

5-1

Prime Factorization (pages 197–200)

A **prime number** is a whole number greater than 1 that has exactly two factors, 1 and itself. A **composite number** is a whole number greater than 1 that has more than two factors. Every composite number can be written as the product of prime numbers in exactly one way if you ignore the order of the factors. This product is called the **prime factorization** of the number.

EXAMPLE

Finding Prime Factorization	Method 1: Use a factor tree.	Method 2: Divide by prime numbers until the quotient is prime. Use a calculator, if necessary.
		$36 \div 2 = 18$; $18 \div 2 = 9$; $9 \div 3 = 3$
The prime factorization of 36 is $2 \times 2 \times 3 \times 3$.		

Try These Together

- Is 23 composite or prime?
HINT: Test for divisibility by 2, 3, 5, 7, and 11.
- Use a factor tree to find the prime factorization of 28.
HINT: You can divide by 2 and then by 2 again.

PRACTICE

Determine whether each number is composite or prime.

3. 51 4. 228 5. 227 6. 73 7. 154

Use a factor tree to find the prime factorization of each number.

8. 64 9. 93 10. 54 11. 125 12. 244

Use your calculator to find the prime factors of each number. Then write the prime factorization of each number.

13. 84 14. 96 15. 150
16. 30 17. 232 18. 245

19. **Maps** Rhode Island is the smallest state in the United States. It only covers 3,188 square kilometers. Find the prime factorization of 3,188.

20. **Standardized Test Practice** Which number is a factor of both 21 and 36?

- A** 4 **B** 3 **C** 9 **D** 12

Answers: 1. prime 2. $2 \times 2 \times 7$ 3. composite 4. composite 5. prime 6. prime 7. composite 8. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$ 9. 3×31 10. $2 \times 3 \times 3 \times 3$ 11. $5 \times 5 \times 5$ 12. $2 \times 2 \times 61$ 13. $2 \times 2 \times 3 \times 7$ 14. $2 \times 2 \times 2 \times 2 \times 3$ 15. $2 \times 3 \times 5 \times 5$ 16. $2 \times 3 \times 5 \times 5$ 17. $2 \times 2 \times 2 \times 2 \times 29$ 18. $5 \times 7 \times 7$ 19. $2 \times 2 \times 2 \times 797$ 20. B

5-2**Greatest Common Factor** (pages 203–206)

The **greatest common factor (GCF)** of two or more numbers is the greatest number that is a factor of each number.

Finding the Greatest Common Factor

To find the GCF of two or more numbers:

- Method 1: List the factors of each number and then identify the common factors. The greatest of these common factors is the GCF.
- Method 2: Write the prime factorization of each number, or divide by prime numbers until the quotient is prime. Then identify all common prime factors and find their product, which is the GCF.

EXAMPLES

- A** Find the GCF of 12, 20, and 36 by listing factors.

factors of 12: **1, 2, 3, 4, 6, 12**

factors of 20: **1, 2, 4, 5, 10, 20**

factors of 36: **1, 2, 3, 4, 6, 9, 12, 18, 36**

The greatest of the common factors is 4, which is the GCF of 12, 20, and 36.

- B** Find the GCF of 27 and 90 by using prime factorization.

prime factorization of 27: $3 \times 3 \times 3$

prime factorization of 90: $2 \times 3 \times 3 \times 5$

The common prime factors are 3 and 3. Their product is 9. The GCF of 27 and 90 is 9.

Try These Together

1. Find the GCF of 12 and 16 by listing factors.

HINT: Circle the factors common to 12 and 16. Then choose the greatest of those circled.

2. Find the GCF of $15 = 3 \times 5$ and $50 = 2 \times 5^2$ by listing common prime factors.

HINT: There is only one common prime factor.

PRACTICE

Find the GCF of each set of numbers by listing factors.

3. 54, 81

4. 72, 90

5. 132, 144

6. 20, 36, 44

Find the GCF of each set of numbers by listing common prime factors.

