Solving Systems of Linear Equations

Two or more equations that have common variables are called a **system of equations**. The solution of a system of equations in two variables is an ordered pair of numbers that satisfies both equations. A system of two linear equations can have zero, one, or an infinite number of solutions. There are three methods by which systems of equations can be solved: graphing, elimination, and substitution.

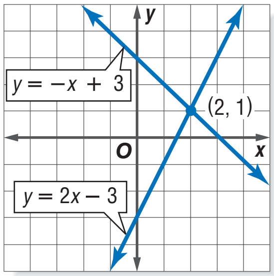
EXAMPLE 1

Solve each system of equations by graphing. Then determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.

a. $y = -x + 3$

$y = 2x - 3$

The graphs appear to intersect at (2, 1). Check this estimate by replacing x with 2 and y with 1 in each equation.



Check $y = -x + 3$ | $y = 2x - 3$

$1 \stackrel{?}{=} -2 + 3$ | $1 \stackrel{?}{=} 2(2) - 3$

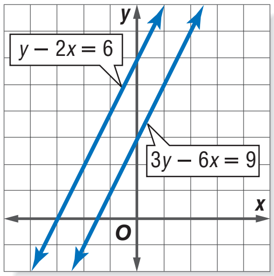
$1 = 1$ ✓ | $1 = 1$ ✓

The system has one solution at (2, 1).

b. $y - 2x = 6$

$3y - 6x = 9$

The graphs of the equations are parallel lines. Since they do not intersect, there are no solutions of this system of equations. Notice that the lines have the same slope but different y-intercepts. Equations with the same slope *and* the same y-intercepts have an infinite number of solutions.



It is difficult to determine the solution of a system when the two graphs intersect at noninteger values. There are algebraic methods by which an exact solution can be found. One such method is **substitution**.

EXAMPLE 2

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Use substitution to solve the system of equations.

$y = -4x$

$2y + 3x = 8$

Since $y = -4x$, substitute $-4x$ for y in the second equation.

$2y + 3x = 8$	Second equation
$2(-4x) + 3x = 3$	$y = -4x$
$-8x + 3x = 8$	Simplify.
$-5x = 8$	Combine like terms.
$\frac{-5x}{-5} = \frac{8}{-5}$	Divide each side by -5.
$x = -\frac{8}{5}$	Simplify.

Use $y = -4x$ to find the value of y.

$y = -4x$	First equation
$= -4\left(-\frac{8}{5}\right)$	$x = -\frac{8}{5}$
$= \frac{32}{5}$	Simplify.

The solution is $\left(-\frac{8}{5}, \frac{32}{5}\right)$.

Sometimes adding or subtracting two equations together will eliminate one variable. Using this step to solve a system of equations is called **elimination**.

EXAMPLE

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3 Use elimination to solve the system of equations.

$\mathbf{3x + 5y = 7}$

$\mathbf{4x + 2y = 0}$

Either x or y can be eliminated. In this example, we will eliminate x.

$3x + 5y = 7$ → Multiply by 4. → $12x + 20y = 28$

$4x + 2y = 0$ → Multiply by −3. → $+ (-12x) - 6y = 0$

$14y = 28$ Add the equations.

$\frac{14y}{14} = \frac{28}{14}$ Divide each side by 14.

$y = 2$ Simplify.

Now substitute 2 for y in either equation to find the value of x.

$4x + 2y = 0$ Second equation

$4x + 2(2) = 0$ $y = 2$

$4x + 4 = 0$ Simplify.

$4x + 4 - 4 = 0 - 4$ Subtract 4 from each side.

$4x = -4$ Simplify.

$\frac{4x}{4} = \frac{-4}{4}$ Divide each side by 4.

$x = -1$ Simplify.

The solution is $(-1, 2)$.