Key Concepts

Slope

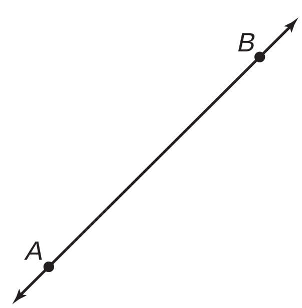
Objective Introduce the concept of the slope of a line and teach students to evaluate it for particular lines.

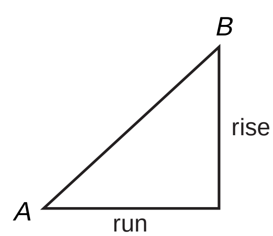
Note to the Teacher *In this lesson, students will be introduced to the notion of the slope of a straight line. It is very important that they be exposed to many examples, and that they do many exercises themselves. In these exercises, they should use the formula for slope so that they become familiar with using it. They should also write and solve their own exercises involving slope. All students should have grid paper for this lesson.*

Slope

First, talk in intuitive terms about what is meant by slope. Give real-life examples of slope such as the slope of the roof of a house, a road going up a hill, or a ladder leaning against a building. Explain that we can assign a number that allows us to measure the *steepness* of a straight line. Also, say that the greater the absolute value of this number, the steeper the line will be.

Next, draw a straight line on the chalkboard with two points on it, A and B. Explain that there are two numbers attached to this pair of points, namely the **rise** and the **run**. The rise is how much higher B is than A in the vertical direction, and the run is how far over from A point B is in the horizontal direction.

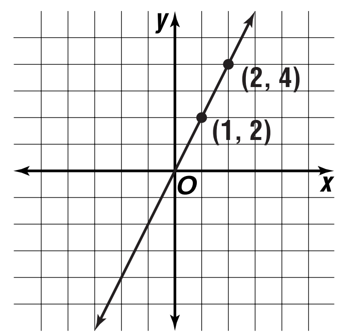


Definition of Slope	**Words** The slope is the value of the quotient $\frac{\text{rise}}{\text{run}}$. **Model** 

Note to the Teacher *So far, we have talked about lines without placing them in the coordinate plane. The next step is to understand slope better by studying lines in the coordinate plane.*

Get students involved in plotting points. Have them plot the sequence of points (1, 1), (2, 2), (3, 3), and (4, 4) and ask them what the pattern is. Ask them to find the value of y in $(-1, y)$. Next, have them plot (1, 2), (2, 4), (3, 6), and (4, 8) on another coordinate plane and ask what the pattern is. Then ask what the two sets of plotted points have in common. **(Both sets of points lie on a straight line.)**

Now draw a line in the second coordinate plane. Remind students that they plotted these points and are now connecting the points with a line.



The two points plotted at the right have coordinates (1, 2) and (2, 4). The rise is the difference between 4 and 2, so it equals 2. The run is the difference between 2 and 1, so it equals 1. The slope is the quotient $\frac{2}{1}$ or 2.

Next, choose two different points on the same line, say (0, 0) and (3, 6), and compute the slope again.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{6-0}{3-0} = \frac{6}{3} = 2$$

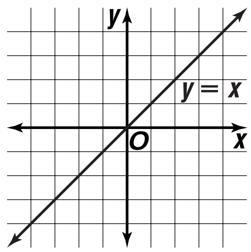
Key Idea	The slope is the same for any pair of points on the same straight line. Therefore, it is not necessary to refer to a particular pair of points when speaking of the slope of a line.

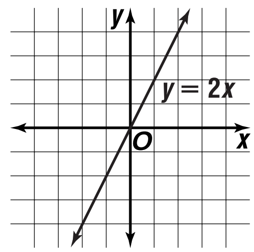
Note to the Teacher *Have students verify this key idea for themselves. Ask them to draw straight lines on graph paper with a ruler, to choose various pairs of points on the graph, and to verify that they get the same answer for the slope for all the different pairs of points. This will not only clarify the key idea, but will reinforce their ability to evaluate the slope. Also, point out that the y-difference is the numerator and the x-difference is the denominator. Since students are used to the x-coordinate coming first, they sometimes tend to put the x-difference in the numerator.*

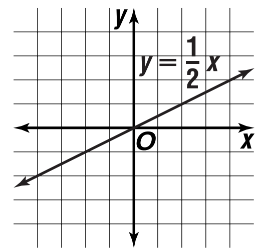
Interpretation of the Slope

Draw three lines whose equations are $y = x$, $y = 2x$, and $y = \frac{1}{2}x$.

These lines look like this in the coordinate plane.

