Key Concepts

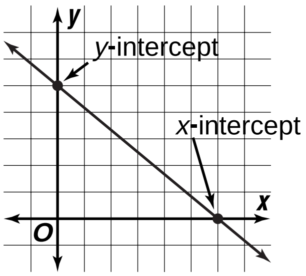
Writing Linear Equations in Slope-Intercept Form

Objective Introduce students to the slope-intercept form of the equation defining a line.

Note to the Teacher *The slope-intercept form is an extremely useful special case of the point-slope form of the equation defining a straight line. It is written as y = mx + b, where m is the slope and b is the **y-intercept**, the point where the line intersects the y-axis.*

Finding the *x*- and *y*-Intercepts of a Line

First, tell the class that *x*- and *y*-intercepts of a line are the points where the line intersects the *x*- and *y*-axes, respectively. Ask, "How can we determine the *x*- and *y*-intercepts of a line that is given in the form of an equation?"



Then show the class how to find the intercepts algebraically by solving linear equations. This can be done whether the equations are in standard form or in point-slope form.

Example 1 **Find the *y*-intercept of the graph of $2x + 3y = 12$.**

Solution This equation is in standard form. The *y*-intercept is the intersection of the line and the *y*-axis, so it is a point that is both on the line and on the *y*-axis. It satisfies the following equations.

$$\begin{aligned} 2x + 3y &= 12 && \textit{Equation of the line} \\ x &= 0 && \textit{Equation of the y-axis} \end{aligned}$$

To find the *y*-intercept, let $x = 0$ in the equation of the line.

$$\begin{aligned} 2x + 3y &= 12 \\ 2(0) + 3y &= 12 && \textit{Let } x = 0. \\ 3y &= 12 \\ y &= 4 && \textit{Divide each side by 3.} \end{aligned}$$

The *y*-intercept of this line is 4, so the line crosses the *y*-axis at (0, 4).

Example 2 **Find the y-intercept of the graph of $2(y - 1) = 5(x + 2)$.**

Solution This equation is in point-slope form. As in the case of standard form, let $x = 0$ and solve for y.

$2(y - 1) = 5(x + 2)$	
$2(y - 1) = 5(0 + 2)$	*Let $x = 0$.*
$2y - 2 = 10$	*Distributive Property*
$2y = 12$	*Add 2 to each side.*
$y = 6$	*Divide each side by 2.*

So, the y-intercept is 6 and the line crosses the y-axis at (0, 6).

To find the x-intercept, let $y = 0$ in the equation, since the x-axis is given by the equation $y = 0$. Have students practice finding the x- and y-intercepts of the graphs of equations. Below are some exercises that can be done in class.

Exercises

Find the x- and y-intercepts of the graph of each equation.

1. $2x + 5y = 20$ **10, 4**

2. $3x - 4y = 36$ **12, −9**

3. $3(y - 2) = 2(x + 3)$ **−6, 4**

4. $7(y + 5) = 5(x - 14)$ **21,−15**

Slope-Intercept Form

Note to the Teacher *The slope-intercept form is a special case of the point-slope form. The given point of the line (x_0, y_0) lies on the y-axis, so $x_0 = 0$. This means that the equation is of the form $y - y_0 = mx$, or $y = mx + y_0$. The equation is therefore given explicitly when both the y-intercept and the slope are known, and is simpler than the more general point-slope form. This is perhaps the most common and most important normal form for the equation of a line.*

First, work the following examples in class or have students work them.

Example 3 **Write the slope-intercept form of the equation for a line that goes through (0, 4) and has a slope of 5.**

Solution

$y - y_0 = m(x - x_0)$	*Point-slope form*
$y - 4 = 5x$	*Replace x_0 with 0, y_0 with 4, and m with 5.*

Now add 4 to each side of this equation in order to express y in terms of x. The result is the following equation.

$$y = 5x + 4$$

Point out that 5 is the slope of the line and that 4 is the y-intercept.

Example 4 **Write the slope-intercept form of the equation for a line that goes through (0, −3) and has a slope of −1.**

Solution

$$\begin{aligned} y - y_0 &= m(x - x_0) && \textit{Point-slope form} \\ y - (-3) &= (-1)x && \textit{Replace } x_0 \textit{ with 0, } y_0 \textit{ with } -3\textit{, and } m \textit{ with } -1. \\ y + 3 &= -x \end{aligned}$$

Now subtract 3 from each side of this equation in order to express y in terms of x. The result is the following equation.

$$y = -x - 3$$

Point out that −1 is the slope of the line and that −3 is the y-intercept.

Note to the Teacher *Now point out that these equations are really much simpler than the general point-slope form, since there are fewer terms. Also point out that in each case, the coefficient of x is the slope and the constant term is the y-intercept. Write the following equation with labels on the chalkboard.*

$$y = \underset{\substack{\nearrow \\ \text{slope}}}{m}x + \underset{\substack{\nwarrow \\ y\text{-intercept}}}{b}$$

Key Idea

An equation for a line is in slope-intercept form if it is of the form

$$y = mx + b,$$

where m is the slope of the line and b is the y-intercept.

Any line that is not vertical has an equation that is in slope-intercept form. A vertical line could not possibly have an equation in slope-intercept form, for the same reason that it cannot have an equation in point-slope form—the slope is undefined.

Converting from Standard and Point-Slope Forms to Slope-Intercept Form

Note to the Teacher *Conversion from standard and point-slope forms to slope-intercept form is achieved by adding and subtracting terms from each side of the equations, and then multiplying or dividing. This is best illustrated using examples.*

Example 5 **Write $6x + 3y = 9$ in slope-intercept form.**

Solution This equation is in standard form. Perform the necessary steps to solve the equation for y.

$$6x + 3y = 9$$

$$6x + 3y - 6x = 9 - 6x \quad \textit{Subtract 6x from each side.}$$

$$3y = -6x + 9$$

$$y = -2x + 3 \quad \textit{Divide each side by 3.}$$

This equation is in slope-intercept form, with a slope of -2 and a y-intercept of 3.

Key Idea	To convert from standard form to slope-intercept form, • move the x term to the right-hand side of the equation, and • divide each side by the coefficient of y.

These steps for converting from standard form to slope-intercept form work whenever the coefficient of y is not zero. If it is zero, the line is vertical and the slope is undefined.

Example 6 **Write $7(y - 3) = 28(x + 2)$ in slope-intercept form.**

Solution To do this, first multiply through all the terms within parentheses using the Distributive Property.

$$7(y - 3) = 28(x + 2)$$

$$7y - 21 = 28x + 56 \quad \textit{Distributive Property}$$

$$7y - 21 + 21 = 28x + 56 + 21 \quad \textit{Add 21 to each side.}$$

$$7y = 28x + 77$$

$$y = 4x + 11 \quad \textit{Divide each side by 7.}$$

This is the slope-intercept form of the equation. The slope is 4 and the y-intercept is 11.

Key Idea

To convert from point-slope form to slope-intercept form,

- use the Distributive Property to multiply through all expressions in parentheses,
- remove the constant from the left-hand side, and
- divide each side by the coefficient of y.

End of Lesson