 - 1 = 14$.

Exercises

Give your class a collection of 10 to 15 test scores, ranging from 0 to 100. Have them compute the mean, median, mode, and range of the data. Ask, "Which of these four measures is affected the most when three more scores are added to the data if all three scores are 100? if all three scores are 50? if all three scores are 0?"

Sample test scores:
{55, 70, 85, 73, 68, 93, 89, 97, 87, 83, 58, 68, 42, 99}