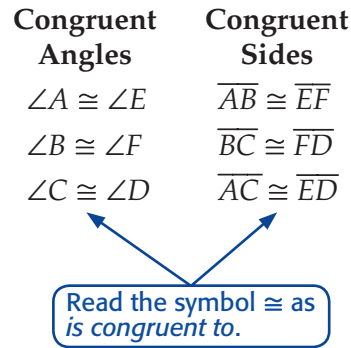
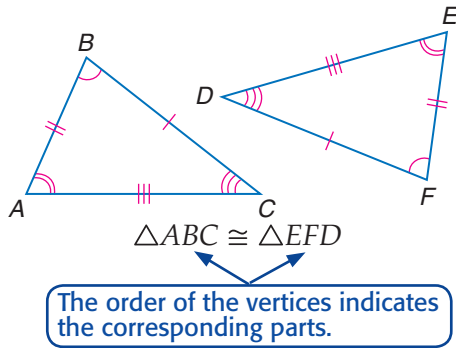


# Congruent and Similar Figures

**Congruent** figures have the same size and the same shape.

Two polygons are congruent if their corresponding sides are congruent and their corresponding angles are congruent.



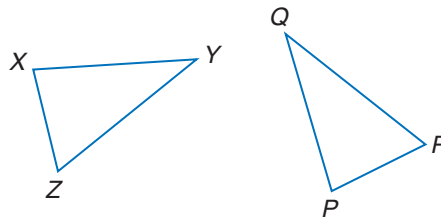
## EXAMPLE

**1** If  $\triangle XYZ \cong \triangle PQR$ , name the congruent angles and sides.

Name the pairs of congruent angles by looking at the order of the vertices in the statement  $\triangle XYZ \cong \triangle PQR$ .

So,  $\angle X \cong \angle P$ ,  $\angle Y \cong \angle Q$ , and  $\angle Z \cong \angle R$ .

Since X corresponds to P, and Y corresponds to Q,  $\overline{XY} \cong \overline{PQ}$ .  
 Since Y corresponds to Q, and Z corresponds to R,  $\overline{YZ} \cong \overline{QR}$ .  
 Since Z corresponds to R, and X corresponds to P,  $\overline{ZX} \cong \overline{RP}$ .



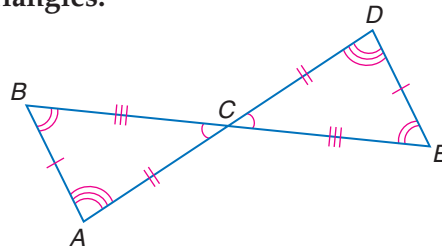
## EXAMPLE

**2** The corresponding parts of two congruent triangles are marked on the figure. Write a congruence statement for the two triangles.

List the congruent angles and sides.

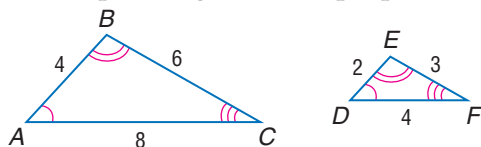
$\angle A \cong \angle D$	$\overline{AB} \cong \overline{DE}$
$\angle B \cong \angle E$	$\overline{AC} \cong \overline{DC}$
$\angle ACB \cong \angle DCE$	$\overline{BC} \cong \overline{EC}$

Match the vertices of the congruent angles. Therefore,  $\triangle ABC \cong \triangle DEC$ .



**Similar** figures have the same shape, but not necessarily the same size.

In similar figures, corresponding angles are congruent, and the measures of corresponding sides are proportional. (They have equivalent ratios.)



**Congruent Angles**

$\angle A \cong \angle D$ ,  $\angle B \cong \angle E$ ,  $\angle C \cong \angle F$

**Proportional Sides**

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

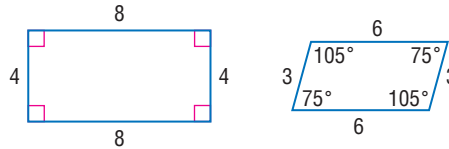
$\triangle ABC \sim \triangle DEF$

Read the symbol  $\sim$  as *is similar to*.

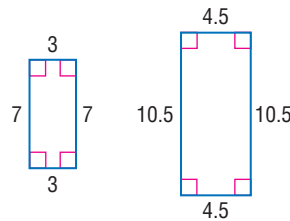
## EXAMPLE

**3** Determine whether the polygons are similar. Justify your answer.

- a. Since  $\frac{4}{3} = \frac{8}{6} = \frac{4}{3} = \frac{8}{6}$ , the measures of the sides of the polygons are proportional. However, the corresponding angles are not congruent. The polygons are not similar.

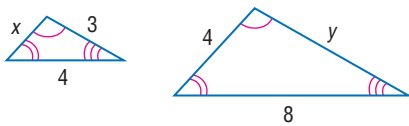


- b. Since  $\frac{7}{10.5} = \frac{3}{4.5} = \frac{7}{10.5} = \frac{3}{4.5}$ , the measures of the sides of the polygons are proportional. The corresponding angles are congruent. Therefore, the polygons are similar.



## EXAMPLE

**4** The triangles are similar. Find the values of  $x$  and  $y$ .



Write proportions using corresponding parts. Then solve to find the missing measures.

$$\frac{x}{4} = \frac{4}{8} \quad \text{Definition of similar polygons} \qquad \frac{3}{y} = \frac{4}{8} \quad \text{Definition of similar polygons}$$

$$x(8) = 4(4) \quad \text{Cross products} \qquad 3(8) = y(4) \quad \text{Cross products}$$

$$8x = 16 \quad \text{Simplify.} \qquad 24 = 4y \quad \text{Simplify.}$$

$$\frac{8x}{8} = \frac{16}{8} \quad \text{Divide each side by 8.} \qquad \frac{24}{4} = \frac{4y}{4} \quad \text{Divide each side by 4.}$$

$$x = 2 \quad \text{Simplify.} \qquad 6 = y \quad \text{Simplify.}$$

## EXAMPLE

**5 CIVIL ENGINEERING** The city of Mansfield plans to build a bridge across Pine Lake. Use the information in the diagram to find the distance across Pine Lake.

$$\triangle ABC \sim \triangle ADE$$

$$\frac{AB}{AD} = \frac{BC}{DE} \quad \text{Definition of similar polygons}$$

$$\frac{100}{220} = \frac{55}{DE} \quad AB = 100, AD = 100 + 120 = 220, BC = 55$$

$$100DE = 220(55) \quad \text{Cross products}$$

$$100DE = 12,100 \quad \text{Simplify.}$$

$$DE = 121 \quad \text{Divide each side by 100.}$$

The distance across the lake is 121 meters.