7. $9 = 3^2$
 $36 = 2^2 \times 3^2$

8. $45 = 3^2 \times 5$
 $81 = 3^4$

Find the GCF of each set of numbers by writing prime factorizations.

9. 12, 48

10. 36, 54

11. 60, 42

12. **Life Science** The smallest adult male gorillas weigh about 135 kilograms. The smallest adult female gorillas weigh about 70 kilograms. What is the greatest common factor of these two numbers?



13. **Standardized Test Practice** Find the greatest common factor of 96 and 360.

A 5

B 12

C 36

D 24

Answers: 1. 4 2. 5 3. 27 4. 18 5. 12 6. 4 7. 9 8. 9 9. 12 10. 18 11. 6 12. 5 13. D

5-3**Simplifying Fractions** (pages 207–209)

You can simplify the fraction $\frac{3}{9}$ by dividing both the numerator and denominator by 3. A fraction is in **simplest form** when the GCF of the numerator and denominator is 1.

Simplifying Fractions

To write a fraction in simplest form:

- find the GCF of the numerator and denominator,
- divide both the numerator and denominator by the GCF, and
- write the resulting fraction.

EXAMPLES

A Express $\frac{6}{12}$ in simplest form.

The GCF of 6 and 12 is 6.

Divide the numerator (6) by 6 to get 1.

Divide the denominator (12) by 6 to get 2.

$\frac{6}{12}$ in simplest form is $\frac{1}{2}$.

B Express $\frac{18}{24}$ in simplest form.

The GCF of 18 and 24 is 6.

Divide 18 by 6. Divide 24 by 6.

$\frac{18}{24}$ in simplest form is $\frac{3}{4}$.

Try These Together

1. Express $\frac{25}{45}$ in simplest form.

HINT: The GCF of 25 and 45 is 5.

2. Express $\frac{3}{15}$ in simplest form.

HINT: What is the GCF of 3 and 15?

PRACTICE

Express each fraction in simplest form.

3. $\frac{82}{94}$

4. $\frac{54}{63}$

5. $\frac{48}{16}$

6. $\frac{55}{105}$

7. $\frac{12}{60}$

8. $\frac{10}{148}$

9. $\frac{14}{62}$

10. $\frac{8}{72}$

11. Life Science There are 2,900 species of jellyfish. They are made up of 2 classes, the hydrozoan and the scyphozoan. There are 200 species of scyphozoan. Express the number of species of scyphozoan as a fraction of all jellyfish species in simplest form.

12. Standardized Test Practice Akikta has \$1,200 in his checking account and \$300 in his savings account. Express the amount of money in his savings account as a fraction of the amount of money in his checking account in simplest form.

A $\frac{1}{4}$

B $\frac{3}{4}$

C $\frac{1}{2}$

D $\frac{1}{3}$

Answers: 1. $\frac{9}{5}$ 2. $\frac{5}{1}$ 3. $\frac{47}{41}$ 4. $\frac{7}{6}$ 5. $\frac{1}{3}$ 6. $\frac{21}{11}$ 7. $\frac{5}{1}$ 8. $\frac{74}{5}$ 9. $\frac{31}{7}$ 10. $\frac{9}{1}$ 11. $\frac{29}{2}$ 12. A

5-4**Decimals and Fractions** (pages 210–213)

Any fraction can be written as a decimal by using division.

Write a Fraction as a Decimal	Use paper and pencil to write $\frac{4}{5}$ as a decimal. $\frac{4}{5}$ means $4 \div 5$. Divide 4 by 5, and the quotient is the decimal you want to find, 0.8.
Repeating Decimals	Decimals like $0.333333 \dots$ are called repeating decimals because the digits repeat. Bar notation can be used to indicate that decimals repeat. $0.666666 \dots = 0.\overline{6}$, $0.277777 \dots = 0.2\overline{7}$, $0.737373 \dots = 0.\overline{73}$ Bar notation is useful because some fractions, when written as decimals, are repeating decimals. For example, $\frac{2}{3} = 0.\overline{6}$.

