Key Concepts

Solving Compound Inequalities

Objective Teach students how to solve compound inequalities.

Note to the Teacher *This lesson builds on the previous one. The difference is that in this lesson, we are dealing with* ***compound inequalities****, expressions in which more than one inequality applies to the same variable. Reinforce the idea that when inequalities are multiplied by a negative number, the inequality symbol must be reversed.*

Compound Inequalities

First, explain the notation and the terminology. A **compound inequality** is an expression like $3 \leq 3x + 4 < 9$ or $1 \geq x - 2 \geq -7$. The two inequality symbols indicate that the variable is to satisfy each part. So, for example, the inequality $2 < 2x - 5 < 4$ is read "two is less than $2x - 5$ and $2x - 5$ is less than four" or "$2x - 5$ is between 2 and 4."

Solving Compound Inequalities

First, remind students that solving an inequality (compound or otherwise) means describing its solution set, since there is more than one solution. Then explain that compound inequalities can be solved using the same methods that are used to solve single inequalities, the Addition and Multiplication Properties of Inequalities.

Example 1 **Solve $3 \leq 2x + 1 < 8$.**

Solution First, subtract 1 from each part so that no constants appear in the expression that contains the variable.

$$3 \leq 2x + 1 < 8$$

$$2 \leq 2x < 7 \quad \textit{Subtract 1 from each part.}$$

$$1 \leq x < \frac{7}{2} \quad \textit{Divide each part by 2.}$$

The solution set is $\left\{x \mid 1 \leq x < \frac{7}{2}\right\}$.

Note to the Teacher *Students may ask why this is considered solving the inequality. After all, we still have an inequality. Why is this one any better than the original inequality? The answer is that the inequalities now apply directly to x, not to an algebraic expression involving x. This means that it is much easier to visualize the solution set, as the points to the right of and including 1 and to the left of* $\frac{7}{2}$*. Emphasize that solving compound inequalities means isolating x in the inequalities.*

Example 2 **Solve $-1 \le 3 - x \le 5$.**

Solution First, subtract 3 from each part to remove any constants from the expression that contains x.

$$-1 \le 3 - x \le 5$$

$$-4 < \ -x \ \le 2 \quad \textit{Subtract 3 from each side.}$$

Next, multiply each part of the inequality by -1 in order to isolate x in the middle. Since -1 is negative, reverse the inequality symbols.

$$4 > x \ge -2$$

The solution set is $\{x \mid 4 > x \ge -2\}$.

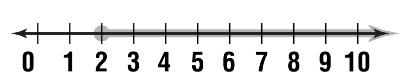
Graphing Compound Inequalities

The solution sets for compound inequalities are typically intervals, rather than half lines. Here is an example to show why.

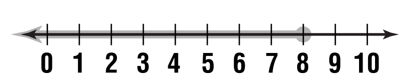
Example 3 **Graph $2 \le x \le 8$.**

Solution Remember that the solution set for the compound inequality is the set of numbers for which both inequalities hold. In the graph, this means that the solution set can be shown as the overlap of the two solution sets for the individual inequalities.

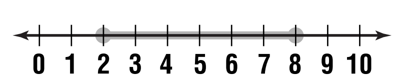
$2 \le x$ or $x \ge 2$



$x \le 8$



$2 \le x \le 8$

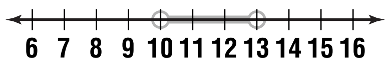


Now have students complete the following exercises.

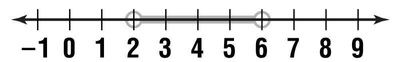
Exercises

Solve each compound inequality. Then graph the solution set.

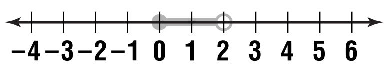
1. $2 < x - 8 < 5$ **$\{x \mid 10 < x < 13\}$**



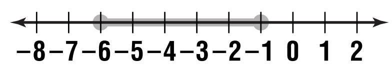
2. $4 < 2x < 12$ **$\{x \mid 2 < x < 6\}$**



3. $4 \leq 3x + 4 < 10$ **$\{x \mid 0 \leq x < 2\}$**



4. $7 \geq -x + 1 \geq 2$ **$\{x \mid -6 \leq x \leq -1\}$**



End of Lesson