Key Concepts

Writing Linear Equations in Point-Slope and Standard Forms

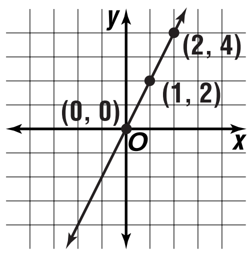
Objective Teach students to write equations for lines given information such as the slope and a point on the line, or two points on the line.

Note to the Teacher *There are a number of ways to write equations representing straight lines. This lesson introduces two of them. The* ***point-slope form*** *can be written if the slope of the line in question and a point on the line are given, or if two points on the line are given. The* ***standard form*** *is also useful because it can be used to represent any line, even a vertical one. Vertical lines cannot be given in point-slope form since their slope is not defined.*

Point-Slope Form

Begin by reminding the class about an important key idea from the preceding lesson.

Key Idea	If we have a straight line, and we choose any two different points on it, we will get the same value for the slope no matter what points we choose.

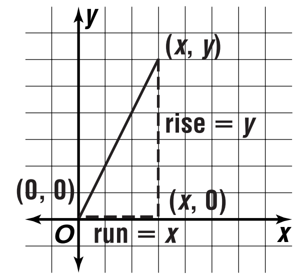


Now draw a line on the chalkboard like the one shown at the right. Display some points on the line, such as (1, 2) and (2, 4), in addition to the origin (0, 0).

Ask students to find the slope of this line, which is 2. Write the general point $P = (x, y)$, where x and y are variables, and ask, "How can we tell from the coordinates x and y whether P lies on the line?"

To answer the question, remind the class what is known. If (0, 0) and (x, y) are used to compute the slope, the answer still must be 2, since the key idea states that we will get the same answer no matter which two points are chosen.

Remind the class that they can find the slope by dividing the rise by the run. Since the rise is y and the run is x,



$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y}{x}.$$

We know that the slope of this line is 2, so $\frac{y}{x} = 2$.

So, the rise is twice the run, and $y = 2x$ for any point on this line. Point out that this equation holds for the coordinates of any point on the line.

Note to the Teacher *There are many equations that describe the same line. We could have computed the slope using (x, y) and any other point on the line, such as (1, 2), as shown below.*

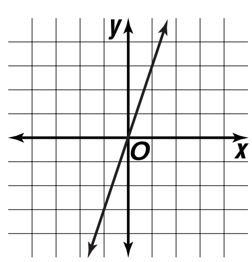
$$\frac{(y - 2)}{(x - x)} = 2$$

$$(y - 2) = 2(x - 1) \quad \textit{Multiply each side by } x - 1.$$

Ask students to write their own equations for the line, using various points, such as (2, 4) or (3, 6).

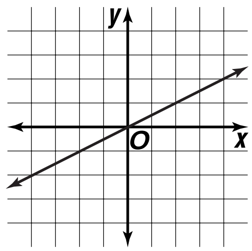
Here are some more examples. Ask students to determine the slope and to write an equation for each line graphed.

Example 1 **Write an equation for the line.**



Solution The line has slope 3. One equation for it is $y = 3x$.

Example 2 **Write an equation for the line.**



Solution The line has slope $\frac{1}{2}$ and an equation for it is $y = \frac{1}{2}x$.

So, if the slope of a line (let's call it m) and a single point (let's call it (x_0, y_0)) on that line are known, then the following equation for the line can be written.

$$(y - y_0) = m(x - x_0)$$

This equation is said to be in **point-slope form** since it uses only information about one point and the slope.

Now have students work on exercises.

Exercises

Find an equation in point-slope form for each line.

1. the line with slope 4 going through the point (1, 2)
   $(y - 2) = 4(x - 1)$
2. the line with slope -1 going through the point (5, 4)
   $(y - 4) = -1(x - 5)$
3. the line with slope $\frac{1}{2}$ going through the point (0, 10)
   $(y - 10) = \frac{1}{2}x$

Note to the Teacher *Point out that the slope and a single point on the line determine the whole line. Intuitively, the steepness and one point determine the line. This can be illustrated by rotating a ruler through a single point, and tracing out many of the possible slopes.*

Finding an Equation for a Line Passing Through Two Points

Point out the fact that two points determine a line and that there is exactly one line that goes through any two distinct points. This can also be illustrated using a ruler, and showing that if two points are specified, then the ruler can only be placed over them in one way.

