

Operations with Integers

The absolute value of any number n is its distance from zero on a number line and is written as $|n|$. Since distance cannot be less than zero, the absolute value of a number is always greater than or equal to zero.

EXAMPLE

1 Evaluate each expression.

a. $|3|$

$$|3| = 3 \quad \text{Definition of absolute value}$$

b. $|-7|$

$$|-7| = 7 \quad \text{Definition of absolute value}$$

c. $|-4 + 2|$

$$\begin{aligned} |-4 + 2| &= |-2| & -4 + 2 &= -2 \\ &= 2 & \text{Simplify.} \end{aligned}$$

To add integers with the same sign, add their absolute values. Give the result the same sign as the integers. To add integers with different signs, subtract their absolute values. Give the result the same sign as the integer with the greater absolute value.

EXAMPLE

2 Find each sum.

a. $-3 + (-5)$

Both numbers are negative, so the sum is negative.

$$-3 + (-5) = -8 \quad \text{Add } |-3| \text{ and } |-5|.$$

b. $-4 + 2$

The sum is negative because $|-4| > |2|$.

$$-4 + 2 = -2 \quad \text{Subtract } |2| \text{ from } |-4|.$$

c. $6 + (-3)$

The sum is positive because $|6| > |-3|$.

$$6 + (-3) = 3 \quad \text{Subtract } |-3| \text{ from } |6|.$$

To subtract an integer, add its additive inverse.

EXAMPLE

3 Find each difference.

a. $4 - 7$

$$\begin{aligned} 4 - 7 &= 4 + (-7) & \text{To subtract 7, add } -7. \\ &= -3 \end{aligned}$$

b. $2 - (-4)$

$$\begin{aligned} 2 - (-4) &= 2 + 4 & \text{To subtract } -4, \text{ add } 4. \\ &= 6 \end{aligned}$$

The product of two integers with different signs is negative. The product of two integers with the same sign is positive. Similarly, the quotient of two integers with different signs is negative, and the quotient of two integers with the same sign is positive.

EXAMPLE

4 Find each product or quotient.

- a. $4(-7)$ The factors have different signs.
 $4(-7) = -28$ The product is negative.
- b. $-64 \div (-8)$ The dividend and divisor have the same sign.
 $-64 \div (-8) = 8$ The quotient is positive.
- c. $-9(-6)$ The factors have the same sign.
 $-9(-6) = 54$ The product is positive.
- d. $-55 \div 5$ The dividend and divisor have different signs.
 $-55 \div 5 = -11$ The quotient is negative.
- e. $\frac{24}{-3}$ The dividend and divisor have different signs.
 $\frac{24}{-3} = -8$ The quotient is negative.

To evaluate expressions with absolute value, evaluate the absolute values first and then perform the operation.

EXAMPLE

5 Evaluate each expression.

- a. $|-3| - |5|$
 $|-3| - |5| = 3 - 5$ $|-3| = 3, |5| = 5$
 $= -2$ Simplify.
- b. $|-5| + |-2|$
 $|-5| + |-2| = 5 + 2$ $|-5| = 5, |-2| = 2$
 $= 7$ Simplify.