EXAMPLES

Express the fractions as decimals. Use bar notation for repeating decimals.

A $\frac{3}{5}$

$$\frac{3}{5} = 3 \div 5$$

$$\begin{array}{r} 0.6 \\ 5 \overline{)3.0} \\ \underline{-30} \\ 0 \end{array} \quad \begin{array}{l} \text{Divide 3 by 5.} \\ \text{Therefore, } \frac{3}{5} = 0.6. \end{array}$$

B $\frac{3}{11}$

$$\frac{3}{11} = 3 \div 11$$

$$\begin{array}{r} 0.2727 \dots \\ 11 \overline{)3.00} \\ \underline{-22} \\ 80 \\ \underline{-77} \\ 30 \\ \underline{-22} \\ 8 \end{array}$$

Divide 3 by 11. The digits 2 and 7 will repeat since 8 and 3 will continue to alternate as the remainders.

Therefore, $\frac{3}{11} = 0.\overline{27}$.

Try These Together

Express each fraction or mixed number as a decimal. If the decimal is a repeating decimal, use bar notation.

1. $\frac{1}{6}$

HINT: Divide 1 by 6.

2. $4\frac{7}{8}$

HINT: The whole number is written to the left of the decimal point.

PRACTICE

Express each fraction or mixed number as a decimal. If the decimal is a repeating decimal, use bar notation.

3. $\frac{3}{6}$

4. $\frac{2}{9}$

5. $\frac{12}{25}$

6. $5\frac{2}{3}$

7. $8\frac{4}{9}$

8. $7\frac{1}{4}$



9. **Standardized Test Practice** Suppose that $\frac{1}{8}$ of D'andre's class scored As on their science exam. Express this fraction as a decimal.

A 0.215

B 0.125

C 0.252

D 0.115

Answers: 1. 0.16 2. 4.875 3. 0.5 4. 0.2 5. 0.48 6. 5.6 7. 8.4 8. 7.25 9. B

5-5**Fractions and Percents** (pages 216–219)

A **percent** is a ratio that compares a number to 100. Fractions and percents are ratios that represent the same number.

Expressing a Ratio as a Percent	$\frac{n}{100} = n\%$ To express a ratio as a percent, first write the ratio as a fraction with a denominator of 100. Then rewrite $\frac{n}{100}$ as $n\%$.
Expressing a Fraction as a Percent	To express a fraction as a percent, multiply both numerator and denominator by the same factor to rewrite the fraction as an equivalent fraction with a denominator of 100.

EXAMPLES

- A** Express as a percent: 37 students out of 100.

Write the ratio as a fraction: $\frac{37}{100}$.

$\frac{37}{100}$ is 37%.

- B** Express $\frac{7}{25}$ as a percent.

To rewrite $\frac{7}{25}$ as an equivalent fraction with a denominator of 100, multiply both numerator and denominator by 4, since $100 \div 25$ is 4.

$$\frac{7 \times 4}{25 \times 4} = \frac{28}{100}, \quad \frac{28}{100} \text{ is } 28\%.$$

$$\frac{7}{25} = 28\%$$

Try These Together

1. Express as a percent: 32.5 square miles in 100.

HINT: Write as a fraction with a denominator of 100.

2. Express $\frac{3}{5}$ as a percent.

HINT: Recall that $\frac{n}{100} = n\%$.

PRACTICE

Express each ratio or fraction as a percent.

3. 62 out of 100

4. $\frac{1}{4}$

5. $\frac{12}{100}$

6. $\frac{2}{5}$

7. \$55 per \$100

8. $\frac{13}{20}$

9. **Computers** 78 out of 100 computers at Tina's school have CD-ROM drives. Express 78 out of 100 as a percent.

10. **Standardized Test Practice** In Enrique's school, 61 out of every 100 students eat a hot lunch. Express this ratio as a percent.

A 3.9%

B 6.1%

C 61%

D 39%

Answers: 1. 32.5% 2. 60% 3. 12% 4. 25% 5. 12% 6. 40% 7. 55% 8. 65% 9. 78% 10. C

5-6**Percents and Decimals** (pages 220–223)

Any decimal can also be written as a fraction. You can use this to express any decimal as a percent.

