Key Concepts

Solving Quadratic Equations by Graphing

Objective Guide students in understanding that the solutions of a quadratic equation occur where the graph of the corresponding function intersects the *x*-axis.

Note to the Teacher *In this lesson, students will use the graphing techniques they learned in the previous lessons to help them solve or approximate solutions to quadratic equations. Begin by stating a definition.*

Quadratic Equations

Quadratic Equation	A quadratic equation is an equation of the form $f(x) = 0$, where $f(x) = ax^2 + bx + c$ is a quadratic function.

The goal in solving a quadratic equation is to find what *x* values make the *y* value of the quadratic function $y = f(x)$ equal to zero. The *y* value of the function will be zero where the graph intersects the *x*-axis. Geometrically, this is because a solution of an equation $f(x) = 0$ occurs when the graph of the function $y = f(x)$ intersects the line $y = 0$. But the line $y = 0$ is the *x*-axis.

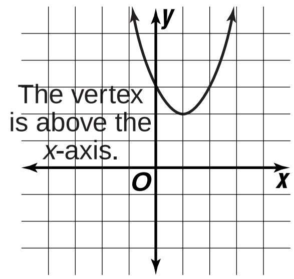
Recall that the graph of a quadratic function is a parabola. So the solutions of a quadratic equation occur where the parabola representing the graph of the quadratic intersects the *x*-axis.

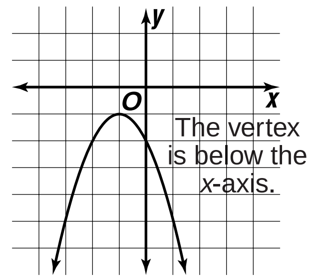
To emphasize this point, draw some parabolas like the following on the chalkboard, to show that a parabola can

- not intersect the *x*-axis,
- intersect the *x*-axis in exactly one point, or
- intersect the *x*-axis in two points.

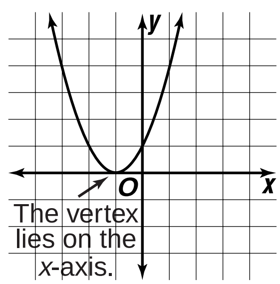
1. **There may be no real solutions.** This will occur if the parabola does not intersect the *x*-axis. Either the parabola opens upwards

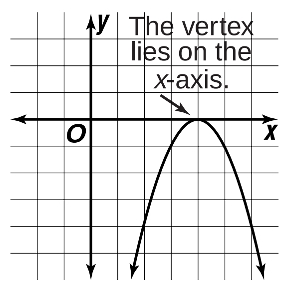
and the vertex (a minimum) lies above the x-axis, or the parabola opens downwards and the vertex (a maximum) lies below the x-axis.



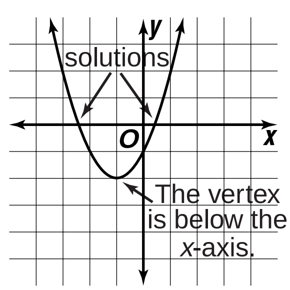


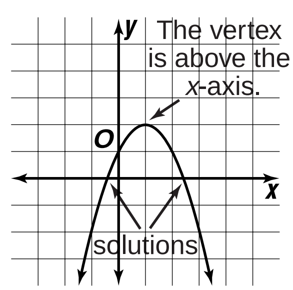
2. **There may be one solution.** This occurs when the parabola intersects the x-axis in exactly one point. This happens when the vertex of the parabola lies on the x-axis.





3. **There may be two solutions.** This occurs when the parabola intersects the x-axis in two points. Either the parabola opens upward and the vertex lies below the x-axis, or the parabola opens downward and the vertex lies above the x-axis.





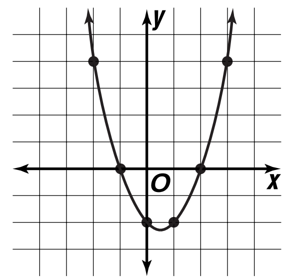
Emphasize that a quadratic equation may have no solutions, one solution, or two solutions, because a parabola can intersect the x-axis in zero, one, or two points.

Now do some examples, where the goal is to find or approximate the solutions to a quadratic equation $ax^2 + bx + c = 0$ by looking to see where the graph of the corresponding quadratic equation intersects the x-axis.

Example 1 **Solve $x^2 - x - 2 = 0$ by graphing.**

Solution **Step 1** Make a table of values, and then graph the function $y = x^2 - x - 2$.

x	$y = x^2 - x - 2$
−2	4
−1	0
0	−2
1	−2
2	0
3	4



Step 2 Find where the parabola intersects the x-axis. In this case, this occurs when $x = -1$ and $x = 2$. Thus, −1 and 2 are the solutions to the equation $x^2 - x - 2 = 0$.

Step 3 Check the answer by factoring.

$$0 = x^2 - x - 2$$
$$0 = (x + 1)(x - 2)$$

Set each factor equal to zero.

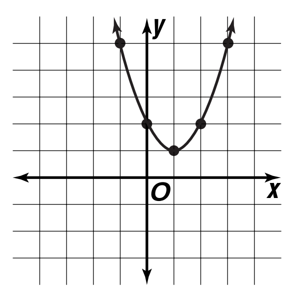
$$x + 1 = 0 \qquad x - 2 = 0$$
$$x = -1 \qquad x = 2$$

So, the solutions check.

Example 2 **Solve $x^2 - 2x + 2 = 0$ by graphing.**

Solution **Step 1** Make a table of values, and then graph the function $y = x^2 - 2x + 2$.

x	$y = x^2 - 2x + 2$
−1	5
0	2
1	1
2	2
3	5

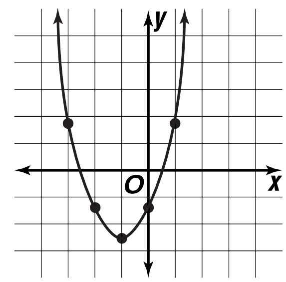


Step 2 Notice that the parabola does not intersect the x-axis. This parabola opens upward and by the techniques described in Lesson 11-1C, we can compute that the vertex is at (1, 1). This vertex lies above the x-axis, so the parabola never intersects the x-axis. We can conclude that there are no solutions to the equation $x^2 - 2x + 2 = 0$.

Example 3 **Solve $x^2 + 2x - \frac{5}{4} = 0$ by graphing.**

Solution **Step 1** Make a table of values, and then graph the function $y = x^2 + 2x - \frac{5}{4}$.

x	$y = x^2 + 2x - \frac{5}{4}$
-3	$\frac{7}{4}$
-2	$-\frac{5}{4}$
-1	$-\frac{9}{4}$
0	$-\frac{5}{4}$
1	$\frac{7}{4}$



Step 2 Try to factor and solve for the roots. In this case, $x^2 + 2x - \frac{5}{4}$ does not have integral factors.

Step 3 Use the graph to approximate the solutions. We can see from the graph that the parabola intersects the x-axis twice: once between -3 and -2, and once roughly halfway between 0 and 1. We can therefore estimate solutions to the equation $x^2 + 2x - \frac{5}{4} = 0$ to be $-\frac{5}{2}$ and $\frac{1}{2}$. (In this case, these two estimates are actually exact solutions.)

In general, graphs provide a good way of approximating solutions to quadratic equations when the corresponding quadratic expressions cannot be factored with integral factors. In order to get exact solutions, we need to use the Quadratic Formula, which will be discussed in Lesson 11-3.