Key Concepts

Elimination Using Multiplication

Objective Teach students how to use the Multiplication Property of Equality to modify systems of equations and then to solve them by using elimination by addition or subtraction.

Note to the Teacher *In this lesson, students learn a method that can be used to solve all systems of linear equations. The methods introduced so far only work in special cases, such as when the coefficient of one variable is ±1, or when the coefficient of a variable in one equation is equal to or opposite of the coefficient in the other.*

Elimination Using Multiplication

Review the three methods for solving systems of equations that students have learned so far. Point out that each method has advantages and disadvantages.

(A) Solving by Graphing This method can be used for all systems, but produces only approximate solutions.

(B) Solving by Substitution This method can be used when the coefficient of one of the variables in one of the equations is ±1.

(C) Elimination by Addition or Subtraction This method works when the coefficients of one variable are equal or opposite.

Now let's develop an additional method, which will allow us to solve all systems of linear equations. Look at the following system of equations.

$$3x + 5y = 11$$
$$6x + 4y = 16$$

Point out that this system of equations cannot easily be solved by substitution, since none of the coefficients is 1 or −1. Also, it cannot be solved using elimination by addition or subtraction. In this case, use the Multiplication Property of Equality to multiply the first equation by 2.

Note to the Teacher *Explain to the class that the reason for multiplying by 2 is that this makes the coefficient of x in the first equation equal to 6, which is the coefficient of x in the second equation. Then the x values can be eliminated by subtracting the second equation from the first. Another way to solve this system would be to multiply the first equation by −2 and then add the equations.*

After multiplying by 2, the first equation becomes

$$6x + 10y = 22.$$

Now subtract the second equation from this modified first equation.

$$\begin{aligned} 6x + 10y &= 22 \\ \underline{(-)6x + 4y} &\underline{= 16} && \textit{Subtract the equations.} \\ 0 + 6y &= 6 \\ 6y &= 6 \\ y &= 1 && \textit{Divide each side by 6.} \end{aligned}$$

Now substitute 1 for y in, say, the first equation.

$$\begin{aligned} 3x + 5(1) &= 11 && \textit{Substitute 1 for y.} \\ 3x + 5 &= 11 \\ 3x &= 6 && \textit{Subtract 5 from each side.} \\ x &= 2 && \textit{Divide each side by 3.} \end{aligned}$$

The solution is (2, 1).

Key Idea	Any equation in a system of equations can be multiplied by a nonzero number to obtain an equivalent equation.

Have students use this fact to complete the following exercises.

Exercises

Use elimination to solve each system of equations.

1. $3x + 6y = 15$
$2x + 7y = 13$
(3, 1)

2. $4x - 3y = 14$
$7x + 2y = 39$
(5, 2)

3. $1.5x + 2.5y = 4$
$0.5x - 3.5y = -3$
(1, 1)

This idea of solving a system of equations by using multiplication can be shown using a four-step procedure. Consider the following system of equations.

$$3x + 2y = 12$$
$$2x + 4y = 16$$

Step 1 Multiply one equation by a number so that a variable in the new equation has a coeffient that is the same as or opposite of the coeffient of the variable in the other equation.

Multiply the first equation by 2. Then the coefficient of y is 4, which is the same as the coefficient of y in the second equation. The new first equation is $6x + 4y = 24$.

Step 2 Subtract one equation from the other or add the equations to obtain an equation in which one of the variables does not appear. Solve for the remaining variable.

$$\begin{aligned} 6x + 4y &= 24 \\ (-)2x + 4y &= 16 \\ \hline 4x + 0y &= 8 \\ 4x &= 8 \\ x &= 2 \end{aligned}$$

Subtract the second equation from the new first equation.

Divide each side by 4.

Step 3 Substitute the value obtained in Step 2 into one of the original equations.

$3x + 2y = 12$ *The original equation*

$3(2) + 2y = 12$ *Substitute 2 for x.*

$6 + 2y = 12$

Step 4 Solve the resulting equation for the other variable.

$6 + 2y = 12$

$2y = 6$ *Subtract 6 from each side.*

$y = 3$ *Divide each side by 2.*

So, the solution is (2, 3).

Note to the Teacher *It is not necessary for the class to memorize this procedure, but presenting it will clarify what needs to be done when solving systems of equations. Make sure to point out that there is more than one way to solve the system. For instance, one might choose the other variable to eliminate, or choose to multiply the first equation by −2 and then add the equations. Have the class work with many examples, including those given in verbal form. Also, it is a good idea to have students graph some of the solutions to further reinforce that concept.*

Multiplying Both Equations to Simplify the System

For some systems of equations, one equation must be multiplied by a fraction in order to make elimination by addition or subtraction possible. Since multiplication of integers is easier, both equations are multiplied by nonzero numbers so that the coefficients of a variable in the equations become equal (or opposite).

Example 1 **Solve the system of equations.**

$$\mathbf{2x + 9y = 7}$$

$$\mathbf{3x + 7y = 4}$$

Solution One approach is to multiply the first equation by $\frac{2}{3}$, and then subtract the resulting equation from the second one. This method works, but involves fractional arithmetic. Another approach is to multiply the first equation by 3 and the second by 2, to get an equivalent system of equations.

$$3(2x + 9y = 7) \rightarrow 6x + 27y = 21$$

$$2(3x + 7y = 4) \rightarrow 6x + 14y = 8$$

$$\begin{aligned} 6x + 27y &= 21 \\ \underline{(-)6x + 14y} &\underline{= 8} \quad \textit{Subtract the equations.} \\ 0 + 13y &= 13 \\ 13y &= 13 \\ y &= 1 \quad \textit{Divide each side by 13.} \end{aligned}$$

Now substitute 1 for y in the second equation.

$$\begin{aligned} 6x + 14y &= 8 \\ 6x + 14(1) &= 8 \quad \textit{Replace y with 1.} \\ 6x &= -6 \\ x &= -1 \end{aligned}$$

The solution is (−1, 1).

End of Lesson