Writing a Decimal as a Percent	To write 0.32 as a percent, multiply the decimal by 100 and add the percent symbol. So, $0.32 = 0.32 = 32\%$.
Writing a Percent as a Decimal	To write a percent as a decimal, divide the percent by 100 and remove the percent symbol. $64\% = 64 = 0.64$.

EXAMPLES

A Write 0.72 as a percent.

$0.72 = 72\%$ *Multiply the decimal by 100 and add the percent symbol.*

B Write 57% as a decimal.

$57\% = 0.57$ *Divide the percent by 100 and remove the percent symbol.*

Try These Together

Express each decimal as a percent.

1. 0.25

2. 0.76

HINT: Multiply by 100 and add the percent symbol.

PRACTICE

Express each decimal as a percent.

3. 0.54

4. 0.67

5. 0.1

6. 0.08

7. 0.42

8. 0.17

Express each percent as a decimal.

9. 48%

10. 75%

11. 9%

12. 23%

13. 35%

14. 99.8%

15. 4%

16. 15.1%

17. What decimal is equivalent to 39.5%?

18. Write the percent that is equivalent to 0.652.

19. **Recycling** In a recent year, the aluminum recycling rate was 62.8%. Write this percent as a decimal.



20. **Standardized Test Practice** In the mid-1990s, 48% of the working people in the world were employed in agriculture. How is this percent written as a decimal?

A 0.048

B 48

C 4.8

D 0.48

Answers: 1. 25% 2. 76% 3. 54% 4. 67% 5. 10% 6. 8% 7. 42% 8. 17% 9. 0.48 10. 0.75 11. 0.09 12. 23% 13. 0.35 14. 0.998 15. 0.04 16. 0.151 17. 0.395 18. 65.2% 19. 0.628 20. D

5-7**Least Common Multiple** (pages 224–226)

When you multiply a number by the whole numbers 0, 1, 2, 3, 4, and so on, you get **multiples** of the number. The **least common multiple (LCM)** of two or more numbers is the least of their common multiples, other than zero.

Finding the Least Common Multiple (LCM)

To find the least common multiple of two or more numbers,

- make a list of several multiples of each number. Then identify the common multiples. The least of these is the LCM.

OR

- write the prime factorization of each number. Write each prime factor as a multiplier the *greatest* number of times it appears in any one of the numbers.

OR

- divide by prime factors until the quotients are prime. Then multiply the divisors and prime quotients to get the LCM.

EXAMPLE

Find the LCM of 6, 36, and 40 by writing prime factorizations.

$$6 = 2 \times 3 \quad 36 = 2 \times 2 \times 3 \times 3 \quad 40 = 2 \times 2 \times 2 \times 5$$

Write each prime factor (2, 3, 5) as a multiplier the *greatest* number of times it appears in any one number. The factor 2 appears three times in 40. The factor 3 appears twice in 36. The factor 5 appears once in 40. The product of $2 \times 2 \times 2 \times 3 \times 3 \times 5$, or 360, is the least common multiple of 6, 36, and 40.

Try These Together

1. Find the LCM of 12 and 30 by listing multiples.

HINT: Look for the least common multiple in the two lists.

2. Find the LCM of 12 and 14 by writing prime factorizations.

HINT: Remember to write each prime factor as a multiplier the greatest number of times it appears in any one of the numbers.

PRACTICE

Find the LCM of each set of numbers by listing multiples.

3. 3, 10 4. 6, 8 5. 9, 12 6. 3, 5, 6 7. 4, 5, 10 8. 5, 15

Find the LCM of each set of numbers by writing prime factorizations or dividing by prime numbers.

9. 6, 9 10. 12, 18 11. 8, 14 12. 10, 36 13. 20, 96 14. 4, 6, 15

15. **Entertainment** Every 10 years, the people of Oberammergau, Germany, put on a special play. Rhonda's family travels to Germany every 3 years. If Rhonda's family was in Germany in the year 2000 and the play was on, what is the next year that the play will be on when Rhonda's family is in Germany?

