# Key Concepts

## Elimination Using Addition and Subtraction

**Objective** Introduce the elimination method of solving systems of simultaneous equations.

**Note to the Teacher** *The main idea in this lesson is that systems of equations may be simplified and solved using the Addition and Multiplication Properties of Equality to add and to subtract equations. It is important to emphasize this idea, and that the goal is to obtain an equivalent equation in which only one variable is on the left-hand side and only a number is on the right-hand side.*

### The Elimination Method for Solving Systems of Equations

Introduce the idea with two examples.

**Example 1** **Solve the system of linear equations.**

$$\mathbf{x - y = 3}$$
$$\mathbf{x + y = 7}$$

**Solution** For $(x, y)$ to be a solution to the system means that both equations hold true for the values $(x, y)$. That is, both equations are satisfied. Add the equations to get another valid equation, using the Addition Property of Equality.

$$\begin{array}{rl} x - y = 3 & \\ \underline{(+)\ x + y = 7} & \textit{Add the equations.} \\ 2x + 0 = 10 & \\ 2x = 10 & \end{array}$$

This is a linear equation in one variable, namely $x$. Solve to get $x = 5$. Then substitute $x = 5$ into one of the original equations and solve for $y$.

$$\begin{array}{rl} x - y = 3 & \\ 5 - y = 3 & \textit{Replace x with 5.} \\ -y = -2 & \\ y = 2 & \end{array}$$

The only solution to the system of equations is (5, 2).

**Example 2** **Solve the system of equations.**

$$\mathbf{2x + 5y = 7}$$
$$\mathbf{2x - 2y = 0}$$

**Solution** Use the Subtraction Property of Equality to subtract one of the equations from the other to get another valid equation.

$$\begin{array}{r} 2x + 5y = 7 \\ (-)\ 2x - 2y = 0 \\ \hline 0 + 7y = 7 \end{array} \quad \textit{Subtract the equations.}$$

Solve $7y = 7$ to get $y = 1$. Then substitute the value of $y$ into one of the equations and solve for $x$.

$$\begin{aligned} 2x - 2(1) &= 0 && \textit{Substitute 1 for y.} \\ 2x - 2 &= 0 \\ 2x &= 2 \\ x &= 1 && \textit{Divide each side by 2.} \end{aligned}$$

The solution is (1, 1).

When the resulting equation involves only one variable, we say that we have eliminated the other variable. In Example 1, $y$ was eliminated, and in Example 2, $x$ was eliminated.

**Note to the Teacher** *It is important for students to practice using this method by working the exercises below. Then, introduce Examples 3 and 4, which are word problems.*

## Exercises

**Solve each system of equations.**

**1.** $x - 2y = 0$
$2x + 2y = 12$
**(4, 2)**

**2.** $x + 3y = 7$
$x + y = 5$
**(4, 1)**

**3.** $x + y = 8$
$-x + y = 6$
**(1, 7)**

**Example 3** **The sum of two numbers is 48, and their difference is 16. What are the numbers?**

**Solution** Let $x$ and $y$ represent the two numbers and write the following system of equations.

$$x + y = 48$$
$$x - y = 16$$

Adding the equations gives $2x = 64$ or $x = 32$. To find the value of $y$, substitute 32 for $x$ in either equation to get $y = 16$. The two numbers are 32 and 16.

**Example 4** **Neal scored 1150 on his SAT. His math score was 250 points greater than his verbal score. What were his scores?**

**Solution** Let $m$ represent Neal's math score, and $v$ represent his verbal score. Then write the following system of equations.

$$m + v = 1150$$
$$m - v = 250$$

Add or subtract these equations to find the solution $m = 700$, $v = 450$. So, Neal's math score was 700 and his verbal score was 450.