

Study Guide and Intervention

6SDAP3.1

Sample Spaces

A game in which players of equal skill have an equal chance of winning is a **fair game**. A **tree diagram** or table is used to show all of the possible outcomes, or **sample space**, in a probability experiment.

Example 1 **WATCHES** A certain type of watch comes in brown or black and in a small or large size. Find the number of color-size combinations that are possible.

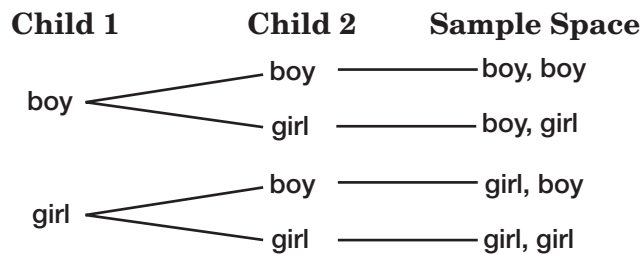
Make a table to show the sample space. Then give the total number of outcomes.

| Color | Size |
|-------|-------|
| Brown | Small |
| Brown | Large |
| Black | Small |
| Black | Large |

There are four different color and size combinations.

Example 2 **CHILDREN** The chance of having either a boy or a girl is 50%. What is the probability of the Smiths having two girls?

Make a tree diagram to show the sample space. Then find the probability of having two girls.



The sample space contains 4 possible outcomes. Only 1 outcome has both children being girls. So, the probability of having two girls is $\frac{1}{4}$.

Exercises

For each situation, make a tree diagram or table to show the sample space. Then give the total number of outcomes.

- choosing an outfit from a green shirt, blue shirt, or a red shirt, and black pants or blue pants
- choosing a vowel from the word COUNTING and a consonant from the word PRIME

Study Guide and Intervention

6SDAP3.1

The Fundamental Counting Principle

If event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by N can occur in $m \times n$ ways. This is called the **Fundamental Counting Principle**.

Example 1 CLOTHING Andy has 5 shirts, 3 pairs of pants, and 6 pairs of socks. How many different outfits can Andy choose with a shirt, pair of pants, and pair of socks?

$$\begin{array}{ccccccc}
 \text{number of shirts} & & \text{number of} & & \text{number of socks} & & \text{total number of outfits} \\
 \underbrace{\hspace{2cm}} & & \underbrace{\hspace{2cm}} & & \underbrace{\hspace{2cm}} & = & \underbrace{\hspace{2cm}} \\
 5 & \cdot & 3 & \cdot & 6 & = & 90
 \end{array}$$

Andy can choose 90 different outfits.

Exercises

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.

1. rolling two number cubes
2. tossing 3 coins
3. picking one consonant and one vowel
4. choosing one of 3 processor speeds, 2 sizes of memory, and 4 sizes of hard drive
5. choosing a 4-, 6-, or 8-cylinder engine and 2- or 4-wheel drive
6. rolling 2 number cubes and tossing 2 coins
7. choosing a color from 4 colors and a number from 4 to 10

Study Guide and Intervention

6SDAP3.1

Permutations

The expression n **factorial** ($n!$) is the product of all counting numbers beginning with n and counting backward to 1. A **permutation** is an arrangement, or listing, of objects in which order is important. You can use the Fundamental Counting Principle to find the number of possible arrangements.

Example 1 Find the value of $5!$.

$$\begin{aligned} 5! &= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 && \text{Definition of factorial} \\ &= 120 && \text{Simplify.} \end{aligned}$$

Example 2 Find the value of $4! \cdot 2!$.

$$\begin{aligned} 4! \cdot 2! &= 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1 && \text{Definition of factorial} \\ &= 48 && \text{Simplify.} \end{aligned}$$

Example 3 BOOKS How many ways can 4 different books be arranged on a bookshelf?

This is a permutation that can be written as $4!$. Suppose the books are placed on the shelf from left to right.

There are 4 choices for the first book.

There are 3 choices that remain for the second book.

There are 2 choices that remain for the third book.

There is 1 choice that remains for the fourth book.

$$\begin{aligned} 4! &= 4 \cdot 3 \cdot 2 \cdot 1 && \text{Definition of factorial} \\ &= 24 && \text{Simplify.} \end{aligned}$$

So, there are 24 ways to arrange 4 different books on a bookshelf.

Exercises

Find the value of each expression.

1. $3!$

2. seven factorial

3. $6! \cdot 3!$

4. $9 \cdot 8 \cdot 7$

5. How many ways can you arrange the letters in the word GROUP?

6. How many different 4-digit numbers can be created if no digit can be repeated? Remember, a number cannot begin with 0.

Study Guide and Intervention

6SDAP3.1

Combinations

An arrangement, or listing, of objects in which order is *not* important is called a **combination**. You can find the number of combinations of objects by dividing the number of permutations of the entire set by the number of ways each smaller set can be arranged.

Example 1 Jill was asked by her teacher to choose 3 topics from the 8 topics given to her. How many different three-topic groups could she choose?

There are $8 \cdot 7 \cdot 6$ permutations of three-topic groups chosen from eight. There are $3!$ ways to arrange the groups.

$$\frac{8 \cdot 7 \cdot 6}{3!} = \frac{336}{6} = 56$$

So, there are 56 different three-topic groups.

Tell whether each situation represents a *permutation* or *combination*. Then solve the problem.

Example 2 On a quiz, you are allowed to answer any 4 out of the 6 questions. How many ways can you choose the questions?

This is a combination because the order of the 4 questions is not important. So, there are $6 \cdot 5 \cdot 4 \cdot 3$ permutations of four questions chosen from six. There are $4!$ or $4 \cdot 3 \cdot 2 \cdot 1$ orders in which these questions can be chosen.

$$\frac{6 \cdot 5 \cdot 4 \cdot 3}{4!} = \frac{360}{24} = 15$$

So, there are 15 ways to choose the questions.

Example 3 Five different cars enter a parking lot with only 3 empty spaces. How many ways can these spaces be filled?

This is a permutation because each arrangement of the same 3 cars counts as a distinct arrangement. So, there are $5 \cdot 4 \cdot 3$ or 60 ways the spaces can be filled.

Exercises

Tell whether each situation represents a *permutation* or *combination*. Then solve the problem.

- How many ways can 4 people be chosen from a group of 11?
- How many ways can 3 people sit in 4 chairs?
- How many ways can 2 goldfish be chosen from a tank containing 15 goldfish?