16. **Standardized Test Practice** What is the least common multiple of 50 and 60?

A 200

B 400

C 300

D 500

Answers: 1. 60 2. 84 3. 30 4. 24 5. 36 6. 30 7. 20 8. 15 9. 18 10. 36 11. 56 12. 180 13. 480 14. 60 15. 2,030 16. C

5-8

Comparing and Ordering Rational Numbers

(pages 227–231)

To compare fractions, rewrite each fraction using the same denominator. Then you only need to compare the numerators.

Finding the Least Common Denominator

A **common denominator** is a common multiple of the denominators of two or more fractions. The **least common denominator (LCD)** is the least common multiple (LCM) of the denominators of two or more fractions. To compare two fractions:

- find the LCM of the denominators, then
- rewrite each fraction using this LCM as the LCD. Compare the numerators.

EXAMPLES

A Find the LCD for $\frac{5}{6}$ and $\frac{9}{10}$.
The LCM of 6 and 10 is 30, so the LCD for $\frac{5}{6}$ and $\frac{9}{10}$ is 30.

B Is $\frac{5}{6} <, >, \text{ or } = \frac{9}{10}$?
Rewrite each fraction with the LCD of 30.
Multiply the numerator and denominator of $\frac{5}{6}$ by 5.
Multiply the numerator and denominator of $\frac{9}{10}$ by 3.

$$\frac{5}{6} = \frac{25}{30} \quad \frac{9}{10} = \frac{27}{30}$$

Since $\frac{25}{30} < \frac{27}{30}$, $\frac{5}{6} < \frac{9}{10}$.

Try These Together

1. Find the LCD for $\frac{3}{4}$ and $\frac{2}{3}$.
HINT: Find the LCM of 4 and 3.

2. Is $\frac{5}{8} <, >, \text{ or } = 0.4$?
HINT: Write both rational numbers as fractions with the same denominator or as decimals.

PRACTICE

Find the LCD for each pair of fractions.

3. $\frac{5}{12}, \frac{3}{8}$

4. $\frac{2}{5}, \frac{4}{7}$

5. $\frac{4}{15}, \frac{1}{3}$

6. $\frac{1}{6}, \frac{1}{9}$

7. $\frac{1}{6}, \frac{5}{7}$

8. $\frac{19}{30}, \frac{7}{10}$

9. $\frac{9}{16}, \frac{1}{4}$

10. $\frac{5}{36}, \frac{11}{24}$

Replace each ● with <, >, or = to make a true sentence.

11. $\frac{8}{9} \bullet \frac{5}{6}$

12. $\frac{2}{3} \bullet \frac{8}{13}$

13. $\frac{5}{6} \bullet 0.75$

14. $\frac{3}{5} \bullet \frac{5}{8}$

15. $\frac{2}{7} \bullet 0.25$

16. $0.7 \bullet \frac{14}{20}$

17. $\frac{5}{11} \bullet \frac{13}{22}$

18. $\frac{15}{48} \bullet \frac{3}{8}$



19. **Standardized Test Practice** What is the least common denominator for $\frac{1}{8}$ and $\frac{5}{6}$?

A 36

B 24

C 18

D 45

Answers: 1. 12 2. > 3. 24 4. 36 5. 15 6. 18 7. 42 8. 30 9. 16 10. 72 11. < 12. < 13. < 14. < 15. < 16. = 17. < 18. B 19. B

5**Chapter 5 Review****Fraction Ladder**

Build a ladder out of the following list of fractions.
Place the fractions in order from least to greatest on
the ladder from bottom to top.

$$\frac{1}{2}$$

$$\frac{19}{20}$$

$$\frac{4}{5}$$

$$\frac{7}{8}$$

$$\frac{3}{4}$$

$$\frac{1}{12}$$

$$\frac{1}{3}$$

$$\frac{3}{10}$$

Answers are located on page 107.