Key Concepts

Substitution

Objective Teach students to solve systems of equations by using the substitution method, and reinforce the geometric concept of what solving the system means.

Note to the Teacher *Make sure that students graph the systems after they have found the solutions so that the geometric idea that the solution corresponds to the intersection of lines is reinforced.*

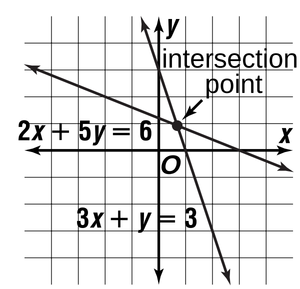
Solving Systems of Equations Algebraically

Remind the class that in Lesson 8-1, they solved systems of equations graphically by graphing the lines corresponding to the two equations and finding the coordinates of the point of intersection by "inspection." Explain that often this method is not good enough because it is sometimes difficult to get an exact answer. Instead, there is an algebraic method for solving systems of equations. Begin with an example.

Example 1 **Find the solution to the system of linear equations.**

$$3x + y = 3$$
$$2x + 5y = 6$$

Solution Remind the class that they can graph the equations to get a rough idea of what the solution set is. The graph of this system of equations is shown at the right.



Use the graph to estimate the coordinates of the intersection point to be about (0.8, 0.9). So, the solution is approximately $x = 0.8$, $y = 0.9$. To find the exact solution, solve the system of equations algebraically.

Key Idea	Use the Addition and Multiplication Properties of Equality to solve systems of equations in two variables, just as equations with one variable are solved.

$$3x + y = 3$$

$$y = 3 - 3x \qquad \textit{Subtract 3x from each side.}$$

This equation is solved for y in terms of x. Since the value of y must be the same in both equations, substitute $3 - 3x$ for y in the second equation.

$$2x + 5y = 6$$

$$2x + 5(3 - 3x) = 6 \qquad \textit{Substitute } 3 - 3x \textit{ for } y.$$

$$2x + 15 - 15x = 6 \qquad \textit{Distributive Property}$$

$$-13x + 15 = 6$$

Point out that this is an equation involving only one variable, namely x. Now solve the equation for x.

$$-13x + 15 = 6$$

$$-13x = -9 \qquad \textit{Subtract 15 from each side.}$$

$$x = \frac{-9}{-13} \qquad \textit{Divide each side by } -13.$$

$$x = \frac{9}{13}$$

The exact value for the x-coordinate of the solution is $\frac{9}{13}$. To find the exact value for the y-coordinate, substitute $\frac{9}{13}$ for x in either of the two equations.

$$3x + y = 3$$

$$3\left(\frac{9}{13}\right) + y = 3 \qquad \textit{Substitute } \frac{9}{13} \textit{ for } x.$$

$$\frac{27}{13} + y = 3$$

Now use the Subtraction Property of Equality to solve for y.

$$y = 3 - \frac{27}{13}$$

$$y = \frac{39}{13} - \frac{27}{13} \text{ or } \frac{12}{13}$$

So, the exact solution is given by $x = \frac{9}{13}$ and $y = \frac{12}{13}$, or $\left(\frac{9}{13}, \frac{12}{13}\right)$.

Note to the Teacher *Point out that this method always works when the system has exactly one solution. Simply solve an equation for one variable in terms of the other by using the Addition and Multiplication Properties of Equality. Now have the class work several exercises of this type.*

Exercises

Solve each system of equations.

1. $x + 4y = 1$
   $2x - 3y = -9$ **(−3, 1)**

2. $2x + y = 6$
   $3x + 2y = 11$ **(1, 4)**

Systems of Equations That Have No Solution or Infinitely Many Solutions

Ask students, "What happens when a system of equations has no solution or infinitely many solutions?" This question is best addressed by examples.

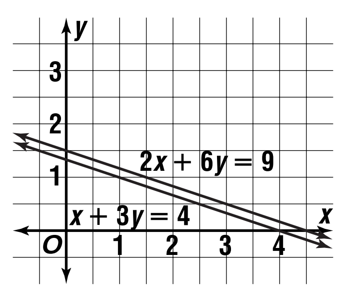
Example 2 **Solve the system of equations.**

$$\mathbf{2x + 6y = 9}$$
$$\mathbf{x + 3y = 4}$$

Solution Solve the second equation for x to get $x = 4 - 3y$. Then substitute $4 - 3y$ for x in the first equation.

$$\begin{aligned} 2x + 6y &= 9 \\ 2(4 - 3y) + 6y &= 9 && \textit{Substitute } 4 - 3y \textit{ for } x. \\ 8 - 6y + 6y &= 9 && \textit{Distributive Property} \\ 8 &\neq 9 \end{aligned}$$

Since this equation is never true, there are no solutions to the system. The graph of this system of equations is shown at the right. Notice that the lines are parallel.



Key Idea	When solving a system of equations, if the final equation does not contain a variable and is false, then there are no solutions.

Example 3 **Solve the system of equations.**

$$2x + 8y = 14$$

$$x + 4y = 7$$

Solution Solve the second equation for x to get $x = 7 - 4y$. Then substitute $7 - 4y$ for x in the first equation.

$$\begin{aligned} 2x + 8y &= 14 \\ 2(7 - 4y) + 8y &= 14 && \textit{Substitute } 7 - 4y \textit{ for } x. \\ 14 - 8y + 8y &= 14 \\ 14 &= 14 \end{aligned}$$

This equation does not involve either variable, but the equation is always true. In this case, there are infinitely many solutions. The two lines are identical.

Key Idea	When solving a system of equations, if the final equation does not contain a variable and is true, then there are infinitely many solutions.

Do the following application problem in class to reinforce how to set up problems like this and how to find the solution.

Exercise

3. At the fair, it costs $1 for each child ride c and $2 for each adult ride a. The Anderson family rode a total of 11 rides and spent $14. How many of each type did they ride? **8 child, 3 adult**