Now, draw two points (with coordinates) on the chalkboard, say $A(1, 5)$ and $B(3, 11)$. Ask, "What information do we need to write an equation describing the line going through A and B?" **the slope and a single point on the line**

We already have a point; we can choose either A or B. To find the slope, we use the formula we developed when we discussed slope. When we compute slope using two points (x_0, y_0) and (x_1, y_1), the formula is

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_1 - y_0}{x_1 - x_0}.$$

In our case, this gives

$$\text{slope} = \frac{11 - 5}{3 - 1} = \frac{6}{2} \text{ or } 3.$$

We now know that the slope of the line through A and B is 3. We also know that $A(1, 5)$ is a point on the line. So, when we substitute into the point-slope formula, we use $m = 3$, $x_0 = 1$, and $y_0 = 5$. This gives us the following equation that describes the line.

$$(y - 5) = 3(x - 1)$$

Here are some exercises that students can work in class.

Exercises

4. Write an equation in point-slope form that describes the line through the points (2, 7) and (5, 16).
$(y - 7) = 3(x - 2)$ or $(y - 16) = 3(x - 5)$

5. Write an equation in point-slope form that describes the line through the points (0, −1) and (1, − 4).
$(y + 1) = -3(x)$ or $(y + 4) = -3(x - 1)$

Note to the Teacher *It is important to point out that there are many different equations in point-slope form that describe the same line.*

It is easy to write a formula for an equation describing the line including the two points (x_0, y_0) and (x_1, y_1). First, remind students about the formula that gives the slope of the line including two points.

$$\text{slope} = \frac{y_1 - y_0}{x_1 - x_0}$$

If we substitute this expression for m in the point-slope formula, we get

$$(y - y_0) = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)(x - x_0).$$

The graph of this equation is the line through the two points.

It is important to have the class practice substituting into this formula. You could simply have them redo Exercises 4-5 using this formula.

Standard Form

Note to the Teacher *This is a different form for the equation describing a line. The advantage of this form is that it can be used to describe all lines, even vertical ones. The disadvantage is that it does not depend as explicitly on a point and a slope or on two points, as does the point-slope form.*

Point out to the class that it is possible to manipulate the equation defining a line using the Multiplication and Addition Properties of Equality, as well as the usual rules of arithmetic, such as the Commutative, Associative, and Distributive Properties.

Suppose a line is defined by $(y - 2) = 5(x - 3)$. Notice that the equation is the point-slope form for the line with slope 5 passing through the point (3, 2). We can make some changes using the properties from above.

$$(y - 2) = 5(x - 3)$$

$$y - 2 = 5x - 15 \quad \textit{Distributive Property}$$

$$y = 5x - 13 \quad \textit{Add 2 to each side.}$$

$$-5x + y = -13 \quad \textit{Subtract 5x from each side.}$$

$$5x - y = 13 \quad \textit{Multiply each side by } -1.$$

This last equation gives the same line as the original equation, since it is equivalent to it. We say it is in **standard form** because all the variables are collected on one side and all the "numbers" (constants) are collected on the other side.

Here are some examples of equations that are in standard form. In each case, the variables are all on the left and the constants are all on the right.

$$5x + 7y = 9 \qquad 4x - 8y = 0 \qquad 0.3x + 0.5y = 1.7$$

Here are some examples of equations that are not in standard form.

$y = 3x + 1$ ← There are variables on both sides of the equation.

$x + y - 4 = 1$ ← There are constants on both sides of the equation.

Key Idea	Any equation in point-slope form can be changed into an equivalent equation in standard form. This is done by removing all constants from the left-hand side and by removing all variables from the right.

Have students work out several exercises.

Exercises

Write each equation in standard form.

6. $(y - 2) = 4(x - 3)$ **$4x - y = 10$**

7. $(y + 4) = -3(x - 7)$ **$3x + y = 17$**

8. $(y - 0.5) = 1.4(x + 3)$ **$1.4x - y = -4.7$**

Remind the class that for vertical lines, the slope is undefined. This means that an equation in point-slope form cannot be written for a vertical line, since there is no slope to insert. To show them that we can write an equation in standard form for a vertical line, do the following example.

Example 3 **The vertical line through (4, 0) can be defined by the equation $x = 4$. Write this equation in standard form.**

Solution This equation is already in standard form, since there is only a variable on the left-hand side and only a constant on the right.

Key Idea	Any line (even vertical ones) can be described by an equation in standard form.

End of Lesson