

Factoring to Solve Equations

Some polynomials can be factored using the Distributive Property.

EXAMPLE

1 Factor $5t^2 + 15t$.

Find the greatest common factor (GCF) of $5t^2$ and $15t$.

$$5t^2 = 5 \cdot t \cdot t, 15t = 3 \cdot 5 \cdot t \quad \text{GCF: } 5 \cdot t \text{ or } 5t$$

$$\begin{aligned} 5t^2 + 15t &= 5t(t) + 5t(3) && \text{Rewrite each term using the GCF.} \\ &= 5t(t + 3) && \text{Distributive Property} \end{aligned}$$

To factor polynomials of the form $x^2 + bx + c$, find two integers m and n so that $mn = c$ and $m + n = b$. Then write $x^2 + bx + c$ using the pattern $(x + m)(x + n)$.

To factor polynomials of the form $ax^2 + bx + c$, find two integers m and n with a product equal to ac and with a sum equal to b . Write $ax^2 + bx + c$ using the pattern $ax^2 + mx + nx + c$. Then factor by grouping.

EXAMPLE

2 Factor each polynomial.

a. $x^2 - 8x + 15$

In this equation, b is -8 and c is 15 .

This means that $m + n$ is negative and mn is positive. So m and n must both be negative.

$$\begin{aligned} x^2 - 8x + 15 &= (x + m)(x + n) \\ &= (x - 3)(x - 5) \end{aligned}$$

b is negative and c is positive.

Factors of 15	Sum of Factors
$-1, -15$	-16
$-3, -5$	-8

The correct factors are -3 and -5 .

Write the pattern; $m = -3$ and $n = -5$

b. $5x^2 - 19x - 4$

In this equation, a is 5 , b is -19 , and c is -4 . Find two numbers with a product of -20 and with a sum of -19 .

$$\begin{aligned} 5x^2 - 19x - 4 &= 5x^2 + mx + nx - 4 \\ &= 5x^2 + x + (-20)x - 4 \\ &= (5x^2 + x) - (20x + 4) \\ &= x(5x + 1) - 4(5x + 1) \\ &= (x - 4)(5x + 1) \end{aligned}$$

b is negative and c is negative.

Factors of -20	Sum of Factors
$-2, 10$	8
$2, -10$	-8
$-1, 20$	19
$1, -20$	-19

Factor the GCF from each group.

Distributive Property

Here are some special products.

Perfect Square Trinomials

$$\begin{aligned} a^2 + 2ab + b^2 &= (a + b)(a + b) \\ &= (a + b)^2 \end{aligned}$$

Difference of Squares

$$\begin{aligned} a^2 - 2ab + b^2 &= (a - b)(a - b) \\ &= (a - b)^2 \end{aligned}$$

$$a^2 - b^2 = (a + b)(a - b)$$

EXAMPLE

3 Factor each polynomial.

a. $9x^2 + 6x + 1$

The first and last terms are perfect squares, and the middle term is equal to $2(3x)(1)$.

$$\begin{aligned} 9x^2 + 6x + 1 &= (3x)^2 + 2(3x)(1) + 1^2 && \text{Write as } a^2 + 2ab + b^2. \\ &= (3x + 1)^2 && \text{Factor using the pattern.} \end{aligned}$$

b. $x^2 - 9 = 0$

This is a difference of squares.

$$\begin{aligned} x^2 - 9 &= x^2 - (3)^2 && \text{Write in the form } a^2 - b^2. \\ &= (x - 3)(x + 3) && \text{Factor the difference of squares.} \end{aligned}$$

The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$. Since 0 times any number is equal to zero, this implies that we can use factoring to solve equations.

EXAMPLE

4 Solve $x^2 - 5x + 4 = 0$ by factoring.

$$x^2 - 5x + 4 = 0 \quad \text{Original equation}$$

$$(x - 1)(x - 4) = 0 \quad \text{Factor the polynomial.}$$

$$x - 1 = 0 \quad \text{or} \quad x - 4 = 0 \quad \text{Zero Product Property}$$

$$x = 1 \quad \quad \quad x = 4$$