Key Concepts

Rational Expressions with Unlike Denominators

Objective Teach students to add and subtract rational expressions with unlike denominators.

Note to the Teacher *In this lesson, students will learn to add and subtract rational expressions. The techniques are analogous to adding and subtracting rational numbers (fractions). In particular, the key step is to find a common denominator by finding a common multiple of the denominators. Point out, and continue to stress this analogy throughout the entire lesson.*

Adding and Subtracting Rational Numbers

Begin by reminding students that to add or subtract rational numbers, first write the fractions using a common denominator. To do this, find a common multiple of the denominators. Finding the *least common multiple (LCM)* is the most efficient way of adding or subtracting rational numbers so that the answer is in simplest form.

Do the following example on the chalkboard.

Example 1 **Find $\frac{8}{9} - \frac{1}{6}$.**

Solution The first step is to find a common denominator. To do this, find a common multiple of the denominators, 9 and 6. To find the least common multiple, it helps to write the prime factorization of the denominators.

$$9 = 3 \cdot 3$$
$$6 = 3 \cdot 2$$

So, 9 and 6 have a common factor of 3. This means that the LCM is a product of the common factor with the remaining factors appearing in the prime factorization of these two numbers.

$$\text{LCM} = 3 \cdot 3 \cdot 2 = 18$$

So a common denominator is 18. Since $18 = 9 \cdot 2$ and $18 = 6 \cdot 3$, we have the following.

$$\begin{aligned}\frac{8}{9} - \frac{1}{6} &= \frac{8 \cdot 2}{9 \cdot 2} - \frac{1 \cdot 3}{6 \cdot 3} \\ &= \frac{16}{18} - \frac{3}{18} \\ &= \frac{16 - 3}{18} \\ &= \frac{13}{18}\end{aligned}$$

Adding and Subtracting Rational Expressions

Addition and subtraction of rational expressions is done the same way. First find a common denominator by finding the least common multiple of the denominators. Do the following example on the chalkboard.

Example 2 **Find $\frac{8}{6m^2} - \frac{3}{4m}$.**

Solution Find the LCM of the denominators, $6m^2$ and $4m$. To do this, factor these expressions.

$$\begin{aligned}6m^2 &= 2 \cdot 3 \cdot m \cdot m \\ 4m &= 2 \cdot 2 \cdot m\end{aligned}$$

These expressions have a common factor $2m$. So, the LCM of these expressions is the product of the common factor and the remaining factors in these factorizations.

$$\text{LCM} = 2m \cdot 3 \cdot m \cdot 2 = 12m^2$$

Since $12m^2 = 6m^2 \cdot 2$ and $12m^2 = 4m \cdot 3m$, we have the following.

$$\begin{aligned}\frac{8}{6m^2} - \frac{3}{4m} &= \frac{8 \cdot 2}{6m^2 \cdot 2} - \frac{3 \cdot 3m}{4m \cdot 3m} \\ &= \frac{16}{12m^2} - \frac{9m}{12m^2} \\ &= \frac{16 - 9m}{12m^2}\end{aligned}$$

Now do another example, where in order to find the LCM of the denominators, some factoring needs to be done. Here is an example of this sort. It's a somewhat long problem, but it would be very worthwhile to go through its solution in a step-by-step manner on the chalkboard, and then assign similar problems for your students to practice.

Example 3 **Find** $\dfrac{x}{x^2 - x - 2} + \dfrac{x - 1}{x^2 + 2x + 1}$.

Solution Find the LCM of the denominators in order to write these rational expressions using a common denominator. To do this, factor these expressions.

$$x^2 - x - 2 = (x + 1)(x - 2)$$
$$x^2 + 2x + 1 = (x + 1)(x + 1)$$

These expressions have a common factor $(x + 1)$. So, the LCM is the product of this common factor and the other factors appearing in these factorizations.

$$\begin{aligned} \text{LCM} &= (x + 1)(x - 2)(x + 1) \\ &= (x^2 - x - 2)(x + 1) \\ &= x^3 - x^2 - 2x + x^2 - x - 2 \\ &= x^3 - 3x - 2 \end{aligned}$$

Now use the following to rewrite the rational expressions using a common denominator.

$$x^3 - 3x - 2 = (x^2 - x - 2)(x + 1)$$
$$x^3 - 3x - 2 = (x^2 + 2x + 1)(x - 2)$$

$$\begin{aligned} &\frac{x}{x^2 - x - 2} + \frac{x - 1}{x^2 + 2x + 1} \\ &\quad = \frac{x(x + 1)}{(x^2 - x - 2)(x + 1)} + \frac{(x - 1)(x - 2)}{(x^2 + 2x + 1)(x - 2)} \\ &\quad = \frac{x^2 + x}{x^3 - 3x - 2} + \frac{x^2 - 3x + 2}{x^3 - 3x - 2} \\ &\quad = \frac{x^2 + x + x^2 - 3x + 2}{x^3 - 3x - 2} \\ &\quad = \frac{2x^2 - 2x + 2}{x^3 - 3x - 2} \end{aligned}$$

Note to the Teacher *This technique is very important because it reinforces previous techniques like addition of rational expressions with like denominators, finding the least common multiples of polynomials, and factoring. Give your students many problems to practice.*