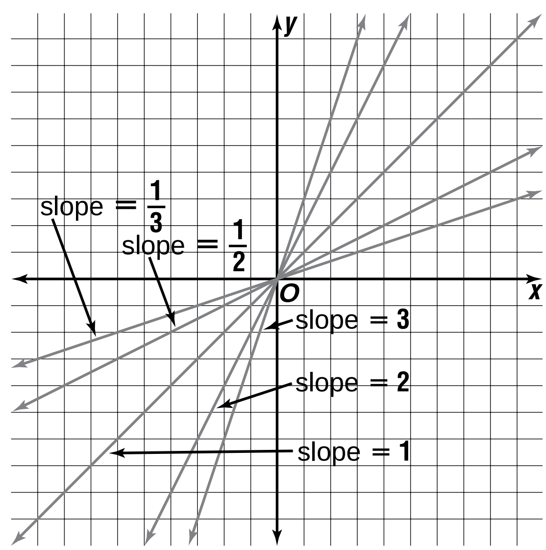






Check the slopes of each of the lines, or better yet, have groups of students calculate the slopes when you have given them points on the lines. Point out that the line with largest slope of 2 is the steepest, and that the line with least slope of $\frac{1}{2}$ is the least steep.

Now sketch lines of various slopes, like those shown at the right. Label the slopes.

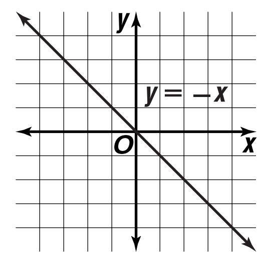


Slopes of Horizontal and Vertical Lines

Make students aware that so far, all the lines have sloped upward from left to right. Ask students, "What slope should a horizontal line have?" Since a horizontal line always has rise equal to zero, the slope will always be zero divided by a positive number, and so the slope is zero. Note that the x-axis has zero slope.

Note to the Teacher *Some students may now ask, "What about vertical lines?" Explain that since the run is always zero and division by zero is undefined, the slope is undefined. It is sometimes useful to think of them as having "infinite slope," but since infinity is* *not a number, this is not a precise statement. If no student asks about this, be sure to introduce vertical lines and their slopes.*

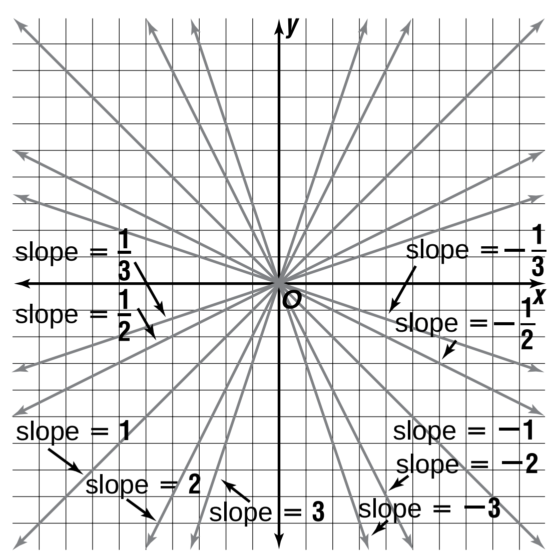
Negative Slope



Point out that so far all slopes have been positive numbers, and all lines have sloped upward from left to right. Explain that for lines drawn in the coordinate plane, the standard direction to move along them is from left to right and bottom to top. Draw a line like the one at the right and ask students what the slope of this line should be. Since the slope is −1, you can now introduce the following important key idea.

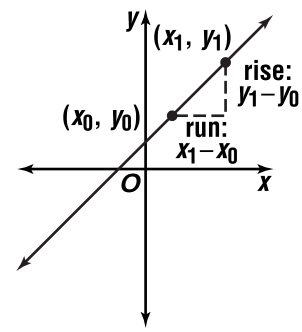
Key Idea	Lines with positive slope rise to the right and lines with negative slope rise to the left. • For positive slopes, the larger the number, the more steeply the line slopes upward. • For negative slopes, the larger the absolute value of the negative number, the more steeply the line slopes downward.

Now draw lines of both positive and negative slopes, like those shown at the right.



Algebraic Formula for Slope

Summarize the whole discussion by introducing the algebraic formula for slope. To do this, draw two points in the coordinate plane that correspond to the ordered pairs (x_0, y_0) and (x_1, y_1), as in the figure at the right.



Point out that the rise is $y_1 - y_0$ and the run is $x_1 - x_0$. Then explain that slope is rise divided by run, which gives the formula

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_1 - y_0}{x_1 - x_0}.$$

Emphasize that values can be substituted into this formula once the coordinates are given. Have the class practice substituting into the formula using different pairs of points. Point out that it does not matter which points are designated as (x_0, y_0) and (x_1, y_1). However, the first x in the denominator must come from the same coordinate pair as the first y in the numerator.

End of Lesson